Principles of Industrial Engineering
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Lecture 46
Network Analysis: Critical Path Method

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering. And you know we are talking about the project management technique that is called the Network Analysis using the two techniques. We have talked at length about the network analysis using the project evaluation and a review technique, in short it is termed as P E R T we have seen that P E R T is a probabilistic approach where we do not know very precisely how many much time an activity will take to complete the entire project.

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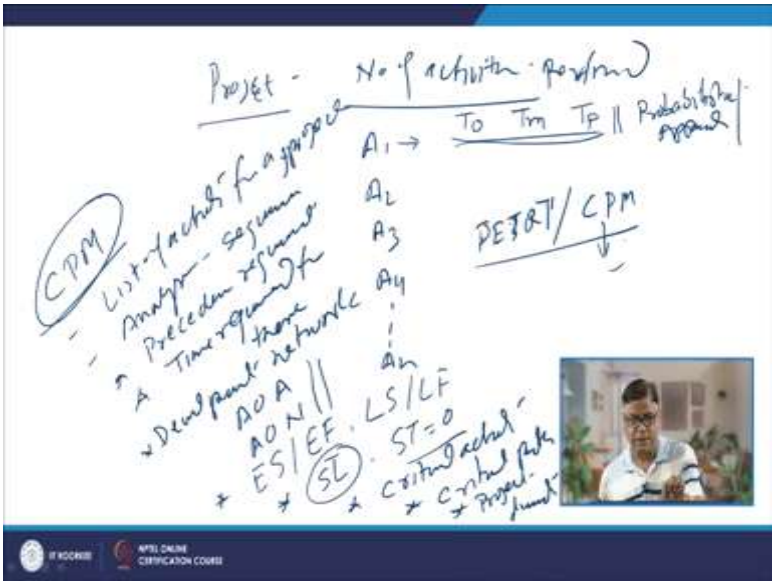

So, you know the project is about the series of activities, number of activities performed in a particular sequence, and once all those activities are completed our project is completed. But in this case say there are a number of activities which are to be performed like say this way.

And but in this case we know the kind of activity to be done but how much time precisely it will take that we do not know and that is why we try to determine or try to estimate the optimistic time, most likely time and pessimistic time. And using these, we try to determine the kind of the average time or meantime an activity will take to complete.

So, this is more like a probabilistic approach and it uses the like it determines the probability of completing the project in a particular time period. So, again apart from this aspect, rest of

the things are same in case of the critical path method also like in critical path method, we prepare a list of the activities to be done for a project.

Then we analyse all those activities, analysis is done primarily to see what will be the sequence of the activities in which these are to be performed and that will help in establishing any precedence requirement, or some of the activities if some of the activities are to be done before others can be taken up.

So, this analysis will indicate the kind of the precedence requirement and will also indicate the kind of time required for these activities. And thereafter, we have to develop the network using this considering the activities to be performed and the time required for performing those activities, the suitable network is developed either activity on Arrow or activity on node approaches.

So, mostly we have used so far the activity on Arrow approach, and here also we will be using the same in network analysis using the critical path method. Once the network is developed, we will be trying to determine the earliest start earliest finish time and the latest start and latest finish time like what we have done earlier in case of the P E R T approach.

And based on this, we will be determining the slack time for each of the activity, and for those activities in which slack time is 0, those are considered to be the critical activities. So once the critical activities are identified in a network, we will be able to have the critical path, and once the critical path is determined, identified, then summing up the time required for completing the various activities falling on the critical path will give us the project duration, the time required to complete the project period.

So, the difference between the P E R T and the CPM is very narrow that is particularly with regard to the deterministic or probabilistic approach. In CPM approach we know with the precision or we know that this is the time a particular activity will take to complete. So, this is not probabilistic in that way and we simply determine the critical path critical activities and the time required to complete the project.

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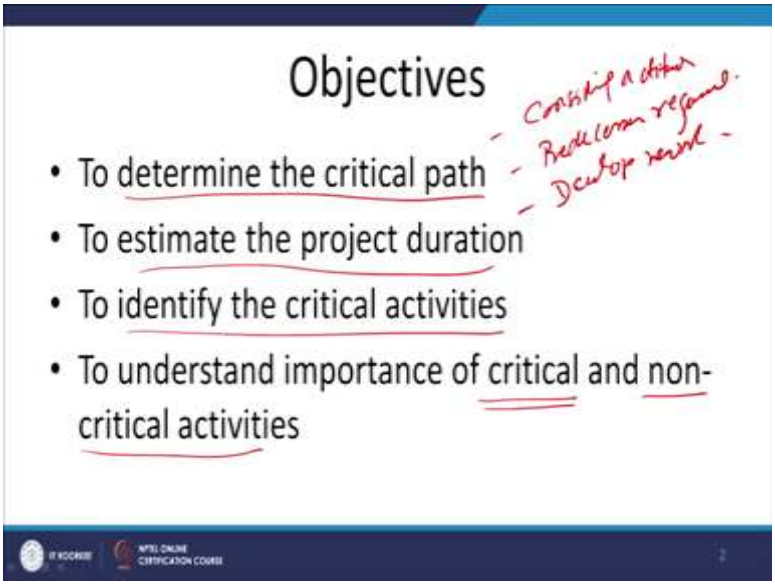

So considering this in mind, these are the various things that will be covered in case of the CPM determining the critical path, which will help in estimating the project duration. And this determination of critical path will also help in establishing or identifying the critical activities.

And this will also help in what are the activities which must be given enough attention so that they are completed in the specified or stipulated time period and there are many activities where in there is some flexibility to delay in completion of those activities without delaying the project period.

So, as I have said, the procedure for CPM is similar to that of the P E R T where in considering activities and the predecessor or precedence requirement, we have to develop a network, this is the first step. So we will see the kind of the activity say for a, in a particular case related with the project.

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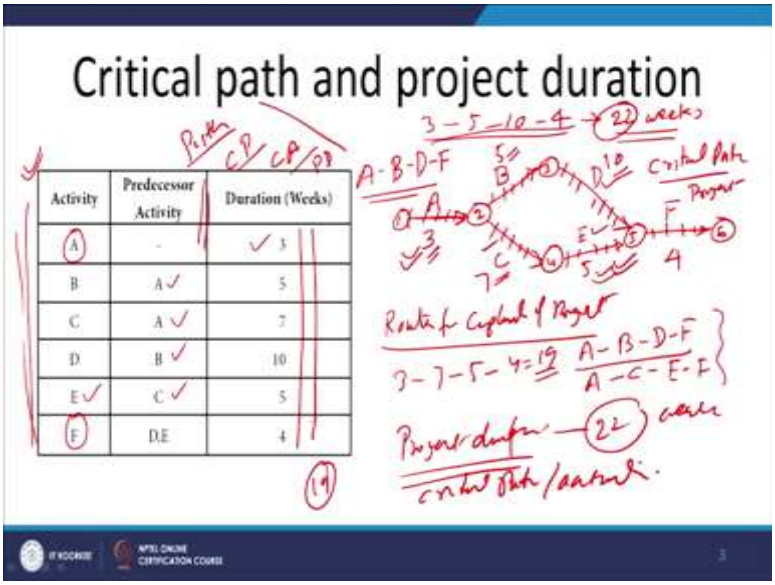

The activities to be performed are 6, activity A is the first activity and it has no predecessor activity so it can be started at any time. Thereafter activity B, activity B and C can be done only after the completion of activity A. So means once the activity A is completed, say this is the arrow for activity A and node 1 and 2.

So, once the activity A is completed then we can take up both the activities in parallel also. So, these are the concurrent activities which can be started in parallel. So, activity B and activity C thereafter we take up. So say here, this is the node 3 and node 4.

Since the predecessor's for activity B and C is the completion of the activity A so we have started both activities at the same time after the completion of A. And activity D can be started only after the completion of the B, so now we have to see that activity D is taken up after the completion of the B, so this will be the node 5.

And similarly, activity E can be taken up only after the completion of the activity C, so now we will take of the activity E after the completion of the activity A. So, this is how we have completed 5 activities. Now, if we see the activity F, activity F can be completed only after the completion of the D and E. So, since now we have completed D and E both activities so thereafter we can take up that last activity that is F.

So, this will be the node 6 and this will be indicating the activity F, this is how network can be developed say the time also is given for completing these activities with the precision, there is no optimistic or most likely time values, but the time required to complete these activities is known with the precision and in very confirm way.

So here activity A takes 3 weeks, activity B takes 5 weeks, activity c takes 7 weeks, activity D takes 10 weeks and activity E takes 5 weeks and then activity F takes 4 weeks. So, if we see, these are the various activities those need to be completed. Now there is another aspect, the routes for completion of the projects, while considering that all activities need to be finished.

So, if we see there are basically two paths, path A 1 to 2 for activity A then another activity B falling between the nodes 2 and 3. So, A, B, D and F this is one path, so if we say A, B, D and F is one path, and there is another path in process of completion of this project, that is activity A then activity C, activity E and activity F, so, A, C, E and F.

So there are two paths in this network, we have to see which path is critical and most important so, the activities on that path can be focused effectively for completion of the project within the minimum possible time period. So, we will consider the time required for completing both these paths separately.

So if we just sum up the time required for completing the activities on each of these two paths, so activity A takes say 3 weeks, activity B takes 5 weeks, activity D takes 10 weeks and activity F takes 4 weeks. So the time required to complete the activities on the path A, B, D, F, is like say 14, 19 and 22.

So 22 weeks, so 22 weeks is the time for completing the activities on one path that is A, B, D and F. And there is another path A, C, E and F. So far, A again 3 weeks, for C it is taking 7 weeks, for E it is taking 5 weeks and for F the same 4 weeks. So, the time required to complete the activities on another path that is A, C, E and F is say 19 weeks. So, since for completing any project it is required that all project activities are completed.

So, we have to choose the path which takes the maximum time because before that the project cannot be completed. So, if we see that then the path A, B, D and F is the path which takes the longest time among these two. So, we will consider that the path A, B, D is the critical path and it will be giving the project duration.

So, the activities falling on this critical path that is A, B, D becomes the critical activities and these activities must be completed in the time given or allotted for these activities like say A activity must be completed in 3 weeks, B must be completed in 5, then similarly D should be completed in 10 weeks and F in 4 weeks.

So, if any delay takes place in completion of any of these activities, then the project period is delayed from the 22 weeks. So, considering this we can determine the project duration, project duration for this kind of combination of the activities and the time periods is the 22 weeks and we have also determined the kind of the critical path and the critical activities.

Other activities like activity C and activity E which are not on critical path, they even if they are delayed by little amount of the time delayed by 1 week or 2 weeks or so, it will not be affecting the project completion period because the time required for completing the activities on A, C, E and F path is lesser than the project duration that is the 19 days.

So, there is a slackness of the 3 weeks, so the in combination activity C and E can be delayed by some time without delaying the project period. So since with these activities, there is some slack time available that is why these are the non-critical activities, how to determine the slack time that we will see again coming examples.

So, this is how we can develop the network and we can identify the different paths, we can identify the critical path and critical activities and the project duration. If set of the activities with their predecessor requirement and the time required for completing those activities is given.

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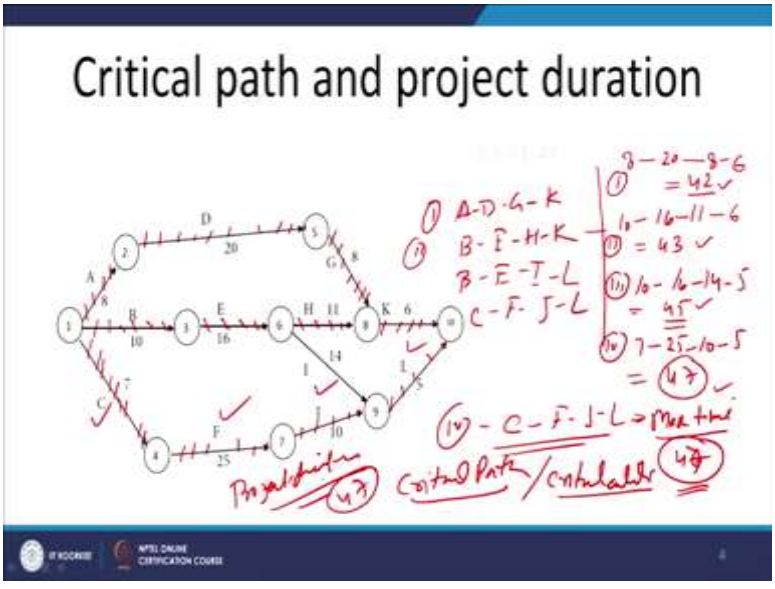

Now, so this is just for another example like, this is more complicated network having the number of paths, like say the path A there is one path is this having the activity A, D, G and K. Another path is B, E, H, K, so another path is B, E, H, K then third path is again B, E then I, L, then there is one more path that is C, F, J, L so these are the, say there are 4 paths.

So, we will be trying to determine the time required for completing the activities on these 4 paths. So, like say the time required to complete the activities on first path A, D, G and K is like say A takes 8, then D takes 20, then G takes 7 and K takes 6, the time period, so the total here is coming 28, 16, 6, 24, so 42 for the first path.

Then second path is B, E, H, K so, for B it is 10, for E it is 16, for H it is 11 and for K it is 6, so the total is coming like say (6, 10 is 16, 32) 43, so this is for the second path is 43. Third path is B, E, B 10, E 16 then I is 14 and L is 5. So, here how much time it takes? Say(16, 14, 13 and 40 and 5) 45 is the time period for third path.

And for fourth path, the activities falling on the C, F, J and L, so for C 7, F 25, J 10 and for L it is 5. So this total will be giving us 35 plus 12 is 47. So if we see there are 4 paths and the maximum time, and the time to complete each of the path is coming like for first path is 44, then 43, 45 and 47. So, the fourth path having the activities C-F-G and L, this is the path which takes the maximum time to complete the activities on this path that is of the 47 time period.

So this path becomes the critical path and activities falling on this path becomes the critical activities. So, any delay on the completion of activities C, F, J and L will delay the project completion period from the 47 weeks. So, the project completion period or the project duration by which it can be completed if the project activities falling on the critical path are completed on the stipulated period then project can be completed in 47 time periods.

So, is there another case wherein there are a large number of activities many paths, so we can identify the critical path as well as the critical activities and determine the project duration.

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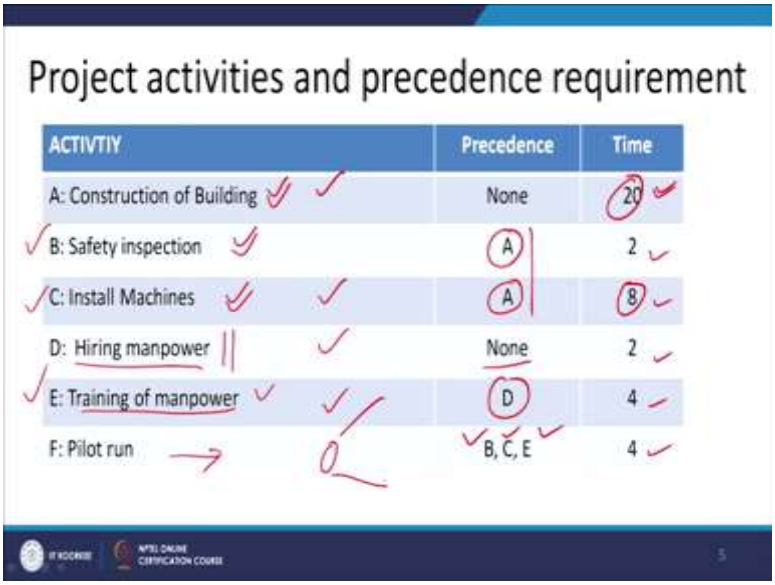

Let us say, this is the another example related with the construction related to the development of one particular hotel. We are in it requires first activity A related with the construction of building, then safety inspection of the building, but that can be done only after the completion of the building.

Then installing the machines ofcourse, you need to have a buildings where machines can be installed so, the predecessor requirement for B and C both is like say A that is a construction of building. Hiring of manpower does not require completion of the construction of the building it is independent activity so there is no predecessor requirement for this activity.

Similarly, the training of the manpower can be done only if they are hired. So, the predecessor requirement for activity E is the completion of the D that is the hiring of the manpower. And once we have got the building, once machines are installed, safety inspection is over, manpower hired and all these activities are done, we can go for pilot run.

So for having the pilot run, we must complete the activity B, C and E means the manpower must be trained, the machines must be installed and the safety inspection has to be completed. So, now and this is the time values are which are required for completing these various activities like say 22, 8, 2, 4 and 4, for these various activities A to F respectively.

So, the maximum activity, maximum time is taken by the activity A that is construction of building and then installing machines say 8 weeks. Now we need to develop the network first considering this predecessor requirement. Since activity A and C, these two activities do not

have any predecessor so both can be started at the same time like this. And thereafter we will see as per the predecessor requirement we will be going ahead.

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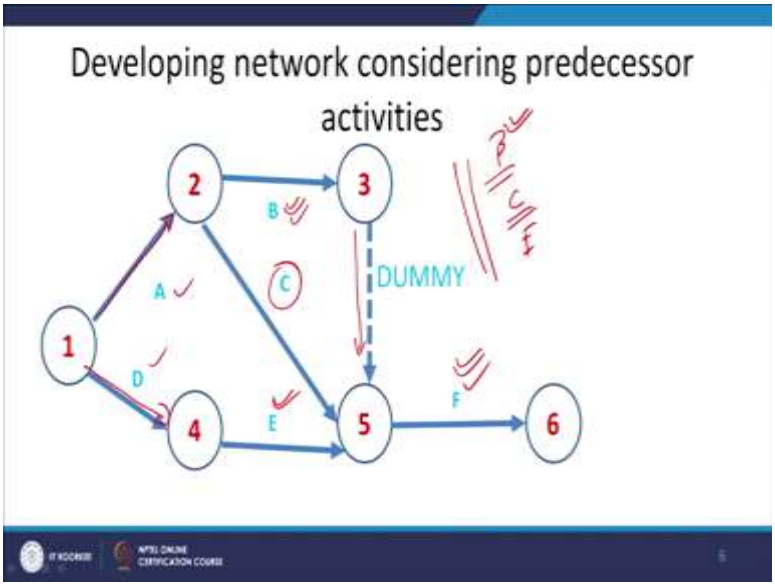

So, since A and D, these two activities that is hiring of the manpower and the construction of building these two activities do not have any predecessor requirements so we can start these two in parallel so these are the concurrent activities. And after the completion of A there will be the safety inspection, means after construction of the building safety inspection B and then activity C, once the building is completed, also the machines can be installed.

Similarly, after hiring of the manpower, their training is possible, so after E is possible only after the completion of the D. We have seen that for doing the pilot run, it is important that the inspection of the building under the activity B is done, the machines are installed and the manpower is trained according to the activity E so all these must be completed.

So to show certainly at the node 5, the presence of the activity C and E indicates that these must be completed, but to show the completion requirement of the activity B also, it is required that one dummy activity is shown which will be connecting the node 3 to 5 to show the predecessor requirement of B before taking up the activity F. So, activity F can be taken only after the completion of the B, C and E is done that is why we need this dummy activity.

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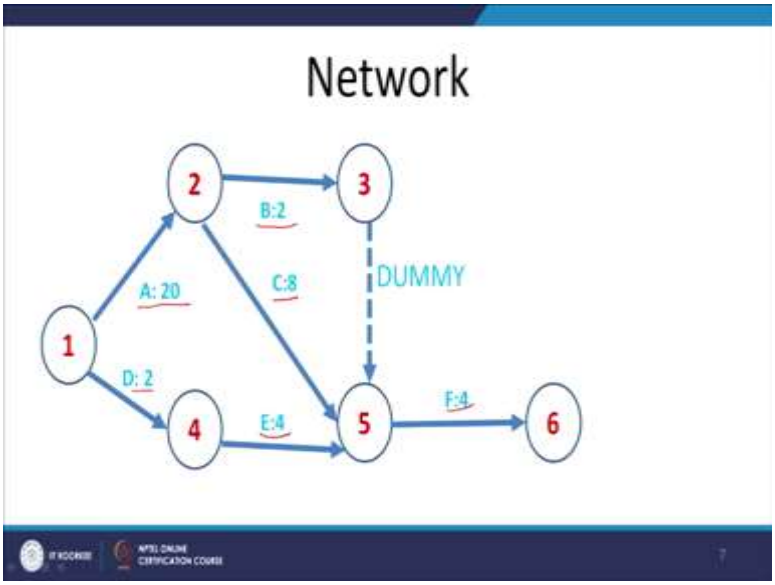

Now we will see, in this network what it shows the different time values. So, this is the same network what we have done is, we have shown the time values 20 say for completion of the activity A, 2 for activity D, 2 time value for activity B, 8 for activity C and 4 for activity E and then 4 for activity F. Now we will see how to determine the earliest start earliest finish and latest start latest finish time.

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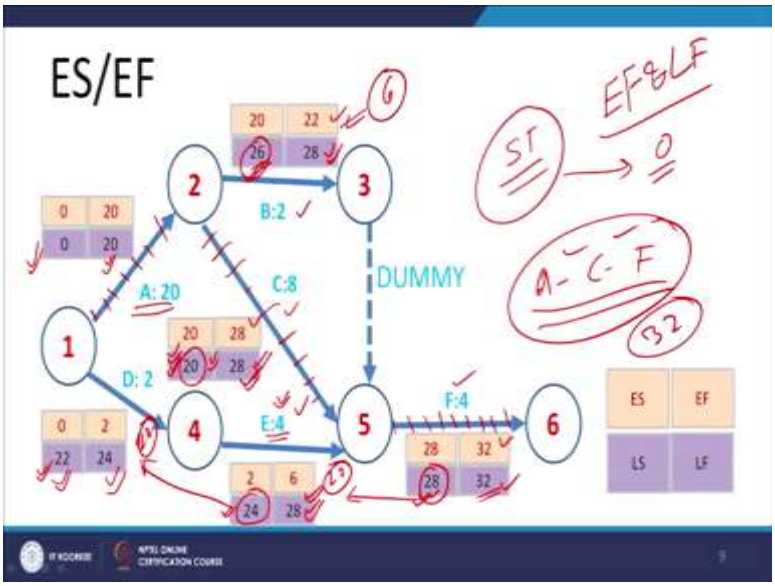

Since the activity A and D these two activities do not have any predecessor requirements, so, they can be started immediately without any delay. So, earliest start is 0 for both these activities. Since activity a needs 20 a time period value before that it can be finished so the earliest finish is the 20 weeks.

Similarly, the earliest finish for the activity D is also 2 weeks since it is a 2 week is the time or 2 time period is a time value which is required for completing the activity D. So after the 20 time period value only the activity B and C can be started. So earliest start for activity B is the 20 since the time period requires for completion of the B is the 2 weeks only, so 22 becomes the earliest finish time for the B activity.

Similarly, the activity C can be started only after the completion of A and so, earliest start for that activity C is the 20 weeks. It is this, since it takes 8 time values, 8 time period so the earliest finish possible is the 28. Here, then we will take up the activity E, activity E earliest start is possible only after the completion of the activity D so, earliest finish time for activity D is 2 weeks.

So earliest start time for activity E will be the 2 since it takes the 4 time period value so it takes the earliest finish for the activity is 6. So, we have earliest finish for activity B is 22, for C is 28, and for E is 6. Now, we know that after the dummy activity does not take any resource and time, it is just used to show the predecessor requirement. Since the activity F can be started only after the completion of the activity B, C and E.

So, we have to see which activity takes the maximum time to complete before that activity F can be started. So, earliest finish time maximum of the earliest finish time of the B, C and E is to be considered, maximum of the earliest finish time. So, earliest finish time for B is 22, earliest finish time is 28 for C, and for E earliest finish time is 6.

So, the maximum of the earliest finish time among these 3 activities is of the C, so this becomes the earliest start time for the activity F that is 28. And since it takes the 4 time periods, so it can be completed in earliest possible is 32 weeks. Now we will be going for the back calculations, so to finish the activity in the 32 weeks, so this becomes the latest finish time 32 weeks.

So here, 32 weeks is the latest finish time, so to finish this earliest, the latest start will be the 28, so what is the latest time for the activity E. So, activity by latest must be finished before the start of this following activity that is F. So, the 22 actually becomes the latest finish time for the activity E.

Similarly, the latest finish time for the activity C also is 28 weeks and latest finish time is also 28 time period for the activity B. So because before that in any case, the activity B-C and E must be completed so that activity F can be started. Since the latest finish time is 28 weeks so

latest start possible for the activity B is 2 less than the 28 that becomes 26. Similarly, 8 less than the 28 that is the latest finish time so latest start time will be the 20. And similarly here the latest finish time for the activity E is 28 so latest start possible is 24 means 28 minus 4 that will be giving us 24. So considering this, latest start time latest finish time for the activity D will be 24.

And the since the time required to do this activity is 2, so the latest start time is 22. So now, likewise we will be going forces in any case we need to finish the activity B by 22 weeks, (22 times) 26 time period, activity C is to be finished by 20 time period. And activity D has to be finished by 22 periods so the minimum of these two will be taken to C by what will be the latest time for start of this activity.

So among these two, so there is a simple comparison of activity B and C. So minimum of these two is what? 22 is the latest start for B activity, and 20 is the latest start for the activity C, so minimum of these two is considered so that will have the latest finish time for the activity A. Since by latest it has the activity has C has to be started by 20 weeks so at 20 time periods the activity A must be finished.

Since the activity time required for completion of the activity A is 20, so the earliest latest start possible is 0. Basically, we have to determine the slack time and that will be gained by the difference of the earliest start and latest start, when the value of the slack time that is the earliest start and the latest start difference of these sorry, earliest finish and latest finish, when the difference of these two is 0 that means slack time is 0 then those activities where slack time is 0, the activity becomes the critical activity.

So, here the latest finish and earliest finish difference is 0, latest finish and earliest finish difference is 0 for activity C. Similarly, for activity A difference of the latest finish and earliest finish is again 0. So the activity A, C and F becomes the critical activities, while there is a difference of 6 here in this case difference of 22 and in this case difference of also 22.

So, there is a huge slack available, so now what we have to do so, this is how we can determine, this is the critical path and the project completion time. These are the various activities on the critical path and the project completion time is 32 time period.

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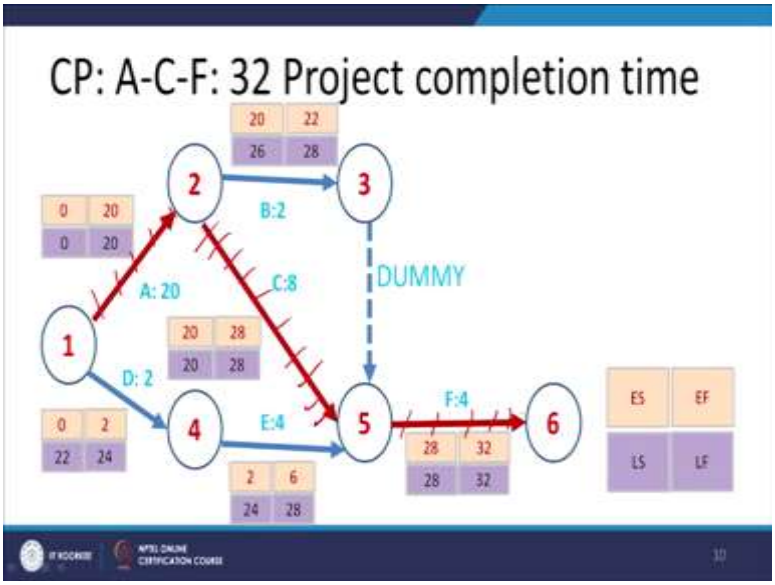

So, this is how we can so the activities on the critical path are highlighted in red and it will be giving us the critical activities and the project completion time. So A, C, F is the project critical path and the 32 time period is the project completion time. Now, I will summarize this presentation. In this presentation basically, I have talked about the critical path method, where in we know precisely the time required to complete the various activities.

So how to develop a network, how to identify the critical path, how to identify the critical activities and how to determine the time required to complete the project in using the critical path method, thank you for your attention.