Principle of Industrial Engineering
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Lecture 59
Quality Control: Control Charts for Attributes

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering and you know we are talking about the quality control. For controlling the quality of the production services, which are being produced by a company or an organization, the performance or the quality of the production services can be checked either in measurable terms, like some of the quality characteristics are checked using the by measuring the certain parameters or by saying simply that yes it is good or bad, acceptable or not acceptable like that.

So we have seen that when the quality characteristic is measurable in numerical terms, then we develop the control charts for variables and X bar R chart is developed under the control charts for variables, while in a situation where we do not measure the quality characteristics but we simply say yes it is good or bad, acceptable or not acceptable, accept or reject, like that, then we develop the control charts for attributes.

Attributes are those characteristics which are not measured in terms of the numerical values but they are simply counted to see really if the things will be in acceptable or non-acceptable forms. So attributes are basically counted and when the control charts are developed for attributes, then basically we try to count the NC items.

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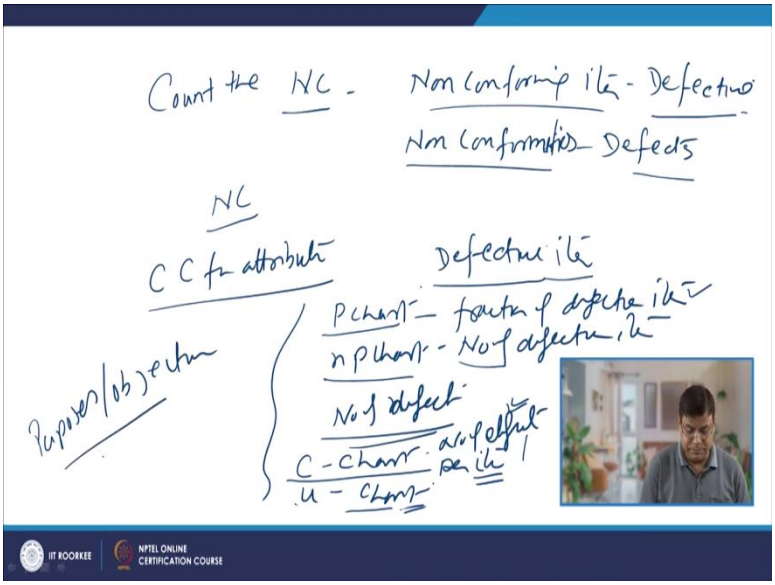

These can be like non-conforming items, which are called defectives, items which are not as per the specifications or there can be non-conformities. These are like the defects, like these may be in form of cracks, dents or certain other features. So non-conformities are defects while non-conforming items will be the defective items. So, when the control charts are developed for attributes, then we basically count either the non-conforming items or non-conformities.

So, control charts for attributes. When these are developed for non-conforming items, like the defective counting is done from the lot for identification of the defective items, then the two types of the control charts are developed using the counting of the non-conforming items or the defective items that is p chart which indicates the fraction of the defective items and np chart which indicates the number of defective items.

On the other hand, when we count the number of defects in the items which have been selected from a lot, then the control charts are developed considering the number of defects. There are two types of control charts; one is the c chart, wherein numbers of defects in per item, per unit are identified, means in a given unit how many defects are there when the sample size is fixed.

And when the sample size is changing and then numbers of defects are counted per item or per unit for variable sample size, then u chart is developed. So c chart for constant sample size and u chart for variable sample size, considering the number of defects, while the fraction of the defective items, we developed the p chart and np chart is developed using the

number of defective items identified through the counting from the items which have been selected from the lot.

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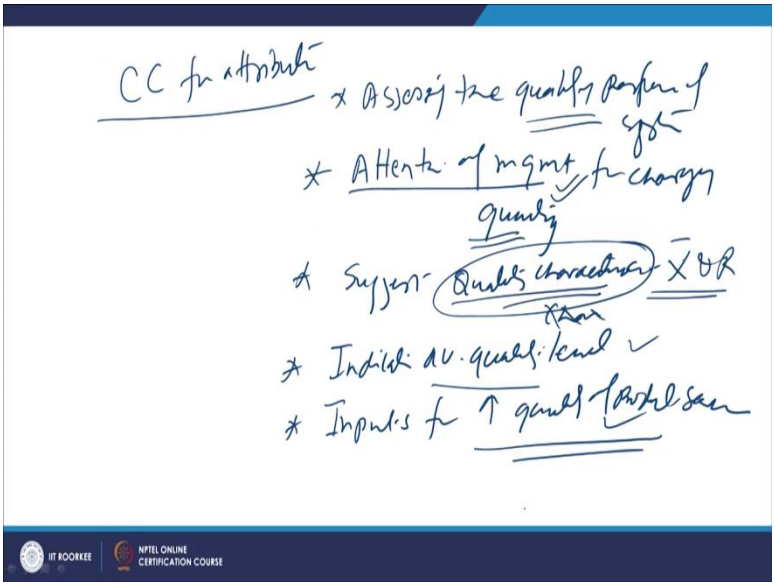

Then, we will see what are the various purposes or objectives that are realized by developing the control charts for attributes. We have seen there are 4 types of the control charts which are developed for attributes. When these charts are developed, then these help in assessing the quality performance of the system, means how good system is to produce the quality products and services.

So it helps in assessing the performance of the system to produce the quality products and services. It also helps in getting or in giving the attention, on taking the attention, of the management for changing quality of the products and services which are being offered. So quality control charts will be highlighting the kind of change in quality of the products and services is taking place and regarding that attention of the management is attracted and it also helps to suggest what kind of the quality characteristics are extremely important for which X bar and R chart can be developed.

So, for developing the X bar R chart for variables it helps in identification the quality characteristics which are important for success of the product. So, control charts for variables if are to be developed, then the quality characteristics are suggested by the control charts for attribute.

It indicates or it shows the average quality level of the products and services which are being produced by the company and when we are able to have the idea about the average quality

level able to measure the performance of the system to produce the quality product and services, then these inputs, means it also helps in giving the inputs for improving the quality of product and services which are being provided by the company.

So improving the quality of the product and services, indicating the average quality level, suggesting the kind of quality characteristics for which X bar and R chart can be developed and giving the attention or taking the attention of the management regarding the changes in the quality level which are taking place and assessing the quality performance of the system which is there in place.

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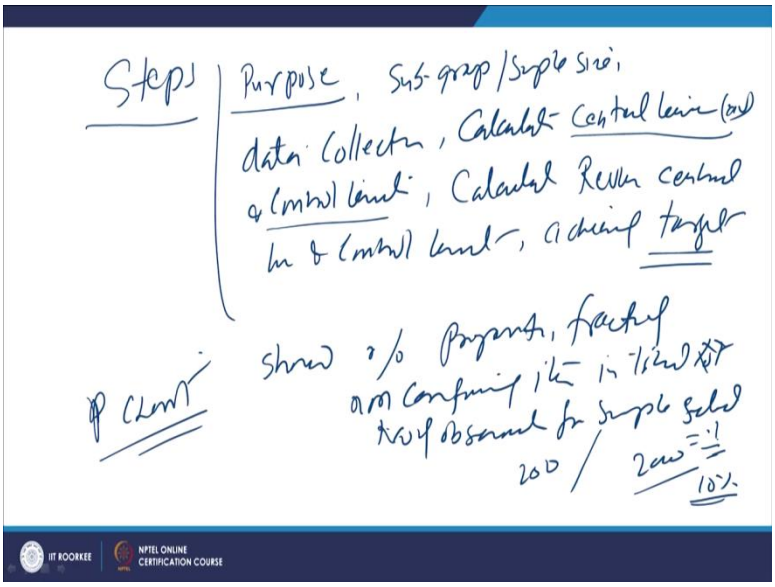

So, there are like say, for developing these charts the certain steps are followed, like whether it is p chart, np chart, c chart or u chart certain steps are followed. First, we identify what kind of the purpose is to be solved, what we want to highlight? So there is a purpose of every chart and that is understood and there after we try to determine the sub group or sample size that should be taken and then we go for the data collection.

And after the data collection, we calculate the central line, that is the average and then the control limits. So, a calculation of the centre line value under the control limits is the next step and after this if we notice that there are certain points, because initially we start with the rough estimates and after the data collection through the rough estimates we calculate the centre line and control limits values.

And thereafter if we find that certain data points are falling out of the limits then those points are discarded and again we calculate, so again calculate the revised a central line and the

control limit values. So this is the next step and after this we plan for achieving the target, whatever is there as far as the improvement is concerned or checking the effectiveness of a particular change towards the improvement or deterioration.

So these are the certain steps as far as the p chart is concerned. So as far as the purpose of p chart is concerned, p chart shows the percentage or proportion or fraction of non-conforming items in a total lot, in a total number of observations from the samples selected. So like say if you have selected the 2,000 items from the different samples and then out of the 2,000 we have got 200 number of the non-conforming items, then the 0.1 or the 10 percent will be fraction of the, p value will be the 0.1 will be indicating that the 10 percent is the fraction of the non-conforming items.

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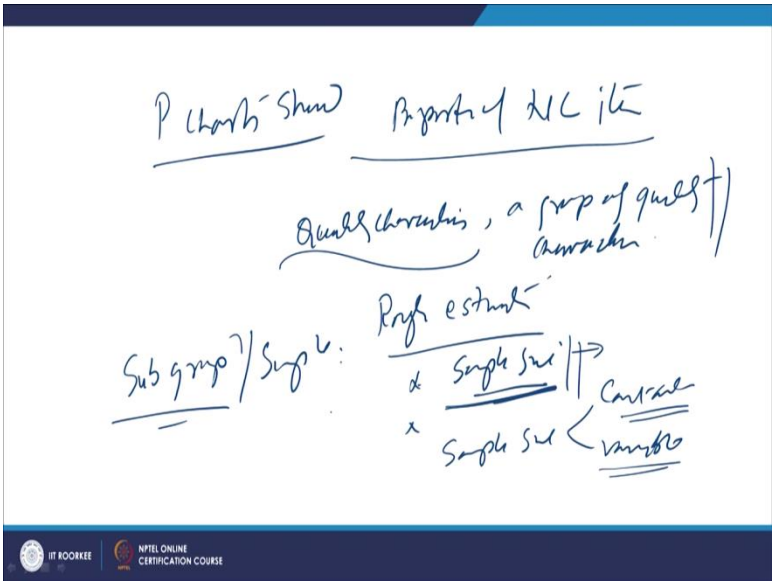

So basically the p charts shows the proportion of the NC items, simple proportion of the non-conforming items, in the lot which has been inspected and this non-conforming items means the items are non-conforming may be with a regard to the certain quality characteristics or a group of quality characteristics.

So non-conformities in a non-conforming item may be due to the single quality characteristic or the number of quality characteristics. So it shows the proportion of the NC items with regard to the one or group of the quality characteristics which are not being matched and because of which it is a becoming the NC item or non-conforming items, so this is one thing.

Then, second is the selection of the sub group size, so sub group or the sample size. When we do not know anything about, we do not have any idea about, the fraction of the non-

conforming item, then we start with this selection of the few number of samples. So rough estimate is made and we select the certain number of samples.

So means, initially the sample size is selected randomly to have the rough idea about the number of the non-conforming items in such a way that roughly we get some idea about the proportion of the NC items in a lot and that will again depend upon and that will give us when the sample of particular size is selected that will give us certain proportion of the NC items. So, through the iterative process we will be reaching to the appropriate value of the central line and the control limits.

Thereafter, it is always preferred that subgroup size, whatever we choose, initially the rough randomly some rough estimate regarding the sample size is made and that is and that samples of that size are selected from the lot, but there can be two possibilities one is the sample size can be constant or it can be variable.

So, in p chart preparation it is always preferred that sample size is constant but even if it is changing, then our approach will be different. So somehow from somewhere we have to start for having the idea about the fraction of the NC items in a lot. So we will be starting with a few samples, so that we can get idea about the fraction of the NC items and thereafter we will be revising the centreline and the control limit values.

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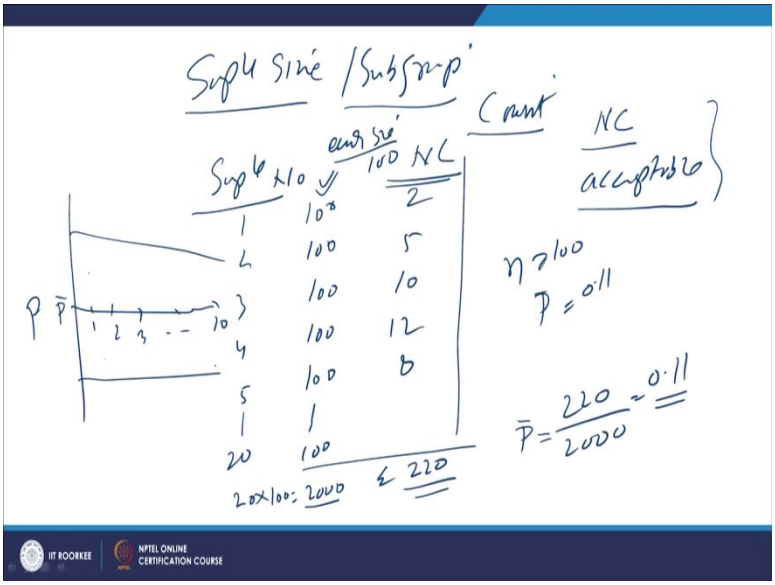

So, once the sample size has been decided and the sample is selected or sub group of the suitable size has been selected, thereafter we have to count, instead of measuring we basically count, count the number of NC items or the acceptable items, non-conforming items or the conforming items, the count is made and then like say we have selected samples.

So these are sample size and the sample number say 1, 2, 3, 4, 5 and like this say 20 samples are selected and each sample size is of the 100 units and then we try to find out the number of the NC items. First the number of NC items will be identified. So each sample since having the 100 pieces, so out of those 100 pieces, every time we will be checking and trying to find out the how many NC items are there.

So like this 2, 5, 10, 12, 8 like this all these things are determined and then will be making sum of all NC items, say it is coming out to be 220 and since the sample size of each sample is 100 and there are 20 number of samples, so total number of items being inspected will be equal to 20 into 100, so this will be 2,000 number of units are inspected and out of the 2,000 if you have 220 number of the NC items, then p bar that is the fraction of the average proportion of the NC items, non-conforming items, will be 220 divided by 2,000, so this will be giving us the value of 0.11.

So this is how do we calculate the value of p bar and here what we have n is 100 and p bar is say 0.1. So how to calculate the initially in the control chart this p here we have p and p bar will be the centre line and then the value of p will be plotted for that different samples here

and there. So here sample 1, 2, 3 like this up to 20 and the p value will be plotted in the control charts and then we have to set the control limit.

So initially we will be getting the first centre line and the first few values of the lower and upper control limits and thereafter we find, if we find, that some of the points are falling beyond the control limits and then a revised centre line and the control limits values will be calculated. So how to come up with?

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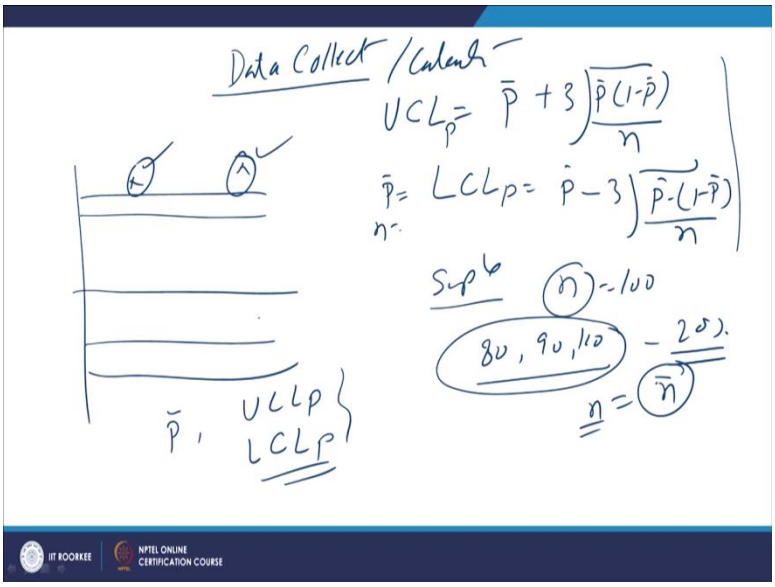

So, after the sample collection go for counting of the NC items. So after data collection the calculations are made for the upper and lower control limits. So UCL for p chart is calculated using the p bar plus 3 times of the p bar minus 1 minus p bar divided by n. So p bar is the average fraction of the NC items and n is the sample size and so this is how we will be getting the upper control limit and likewise we can calculate the lower control limit.

Like p bar minus 3 into the sigma, actually this is the standard deviation for the p bar value. So here p bar minus 1 minus p bar divided by n. So this is how we calculate the upper and lower control limit values. When we find that, in this case, we see that the sample size is constant then n value is a say constant in this case 100.

But if the sample size is fluctuating say sometimes it is 80 then 90 then 110. So if the sample size variation is within 20 percentage, then the n value in the above equation is replaced with the n bar that is the average n value is used for calculating the upper and lower control limits. So, on plotting this if we find that certain data points are falling beyond the acceptable limit,

then those data points will be discarded and again we will be calculating the revised control limits.

So basically, after discarding we again follow the same method for calculating the p bar and calculating the UCLp and LCLp, upper and lower control limits and p bar is calculated again after discarding the certain points which are falling beyond the acceptable limits and then we develop the control charts again. So this is the kind of continues process. In this process you will see that the control lines will be coming closer.

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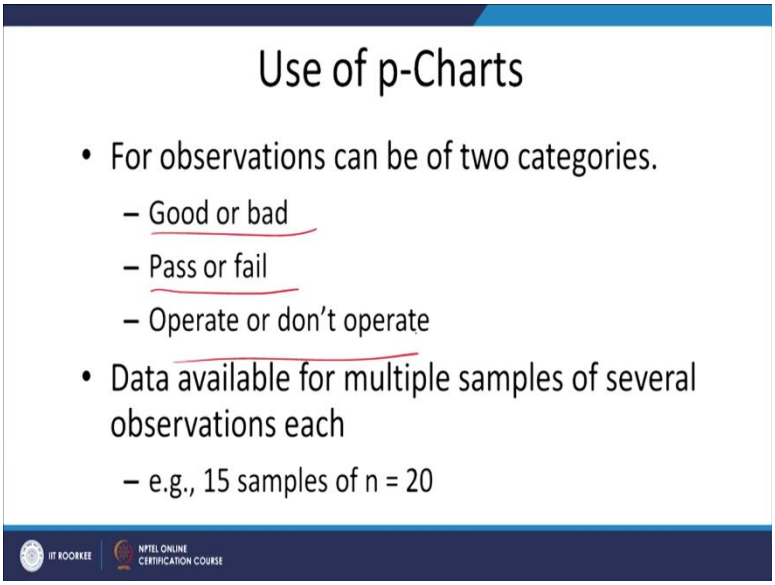

Now, we will see 1 example wherein we have got 1 example showing how we can calculate the control limits and p bar values for developing the p chart. So as I have said in p charts, we just count the things like the good or bad, pass or fail, operate or does not operate kind of thing and then about this I will talk little later.

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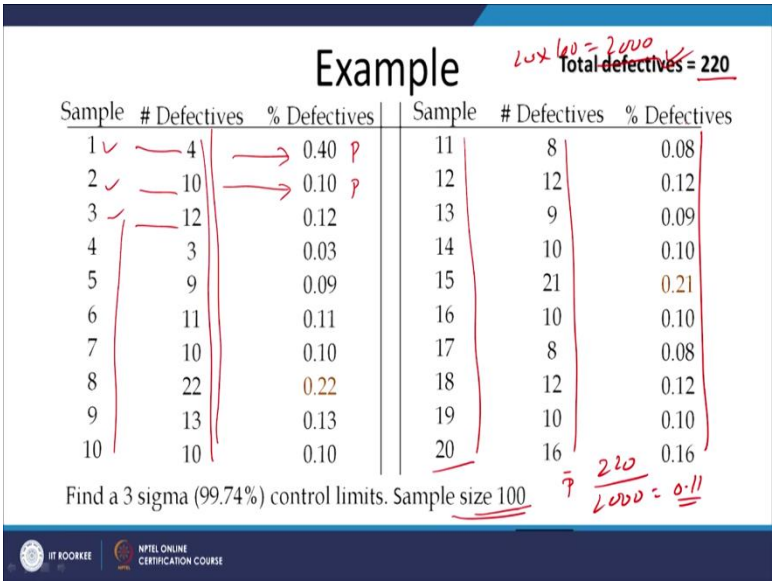

So here, if we see this is just an example where like say, there are the samples, sample 1, 2, 3 up to say 10 and then 11 to 20. Say the 20 samples are selected and each sample is of size is 100. So the total number of the units being inspected and observed is 20 into 100, that is 2,000 and then it is showing that defective items are non-conforming items in each sample like this and then fraction of the defective items.

So these are basically p values and when we make sum of all these defective items divided by the total number of the items inspected, then that will be giving us the p bar value. So in this case the total defective item is 220 and the total number of items inspected 2,000, so 0.11 will be the p bar value and these are the basically p value, the fraction of the defective items and these will be plotted on the control charts.

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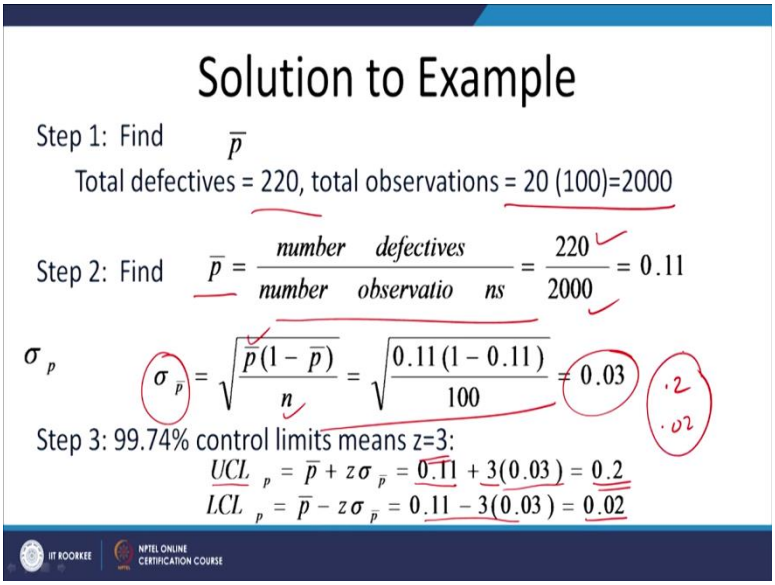

So here, basically to find the total defective items, total number of observations and p bar that is number of defective items divided by the total number of observations. So that is 220 divided by 2,000, so this will be giving us the value of 0.11 and then we can calculate the standard deviation.

So by simply putting the value of the p bar and n will be getting the value of 0.03 for the standard deviation for the, fraction of the defective items and assuming then standard normal deviate 3 we can simply calculate the upper control limit, like this p bar is 0.11 and 3 into the standard deviation for the fraction of the defective item that is 0.03.

So this will be giving us the 0.2, the upper critical limit and then lower critical limit likewise is calculated like this and it will be giving us the value of 0.02. So 0.2 and 0.02 are the 2 upper and lower control limits respectively. So if we see here in previous example, what we will notice?

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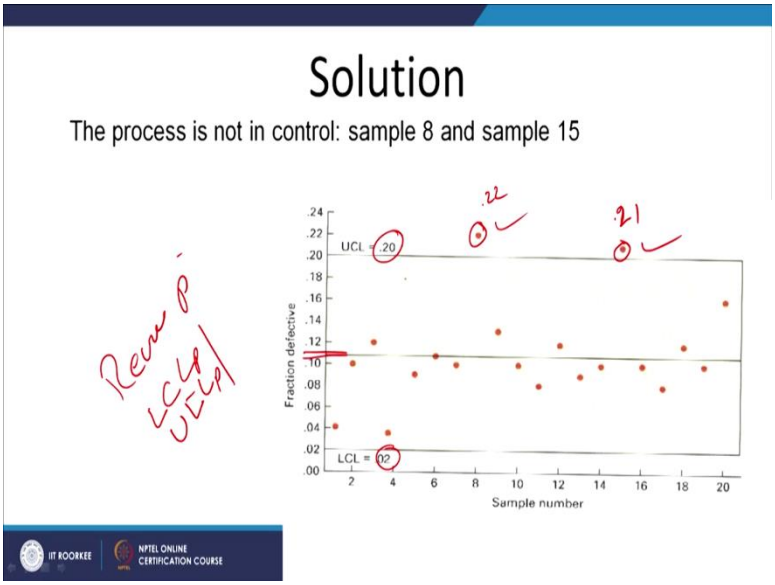

So these values are plotted, so what we will notice that for sample number 8 the value is 0.22 and for sample number 15 the value of the p is 0.21. So since the upper control limit is 0.2 and lower control limit is 0.02, so these 2 data points are falling beyond the control limits so we need to discard these for calculating the revised p bar giving the centreline value and the lower control limits and upper control limit values. So those are calculated again.

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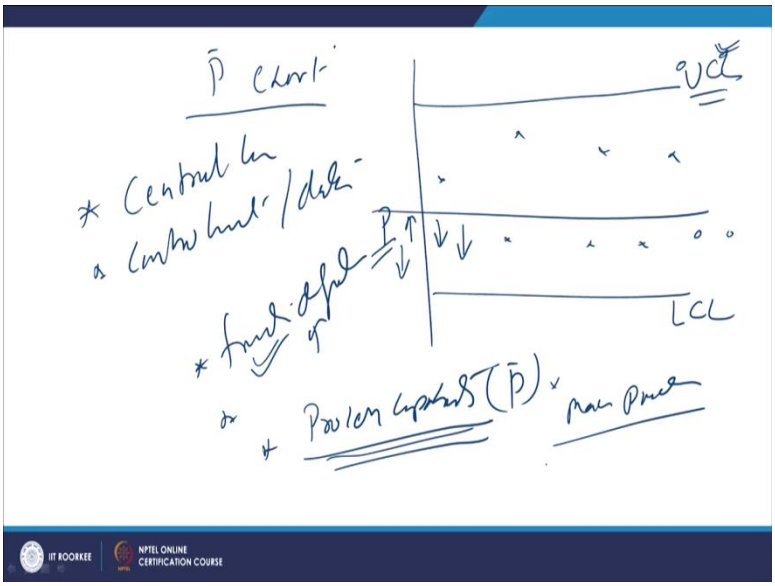

Now, we will be talking more about certain inferences that can be obtained from the p chart, like say, when we have the values. So there are certain inferences about the p chart that is obtained from the central line and the control limits and the data points which are falling.

So, since here we have the p value that the fraction of the defectives, lower control limit upper control limit and when we see that the value of the p bar is higher means the fraction of the defective items is increasing, means the quality of the things or products or services which are being provided is a deteriorating, which means that the management now needs the intervention in form of expertise, in form of better machinery, in form of better procedures.

So, some major intervention is needed when it happens that the average p bar value is high and when it is reduced indicating that the average fraction of the defective item is decreasing. So, that is 1 inference and it indicates the process capability, central-line value that is the p bar value indicates the process capability, how capable a process is.

So, lower the value of p bar it will be indicate indicating the better capability of the process, showing the lesser well fraction of the NC items and so that will be indicating that process can be used to produce large number of the goods without producing many NC items. So, that is one inference that the centreline value indicates the kind of fraction which will be their average fraction of the NC items and the process capability and if the p value is high then it needs the intervention of the management.

On the other hand, if there is a process giving the p value randomly here and there, means non-random pattern is not there but random pattern is being indicated then that will be showing that the process is under control but the movement the points start falling beyond the control limits, then it will suggest that for the same process which was doing well producing the fewer NC items certainly it has a started to produce the things, so that the points have started to fall beyond the control limits, that suggests that there is some problem of the manpower.

So if the points in the p chart are falling beyond the control limits, then the intervention with regard to the manpower is needed and if the process average or the centreline value is a shifting from its position towards the higher side then the intervention of the management is needed which may be in form of the better machinery, better procedures, better manpower and better skill to reduce the proportion of the NC items.

So now, we will see the another interesting control charts for attribute that is the c chart which is used for the number of defects per unit. So each unit is checked and the number of defects present in unit is quantified and then they are plotted.

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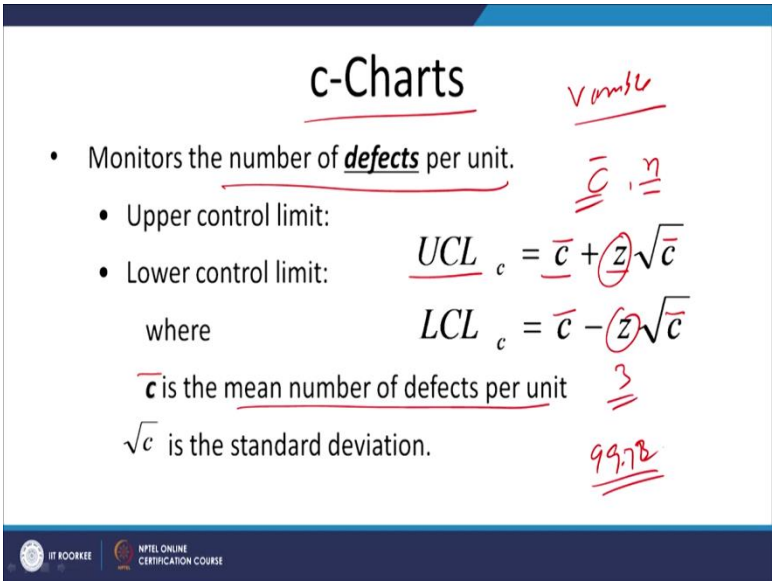

So c chart showing the number of defects per unit, there can be two situations either the sample size is fixed or it is variable. So when sample size is fixed the c chart is plotted and when the sample size is variable u chart is plotted, so this is the case of the constant sample size. So number of defects per unit is counted which will be and then so that will be examine the number of samples are there and then average number of defects per unit is calculated in form of c bar and then depending upon the sample size, then we will be calculating the upper and lower control limit.

So basically, here use upper control limit for c chart is indicated through c bar, z is the standard normal deviation and square root of the average number of the defects per unit and similarly the lower control limit is calculated using the c bar minus z into square root of the c bar. So c bar here is the mean number of defects per unit and here we will be putting the value of 3 that is normally taken for 99.72 fraction of the normal, for the normal distribution.

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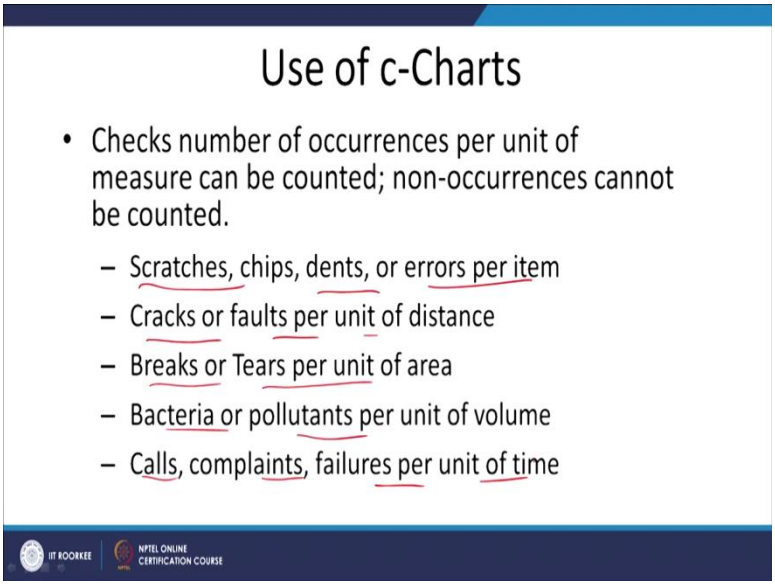

The c chart, use of the c chart is basically when we check the number of the discontinuities or non-conformities or defects which are present and these may be in form of like a scratches or dents or errors per item, cracks or faults per unit distance, breaks or tears per unit area, bacteria or pollutants per unit volume, calls complaints and failures per unit time.

So this is how we count the number of defects or undesirable features which are present in a unit and then their average values are identified. Now, we will summarize this presentation. In this presentation basically I have talked about the importance of the control charts for attribute and what are the procedural steps for developing the control charts and the p chart and the c charts I have talked in detail with the example. Thank you for your attention.