Principle of Industrial Engineering
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Lecture – 58
Quality Control: Control Charts II

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering and you know we are talking about the control charts for quality control. We have seen how to develop X bar R chart under the control charts for variables. And in this presentation we will see the few examples related with the control charts for variables. And then, we will also talk about the process capability and what is the significance of the process capability with respect to the specifications of the units which are to be produced.

So, what we have seen so far, like whenever a product is manufactured the two units will never be same, there will always be some kind of variation, whether the variation is within the same product or variation in the products being manufactured in the same shift or many variation in the dimensions are the characteristics of the products manufactured in the different shifts. So, lot to lot variation sample to sample variation is always there.

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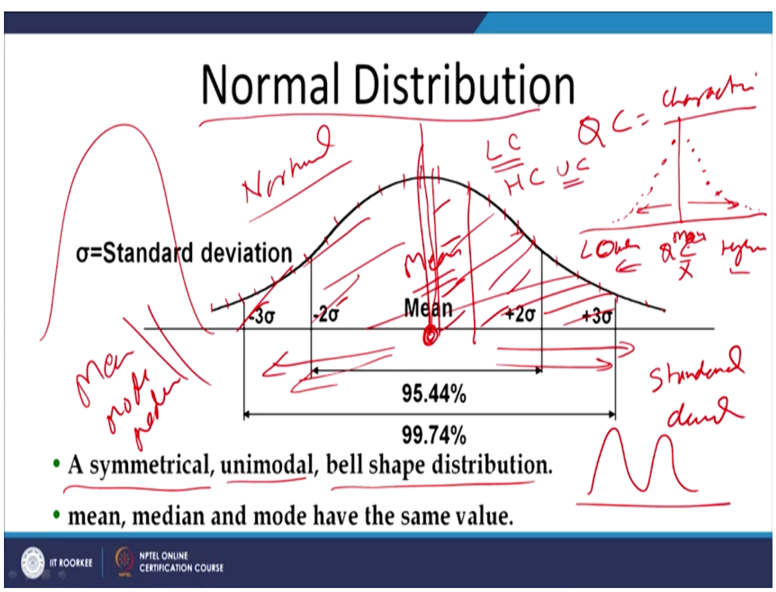

And so, whenever the quality characteristic of the importance is characterized, like it is quantified and then it is plotted, then what we notice like most of the data values will be segregated somewhere in between and very few data points will be falling at the extremes like this. So, this kind when the distribution of the quality characteristic is plotted, then what we notice that the data is trying to segregate the quality characteristic data is trying to segregate somewhere close to the mean.

So, this is say the mean value where the data is trying to segregate, and then extent of the dispersion which exists the extent of dispersion from the deviation from the mean value that we can see from the from the increasing distance of the points from the mean value. So, here this will be like say this is the average value and here we have the lower value this is the lower value of the quality characteristic this side and this side we have the higher value of the quality characteristic.

So, of course, we will be accepting the product of the certain up to the certain lower value and up to the certain higher value and that is what is identified in form of lower value of the acceptable characteristic or higher value of the higher or upper value of the acceptable characteristic and if the quality characteristic which has been quantified and plotted, plotted in a chart, then we see the kind of the distribution which is there is mostly the normal distribution like most of the data values are close to the center then the proportion of the values falling away from the center is decreasing.

So, there are two ways to know to understand the normal distribution, where its mean is and how much scatter or the deviation from the mean is taking place. So, that is what is quantified in terms of the standard deviation. So, with respect to this mean value, what is the proportion of the thing up to the 2 sigma or it may be 1 sigma also 2 sigma or up to 3 sigma on the higher side or on the lower side.

So, in most of the cases are normally what we get a symmetrical, means with respect to the center value or average value, the scatter or the division on the highest side and on the lower side is almost equal and unimodal means, there is just one peak, otherwise there can be two peaks like this in case of the BI model and the typical normal distribution follows a bell shape distribution like inverted bell shape distribution.

And when the distribution is normal what we notice, like all the three ways by which the average is quantified mean, mode or median, all three fall at the same point at the same level. So, that is a situation when we have the normal distribution.

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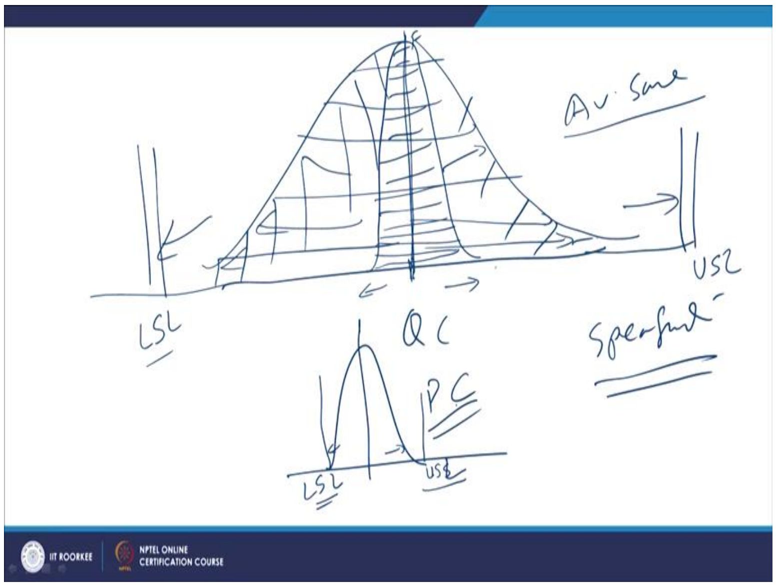

So, it is always a desire like if we have the quality characteristic in the x axis and the values and the frequency in the y axis how many the data points are falling, and then the when it is plotted what we notice that it is possible that most of the values are very close to the average one like this. So, the variation away from the center or the mean is very less or we may have a situation where the average is same, but that the deviation from the center line or the average is too much.

So, these are the two different situations where average is same or the mean value is same, but there is a lot of variability in the characteristic which is being produced. So, in this case we may have the situation where many things are beyond the acceptable limit. So, to see if the product is within the acceptable limit or not. Now, we try to see or try to compare these variations in the quality characteristic with the specification of the product.

So, the product specification if is lying far away from the values where it is a falling say upper specification limit, lower specification limit and all the things anything and everything which is being produced in this case will be within the acceptable limit, but with a lot of variation in the quality characteristic, but in the second case in the first case, where what we have seen?

Most of the values are centered about the near the average. So, the variation or the variability in the quality characteristic is very limited. So, the dimensional features or the quality characteristic feature is very closely control control average is this one and the variation is also less while in the second case, there is a lot of variation, while the process average is still same.

So, and despite of the lot of variation if the specifications upper and lower specifications are still further away, then whatever will be producing that will be within the acceptable limit. On the other hand, if we have a situation like this, this is the kind of the distribution, this is the mean and if the lower specification and upper specification limits are very close to these points.

The upper specification limit and lower specification limit if these are very close to the data points which are falling at the extremes, then the process may become sensitive for having the points falling apart from falling beyond the upper and lower specification limit and in that case we will be producing the things which are not acceptable limit. So, that is about the process capability we will be talking in detail about this.

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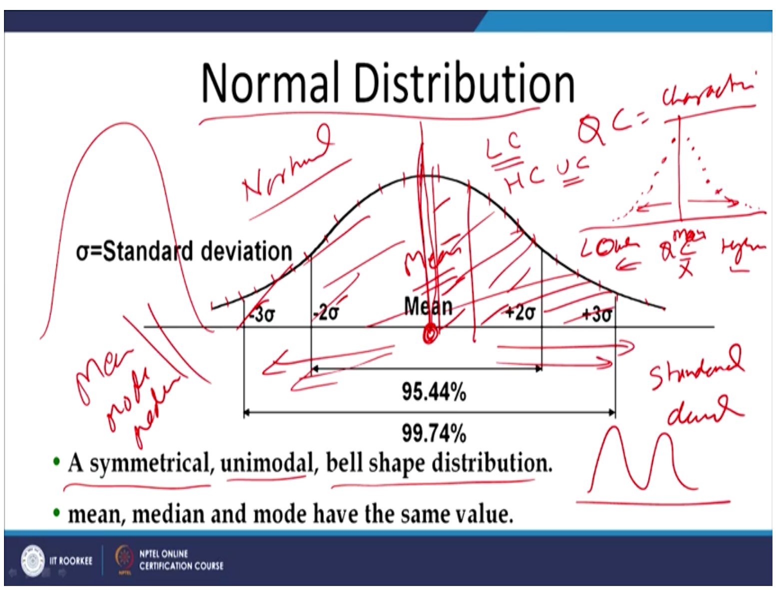

So, what we have seen that whenever number of things are produced, you will always see that there is somewhere average under the process and the quality characteristics are few and the quality characteristics values are falling a few in the highest side and few on the lower side with respect to the mean value and that most of the time follows the normal distribution.

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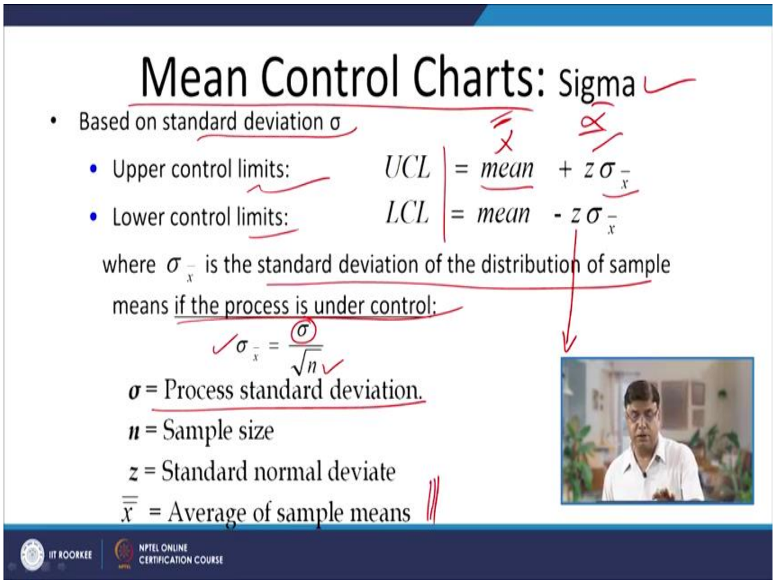

So, as far as the plotting the mean control charts for x bar was concerned using the x sigma value for the process standard deviation. So, based on the standard deviation, upper and lower control limits can be plotted like this upper control limit the x double bar plus z into the x sigma x bar. So, this sigma x bar is the standard deviation of the distribution of samples, if the mean process is under control. So, sigma is the sigma is the like say the processes under the division and n is the number of samples in a number of units or the sample size number of units in a sample or the sample size which will be 2, 3, 4, 5 and 10 like that and accordingly this value of n is identified.

So, sigma x bar is calculated and then the z is that standard normal deviate, which can be out which is normally 3 sigma but it can be 2 sigma or otherwise then x double bar is that x double bar is calculated, which is basically the average of the or mean of all samples, which has been taken from the lot for developing the control charts.

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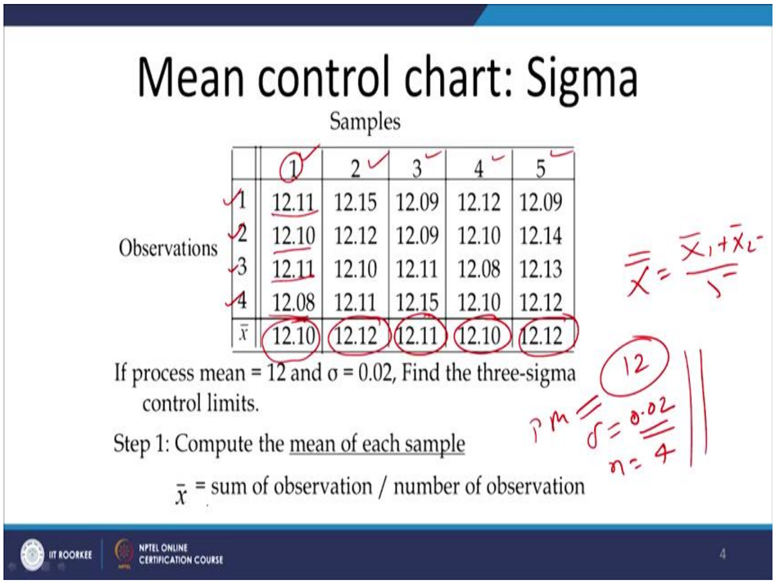

Now we'll see one example, like say, there are 5 samples which have been taken sample 1, 2, 3, 4, 5 and in each sample, sample 1 in each sample, there are 4 units sample 1 has four units 1, 2, 3, 4 and these are the quality characteristics which have been quantified and the average of the quality characteristic for sample 1 is say 12.1. Similarly, the average of the quality characteristic for the sample 2 sample 3 sample 4 and sample 5 is identified and x double bar will be identified from this x bar 1 plus x bar 2 like this and divide by 5.

So, this will be giving us the x double bar say in the process has been set to produce the things corresponding to the 12 dimensions, 12 numerical value of the quality characteristics. So, the process mean process mean is 12 and say the process standard deviation is 0.02 and the number of samples number of units in a sample and here is 4.

So, these are the three things that we have to use to develop the control chart mean control mean charts using sigma.

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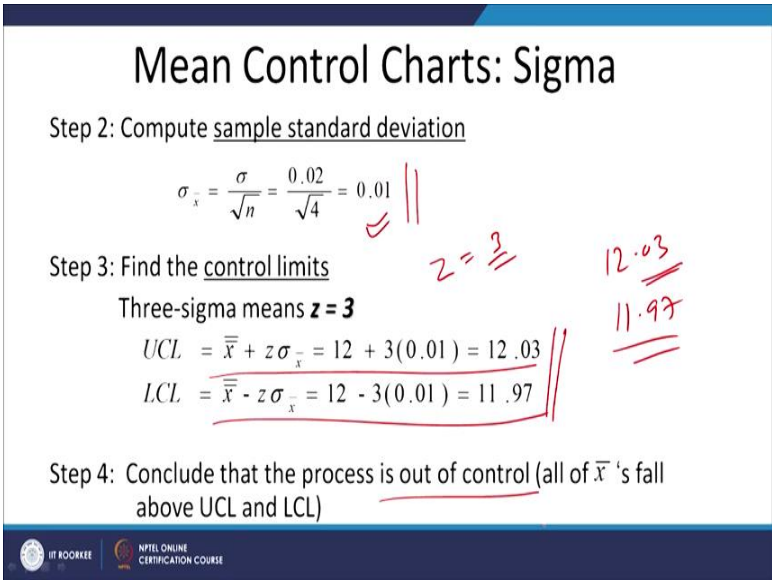

So, x bar we know how can determine first establish to determine the x bar and then compute the sample standard deviation using the sigma an n values. So, the sigma is 0.02 and divided by the square root of the 4, 4 is the number of samples in a number of units in each sample. So, we have the sigma x bar is 0.01. And then, considering the standard normal deviate z value 3, the upper and the lower control limits can be calculated.

So, in this case, what we can see? The lower upper control limit is at 12.03 and lower one is that 12.97. Since a few values are beyond the 0.03 value, so, it will be assumed it will be concluded that processes out of control as all the values will be falling beyond the.

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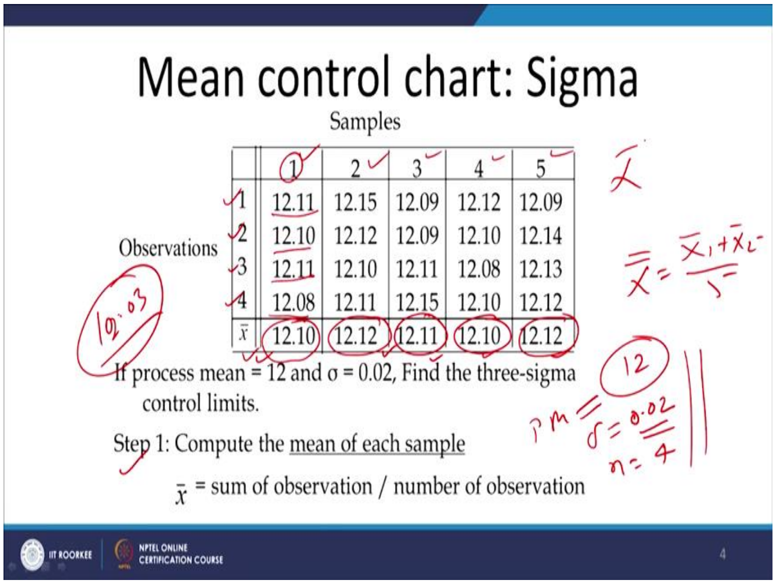

	1	2	3	4	5
1	12.11	12.15	12.09	12.12	12.09
2	12.10	12.12	12.09	12.10	12.14
3	12.11	12.10	12.11	12.08	12.13
4	12.08	12.11	12.15	12.10	12.12
$\bar{x}$	12.10	12.12	12.11	12.10	12.12

So, upper control limit is our 12.03. So if we determine the x bar for 1 sample 1 is 12.1, which is higher than the 12.03. Likewise, the other values are also higher than the 12.03.

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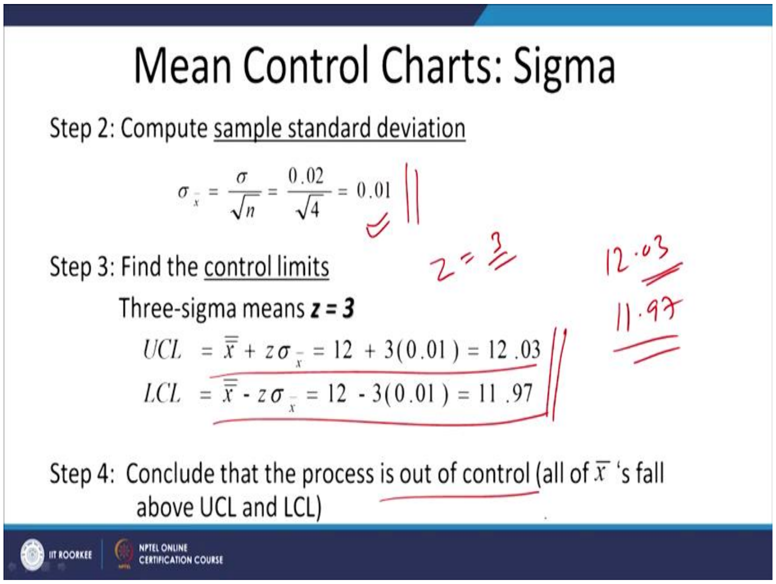

So, since all the x bar values for the different samples are falling beyond the upper control limit, so, the processes out of control.

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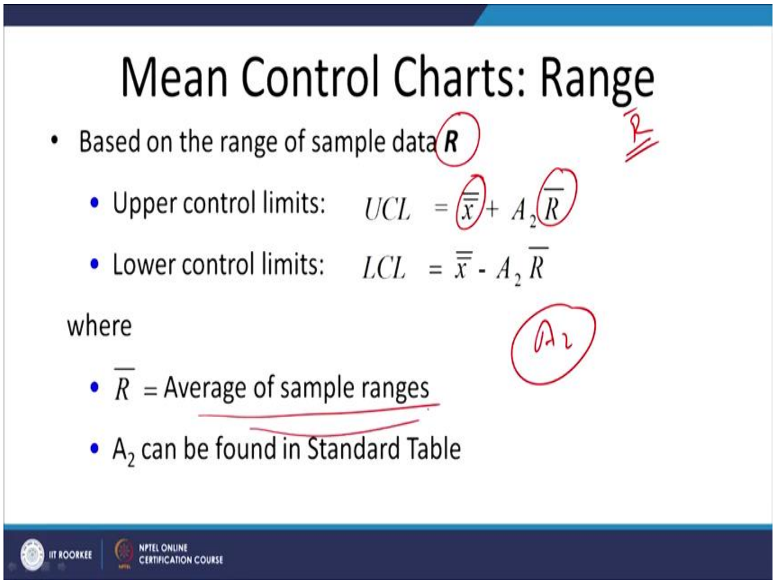

Then, we have also seen how to develop the mean control charts using the range. So, the range for each sample is calculated and then average range is average range R bar is identified, then using the value of x double bar and the R bar the upper and lower control limits are identified. The A2 is the coefficient which is identified based on this number of units in a sample or the sample size using the standard table.

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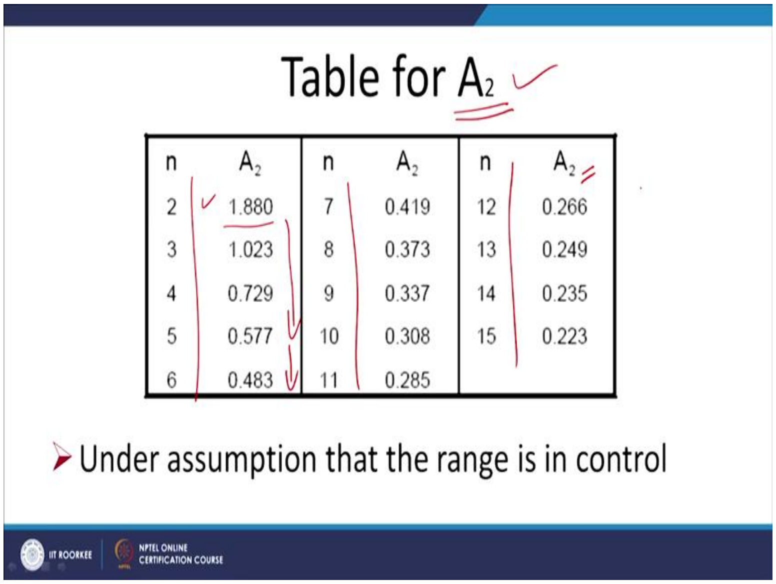

n	A_2	n	A_2	n	A_2
2	1.880	7	0.419	12	0.266
3	1.023	8	0.373	13	0.249
4	0.729	9	0.337	14	0.235
5	0.577	10	0.308	15	0.223
6	0.483	11	0.285		

So, this is the typical table which will help us in getting the value of the A2 using the different sample sizes. So, when the sample size is 2, the value of A2 is 1.28 and this value will keep on decreasing we can see increasing the sample size, the decreasing the value of A2, which means, when the sample size is the lower, the coefficient value is higher, which means the upper and lower control limits will be very wide and far away from the center line.

Value while the higher sample size will be leading to the reduced value of the A2 and that in turn will be decreasing the bringing the upper and lower control limits close to the center line and that in turn will be making the control charts sensitive for fluctuations.

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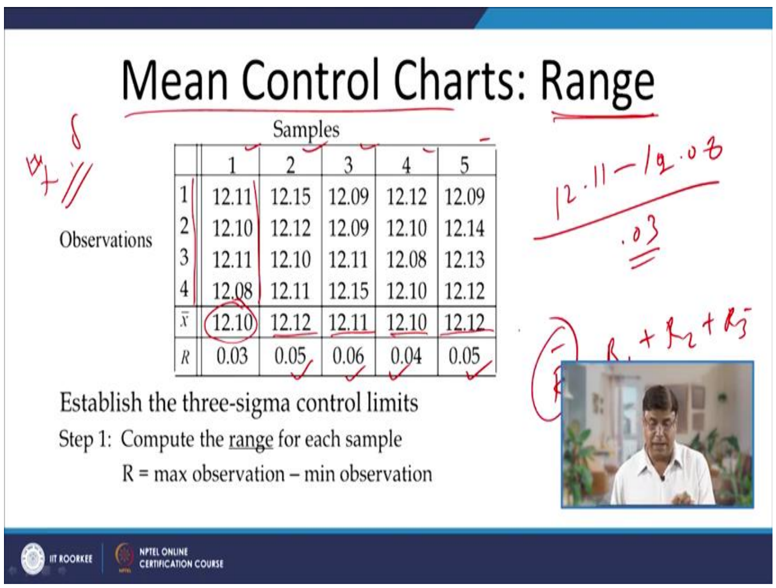

This is another example where it has been explained how to develop the mean control charts using the range. Earlier, what we have seen we are using the control charts further away the process average and the external division has been set and here we will be using the developing the control charts. In this example, we will be developing the mean control charts using the range.

So, the, the again the 5 samples and in each sample four units, the average of the characteristic value x bar for each sample is identified, let us say calculated that is 12.1 1 then 12.1 to 12.11, 12.10 and 12.12. So, this is about the average quality characteristic of each sample, which is having the four units and then range is identified difference of maximum and minimum.

So 12.11 minus 12.08 this will give you the range of 0.03 for sample 1. Likewise, the range for the different samples is calculated, calculated, and then R bar is calculated from the range for sample 1 range for sample 2 range for sample 3, up to R5 divided by 5. So R4 plus divided by R5 this will be giving us the average range.

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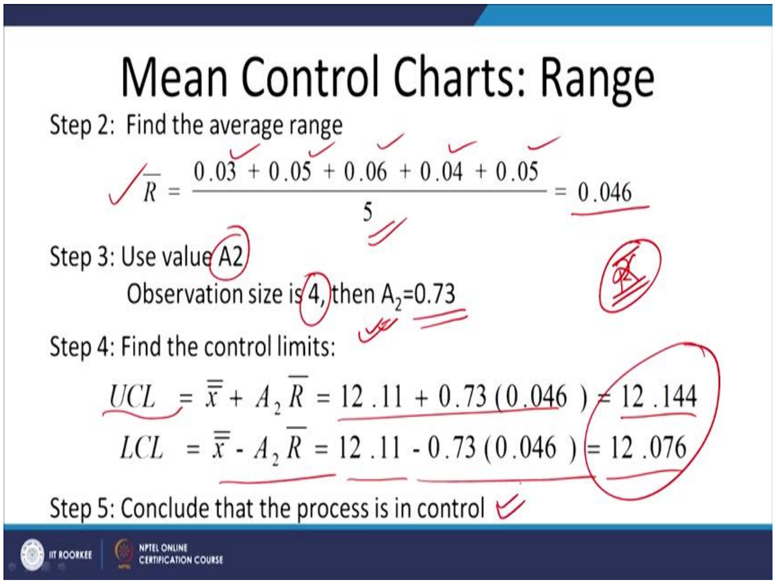

So, this average range will be used to calculate the control limits. So, R bar here using these ranges for each sample divided by the number of samples that is 5 is calculated and the R bar is coming 0.046. And then, using the value of A2 from the standard data table for the sample size 4, we will get the value of the 0.73.

So based on the sample size 4 the A2 value of the coefficient A2 is identified and then upper and lower control limits are calculated upper control limit x double bar plus A2 into R bar and the lower control limit x bar minus A2 to R bar and we will have to put the values and then calculate the upper and lower control limit.

So, what it will suggest that upper and lower control limits are such that all average, all values of the quality characteristic x bar values of the quality characteristic are the falling within the limits, so the process is under control.

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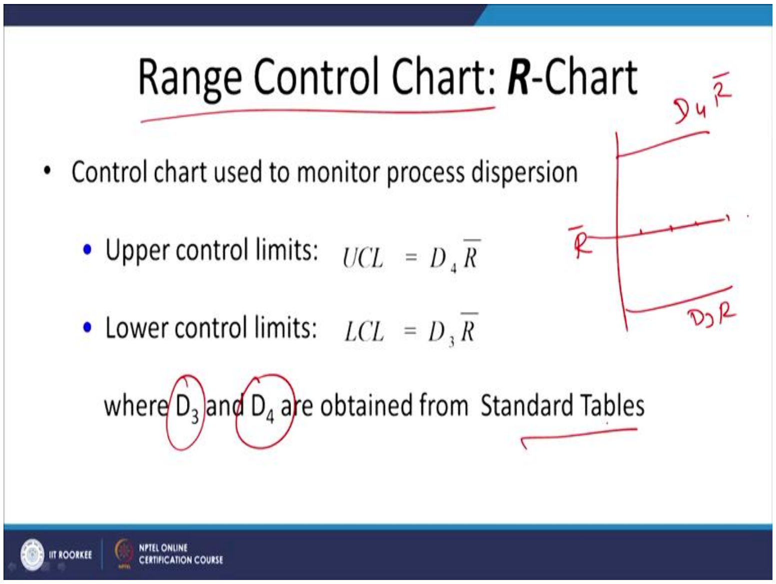

Now, we will see the range control charts, range control charts are developed, again developed in the same way that is also important and both mean control chart and range control charts have to be seen together to make the effective inference about if the process is under control or not. So upper and the lower control limits and there is a some the range value and then upper control limit is identified through the D4 into R bar and D3 R bar gives us the lower control limit and the range values are plotted for the different samples D3 and D4 are the coefficients which are obtained as per the sample size from the data table or standard a table.

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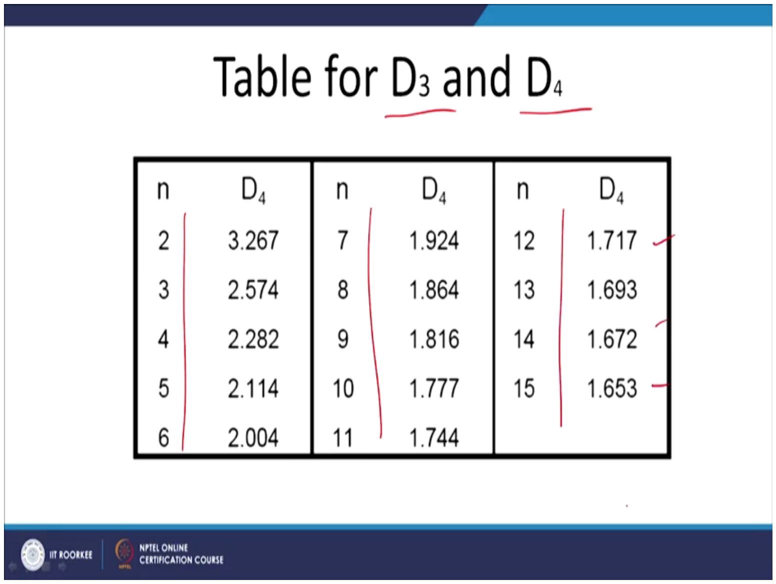

Table for D_3 and D_4

n	D_4	n	D_4	n	D_4
2	3.267	7	1.924	12	1.717
3	2.574	8	1.864	13	1.693
4	2.282	9	1.816	14	1.672
5	2.114	10	1.777	15	1.653
6	2.004	11	1.744		

So, the table for D3 and D4 here in gives this primarily shows the D4 likewise we can get it for the D3. So here, different samples sizes and the corresponding values of the D3, oh sorry D4.

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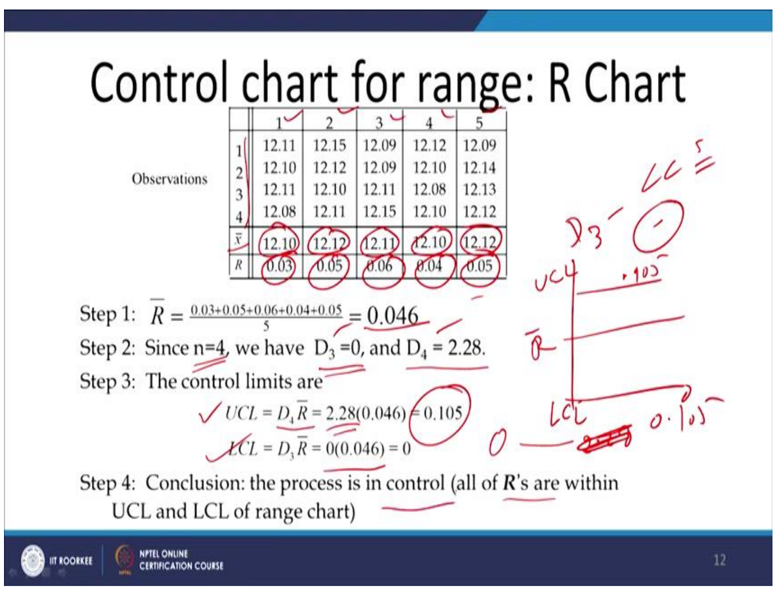

Say, when the sample size value is less than certain limit, then we find that the values are negative, then in that case the D3 value if is less than 5 or 6 sample size, the D3 value is comes out to be the negative. So in that case, the lower control limit is set at the 0 level. So, say since in this case say the value of n is 4 for which D3 value is taken 0 and D4 value is 2.28.

So, in the same example, the 5 samples each sample is having four units, the x bar value for each sample is calculated, the range for each sample is calculated. And using these range values, we determine the R bar value. And once we have got the R bar value then we using the D3 and D4, we can calculate the upper and lower control limits. So upper control limit D4 into R bar D4 for the sample size 4 is 2.28 into 0.046 that gives us the upper control limit for R chart 0.105 and the lower control limit, since it is 0 so, the lower control limit will be set at 0 LCL, UCL, and here they will have the R bar.

So, conclusion from this is that the process is in control since the all R values will be falling within the acceptable limits. So, here 0.03, 0.04, 0.05, 0.06, 0.04, 0.05 so, all these values are well within the limit of the 02.105. So, this is the upper control limit 0.105 and 0, all the values are falling well within this range, so, the process is under control, but there can be a very different situations if we see just the mean control chart or just see the R control charts, then we may have the misleading or incorrect inference about the situation of the process whether it is under control or not.

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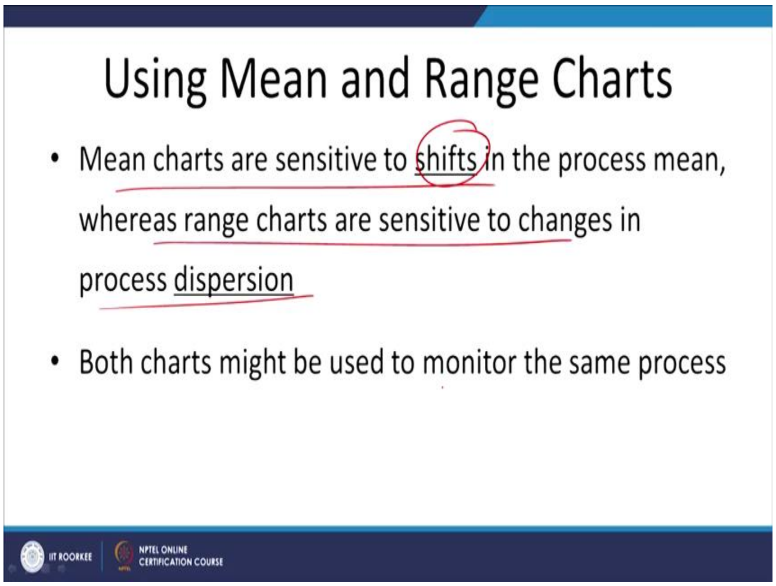

There is a very much possibility that, that the mean control chart data is showing that the data points are falling well within the limits, but R control chart may show that the data points are going beyond the acceptable limit. So, the mean control charts are shift for sensitive in the process mean, process mean itself may shift and while the range chart data is sensitive for the changes in the process dispersion.

So when the process mean shifts, that can lead to the out of control situation and when the dispersion shift beyond the acceptable limit that also can lead to the acceptable beyond that can lead to the situation where processes is out of control and both control charts should be used to monitor the process.

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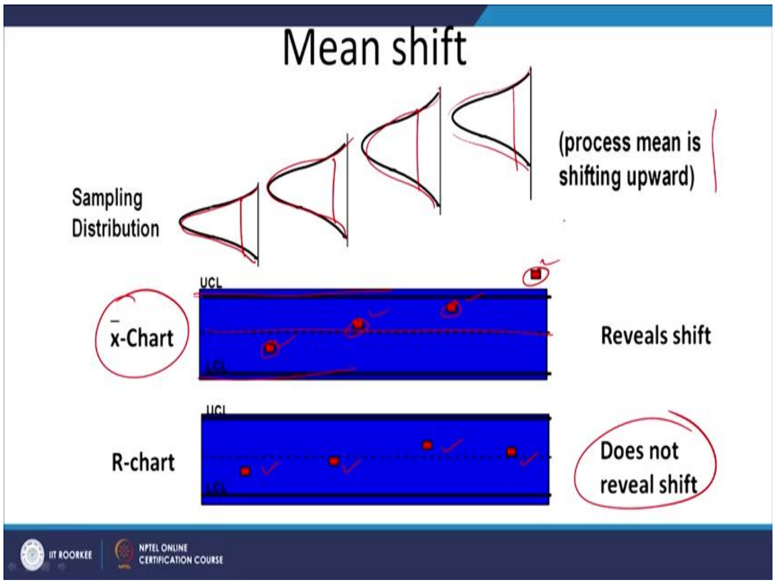

For example, in this case, this is the x bar chart, upper control limit, lower control limit and this is the centerline value and what we see here, the mean is continuously going showing a trend and then the values go beyond the acceptable limit. On the other hand, this will not be shown by the R chart, R chart is still showing that the variation is the range is acceptable is still acceptable limit.

So, the R chart does not reveal this the shift in the process mean, process mean is shifting in upward direction. So here, what it is showing? That the data values are shifting in upward direction like this. So, that is why the process mean is shifting, dispersion here is still say within the acceptable range, but the process mean as a whole is shifting.

So, the process mean is shifting in this case in upward continuously and that will be leading to the situation your processes out of control. So, just seeing one chart will not help in making an effective inference about if the process is under control or not. We need to see both the charts together.

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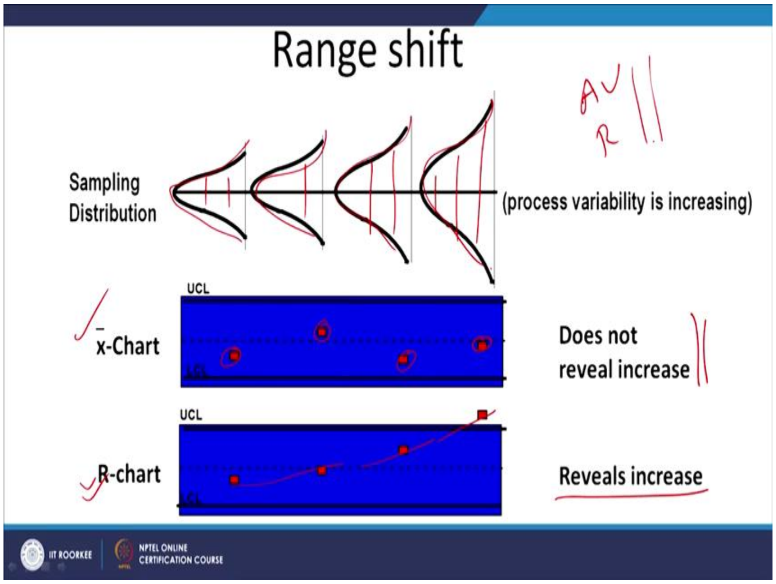

There will be another example where process mean is intact, but the process variability has increased and that can lead to the situation of the process out of control, like say the process mean is still at the same level, but the dispersion has increased significantly here it is increasing dispersion situation.

So, and that is what has been plotted the process mean the average x bar values, average of the quality characteristic of each sample is still falling within the control limits and a non random distribution is being shown so, it does not reveal anything. And the process variability increasing process variability is also not being shown by the x bar chart.

On the other hand, the R chart will show the kind of the trend which is they are increasing variability is shown by the R chart. So, we need to see the average and the range charts both together to make an effective inference about the situation if the process is under control or not.

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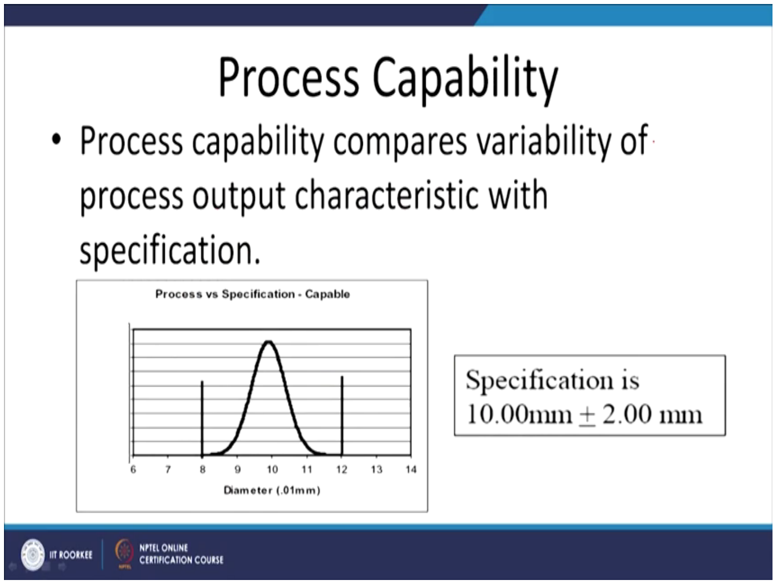

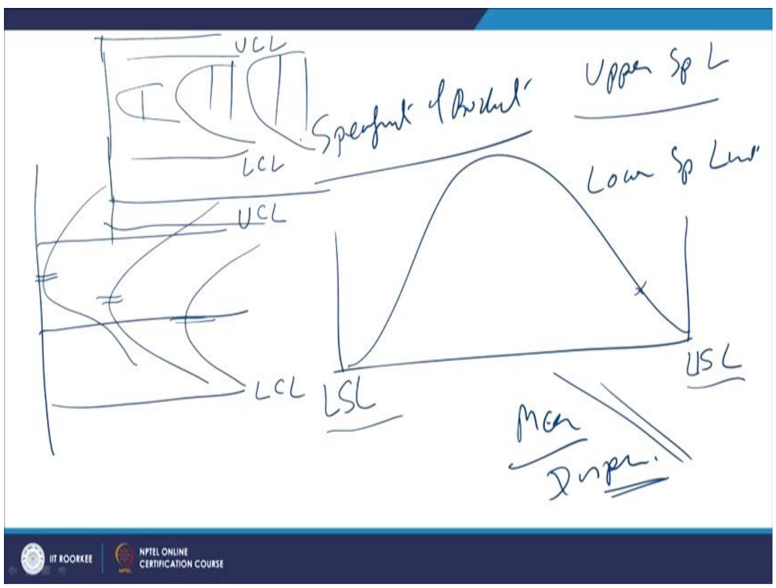

Another important aspect is the process capability. As I have said the specification of product characteristic quality characteristics related with the product is indicated with the help of some upper specification limit and the lower specification limit. So, there will be certain higher value which will be acceptable.

And there will be certain lower value which will also be acceptable and beyond these higher and lower values, say upper specification limit and lower specification limit and whatever we manufacture despite of all averages and dispersions the values of the all products which are being values of the quality characteristics of the products, which are being manufactured, those must fall here within these two limits upper and lower specification limit.

Because, if the quality characteristic value is falling within these two limits, then the product will be acceptable. So, we need to see both mean and the dispersion or the deviation which is there in during the manufacturing of a product to make an effective assessment, if the given process will be capable to manufacture the product of given specification or not all there will be possibility of may manufacturing the products which are beyond the specification limits.

So, how to make that inference about, let us say, so if the process upper and lower control limits upper control limit and lower control limits are these, so the process may be making most of the units with the variability like this, where most of the things are being manufactured say there is a process average, this is the shifting process average. Here, what we can see and the dispersion is still same, there is another possibility where the upper and lower control limits and the dispersion is extremely close or dispersion is increasing like this.

But still, if you see upper if the up so, these are the like a lower control limits upper control limit and if the upper specification limit is further away from, then its fine. So, all these the mean shift or the shift or the change in the dispersion or the standard deviation related with the quality characteristic that will not adversely affect the things and whatever will be manufactured that will be in the acceptable limit.

So, to see if the process is capable to manufacture capable to manufacture certain product or not we try to check the capability of the process.

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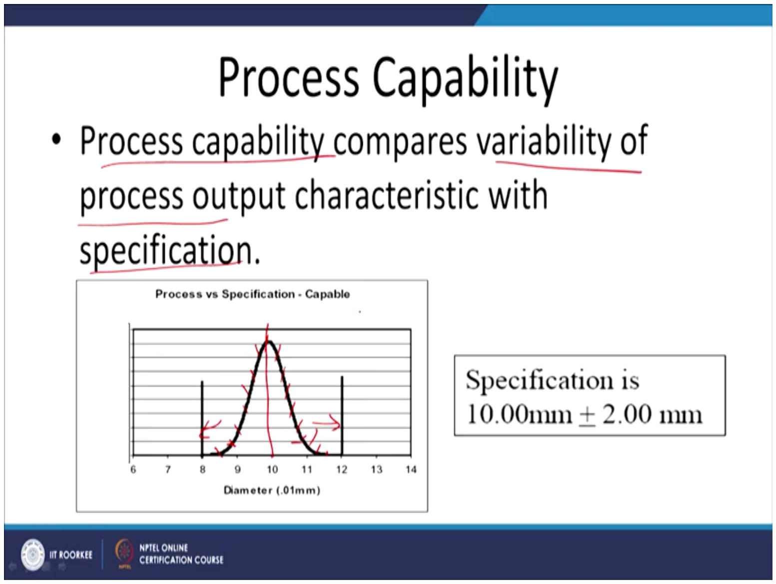

So, the process capability compares the variability of the process output characteristics with the specification. So, if the process is a manufacturing, the product having the quality characteristics are spread like this. Where there somewhere there is a average and then dispersion is aspect like this and the upper and lower specification limits are set further at the higher side, then this will suggest that the process will be capable to process will be capable to manufacture the products within of the acceptable specification limits.

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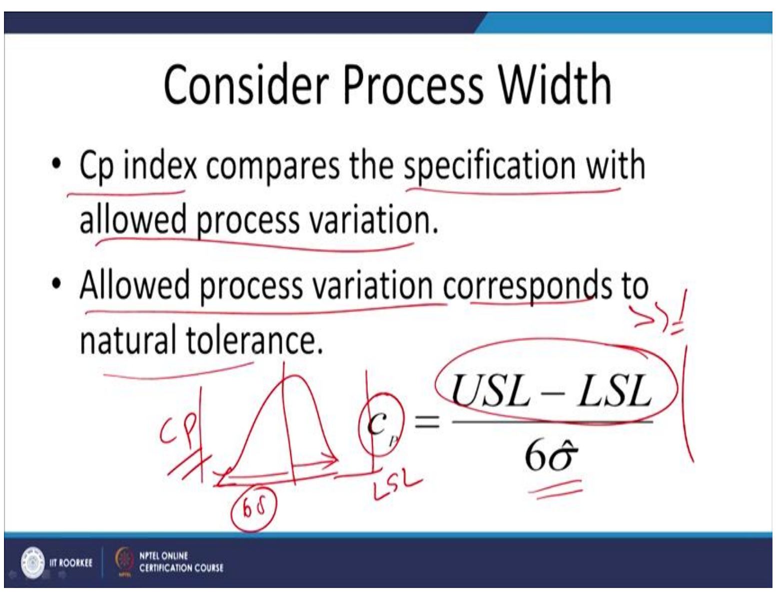

But, if the if the process average is shifting either on the higher side or on the lower side, then to make sure that is still despite of shift in average if the product will be capability to manufacture the things within the specification limits or not, that is what is checked using the different equations. So, CP is the one parameter that is a process capability index CP this compares the specific compare the specification with the allowed process variation.

So, this is the specification, upper difference of upper and lower specification and divided by the process allowed process variation which is seen from the like say 3 sigma, higher side 3 sigma lower side. So, it is 6 sigma and upper specification upper and lower specification limits may be further higher. So, if this value of CP is greater than 1, then we assume that the product or the process will be able to manufacture the things will as per the specified specification limit. So, allowed process variation corresponds to the natural tolerance and in that case you see the process will be capable to do will be capable to manufacture the things.

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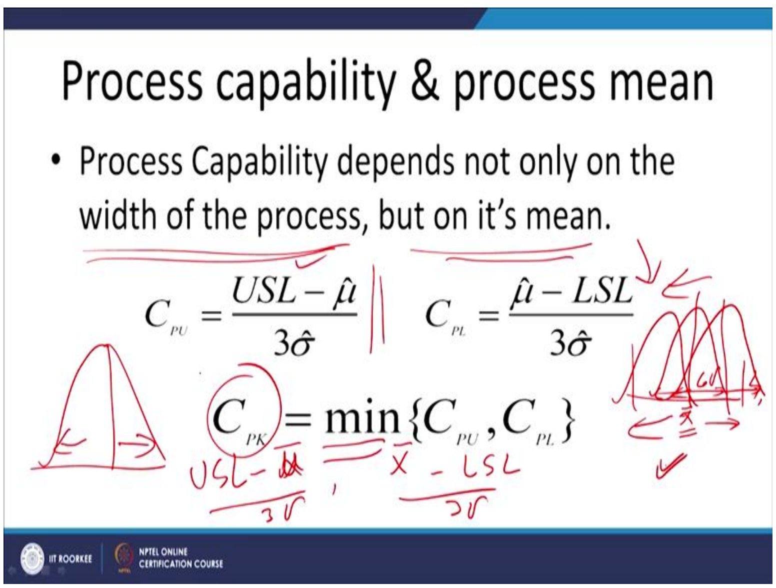

So, process capability depends not only on the width the process but also its mean sometimes what we see that the process variation is like this and everything is fine, but that is true only up to a limit. Like say, this is the process average and this is this dispersion of the 6 sigma, but if the process average is shifted for the same 6 Sigma, if the process average has shifted either on the higher side or in the lower side, then despite of the same sigma 6 sigma value of the dispersion or variability, we may see that some of the things are being manufactured beyond the upper specification limit or beyond the lower specification limits.

So, to see even if there is if there is a shift in the mean. So, then to see if the things can be manufactured within the process will be capability to manufacture the things as per the specification or not. So to check that the process capability does not depend only on the width of the process that is 6 sigma, but also on its mean.

So, to consider this aspect the process capability is calculated little differently where we try to see if the process average value like say upper specification limit minus the process average mu or the x bar divided by 3 sigma. So, this is one thing and another thing is this. So, this is considering the upper specification limit, likewise x bar minus lower specification limit divided by 3 sigma.

So, minimum of these two is another way to consider the process capability and this will ensure that if the process will be capability to will be capable to manufacture the certain products within the specification limits or not despite of shift in its mean.

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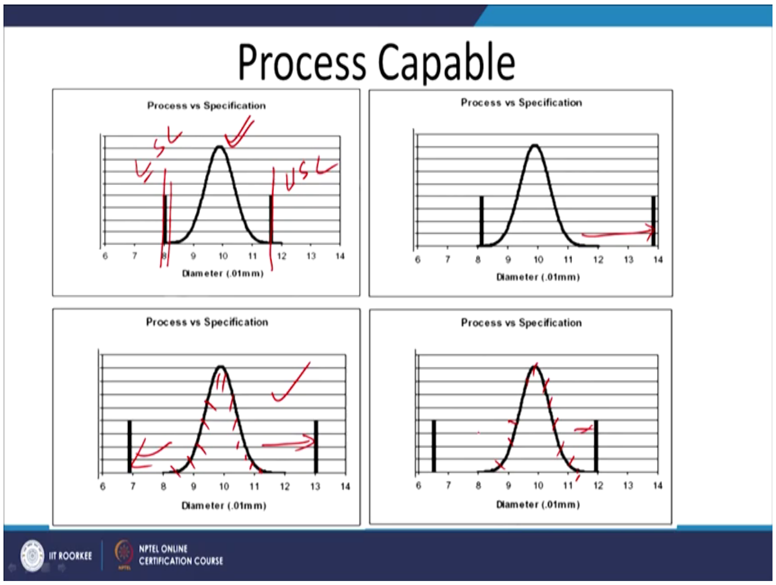

So, these are a few examples where it has been shown whether the process is capable to manufacture certain products within the specification limits or not. So, here specification limits are very close to the process variability. So, here but is still everything whatever will be produced by the process that will be within the upper, upper specification and lower specification limits.

In this case, the upper specification limit is far away from the maximum variability of the process. And upper and lower specification limits are far away with the kind of the characteristics which will be offered by the product. So, in all these things cases we will see that the whatever variation in the characteristics of the product are taking place those are within the acceptable limits.

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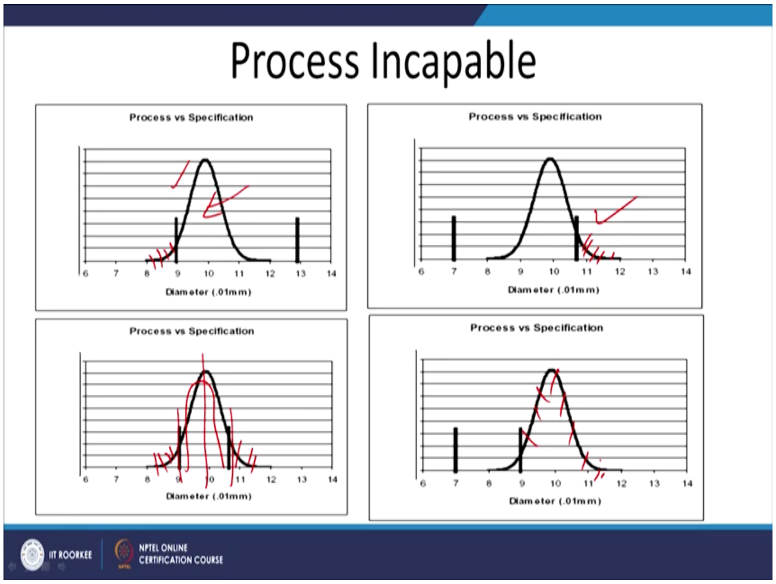

But we may have a situation variability is same, but if the process mean has shifted towards the lower side, then we will see some other things are being manufactured beyond the lower specification limit in this case some other things are being manufactured beyond the upper specification limits. So, this is happening because of the shift or the process is a centered in such a way process is centered close to the upper specification limit or close to the lower specification limit or here the specifications are very especially upper and lower specification limits are very close to the average and it the process needs very tight and very close control.

So, if the process is controlled like this, then everything will be within the control, otherwise we will see that few things are being manufactured beyond the lower specification limits as well as upper specification limits and as a fore case, where most of the things will be manufactured and in form of the products having the specifications beyond the upper specification limit.

So, these are the different examples showing where in the process can produce a number of things beyond the specification limits. So, the process will be considered not capability capable to manufacture such kind of the products where specification limits are such that the process is not able to manufacture those products or the process needs to be properly control. So that the products of the required specifications can be produced.

Now I will summarize this presentation. In this presentation basically I have talked about the methodology for developing the control charts mean control charts and R control charts with a few examples, and I have also talked about the idea of the process capability. Thank you for your attention.