Principles of Industrial Engineering
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Lecture - 45
Network Analysis: Crashing Network and CPM

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering. And you know we are talking about the network analysis, wherein we have talked about the program evaluation and review technique. In this presentation we will be talking about the two aspects, one is about the crashing the network and another is critical path method. So initially, we will start with the crashing of network which can be applied for both the approaches of the network analysis, that is the PERT or CPM.

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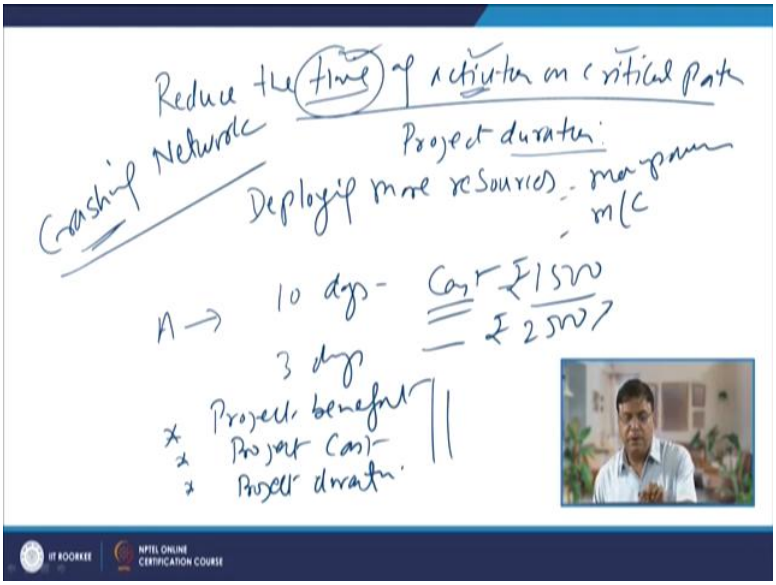

So, the crashing of the network, primary objective of crashing the network is to reduce the time of activities on critical path, because this is the, these are the things that dictate the minimum time a project will take to complete. So, project duration is directly affected by the time to complete the activities on critical path. So, through the crashing of network, efforts are made to reduce the time of activities falling on the critical path.

And this is done by deploying more resources, like say we will be providing more manpower to complete the work earlier or we will be providing the more machines or whatever additional resource needed to complete the activities earlier, those will be provided. And this concept is very effectively used in number of situations, like a job A is to be completed in 10 days, then, it costs less. Like say in passports if the time to complete a passport or get a

passport they may charge like say 1500 rupees, but if you want the passport in 3 days, then they may charge like 2500 rupees.

So, if we want that the things are done in very less time, then we need to pay more. So, that higher cost takes into account in crashing the network. So, whatever activities need to be done to complete a project, we need to deploy more resources, we need to give the special attention and that costs more. So, there may be various objectives of crashing the network, so which includes like for business interests, like we may earn more if we are able to provide the services in less time.

So, like say the project benefits are realized through this, apart from like reducing the project cost also sometimes help to the crashing of network, then it helps in reducing the project duration. So, these are some of the like say secondary benefits which are realized when the time to complete the activities on critical path is reduced. So, once the time is reduced, then the project duration is reduced and that will help in completing the job earlier.

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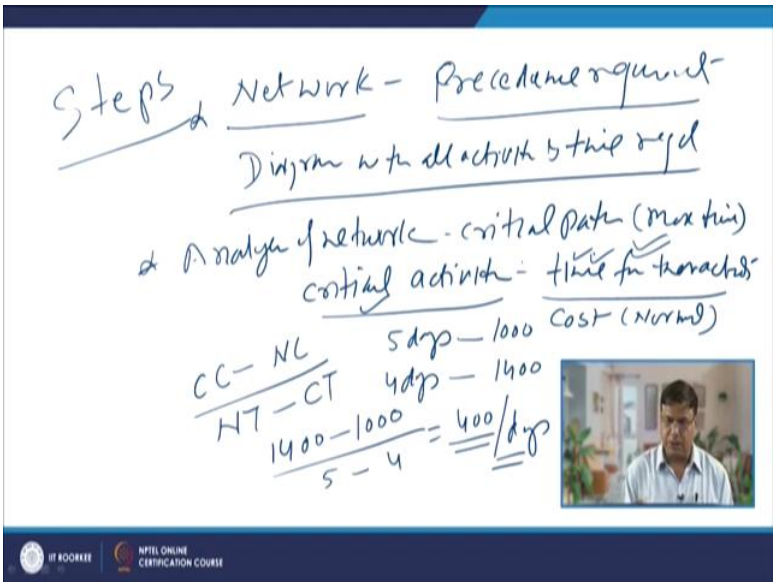

So, now, we have seen what will be the focus and how it is done. So, as far as the steps of crashing the network is concerned, in any case, we have to develop the network considering the precedence requirement for the different activities required to complete the project, considering the precedence requirement a network is developed. So, it is basically diagram with all activities and the time required to complete those activities, that is what is mentioned.

And once it is mentioned, then the second step, after developing the network, second step is analysis of the network. So, network will be analyzed to see the critical path or the path of

network that takes the longest time or the maximum time. So, path of maximum time is identified and whatever activities falling on that critical path will be defined as a critical activities.

And we will be noting down the time required for those critical activities, time required to complete those critical activities. So, time for these activities is identified. So, there can be a situation where in normal course, when the critical activities are performed, it takes certain time, and there will be certain cost to do those activities in that time period. So, we can say it as a normal cost. When in normal course, that critical activities are performed, they take certain time and that will be costing by certain amount.

Say, if a certain activity is completed in 5 days and it costs 1000 rupees, if the same activity is completed in 4 days, and if it costs say 1400 rupees to complete the same activity in 4 day, then we will see that the crash cost minus the normal cost divided by the normal time minus the crash time. If we see these, then we will be able to identify how much crashing is costing us per day. So, considering this example, crash cost is 1400, normal cost is 100, sorry 1400 and normal cost is 10 hundred, that is 1000 normal time is 5 days and crash time is 4 days.

So, here rupees 400 per day is kind of the cost that is occurring on account of the crashing. So, this is one aspect. Crashing will always be costlier than the case when it is done in normal ways. So, identification of the critical activities, time required to complete those activities and cost per unit time for performing those activities in normal situation.

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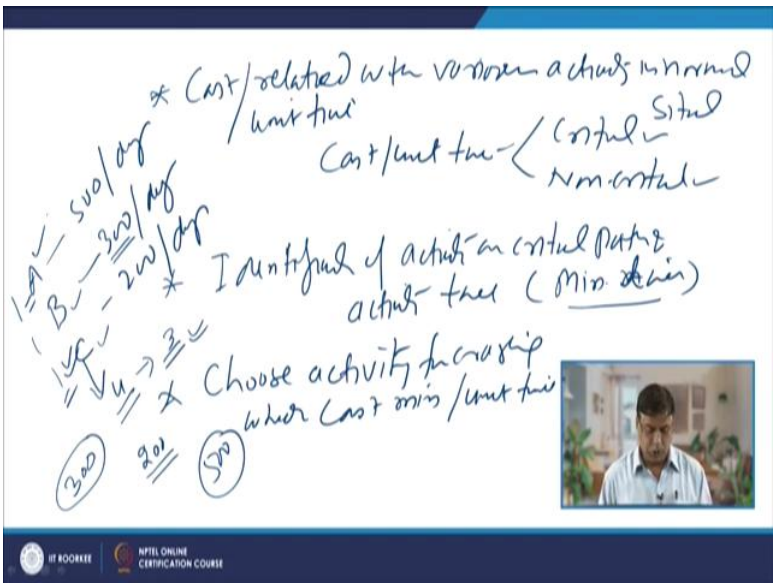

So, the third point is about the cost related with various activities in normal case, in normal situation. So, this is basically cost per unit time, it may be days or weeks or months like that. So, when we identify this, then we will be able to identify the cost per unit time for both, critical as well as non critical activities. Because later on on crashing the network we may find a situation where they maybe a change in critical path due to the reduction in time. So, the activities which were non critical earlier may become critical after crashing the activities on certain paths.

So, it is important that we consider the cost per unit time for both critical as well as non critical activities. So, the next is identification of the activities on critical path and the activities times, basically we focus, start our crashing from the activities that take minimum time to complete or the cost for crashing is minimum. So, these are the two aspects which are kept in mind.

So, identification of the activities on critical path, then choose the activity for crashing or for deployment of more resources so that the time required to complete those activities can be reduced at a minimum possible cost. So, choose those activities on critical path which cost minimum per unit time for crashing. Say there are three activities on critical path and the crash cost per unit time is 500 per day for activity A, 300 rupees for activity B per day and 200 rupees per day for activity C.

And so means reduction of the critical activity by 1 day for activity A costs 500, for activity B reduction in time by 1 day costs 300 extra and for C it costs 200 rupees. So, if there is a choice to, if there is a, if you have to choose that which activity will be crashed first to reduce the project duration to crash the network, then it will be activity C that will be chosen. So, once the activity C is chosen, so say how many days, what is the maximum possible crash.

Say activity C needs the 4 days, so how much crashing is possible. It is not possible like, it may not be possible like activity can be crashed from 4 days to 1 day. So, there may be the maximum possible crashing that will be possible. So, if it is possible that activity maximum can be crashed for from 4 to 3 days, then this will be a limit, this activity Can be crashed just by 1 day by paying extra, deploying more resources of 200.

Then we have to focus on the next activity falling on the critical path, but costing more than the activity which has already been crashed at the minimum possible price. So, the next higher activity, next activity costing higher is this activity B. So, this also if can be crashed by

1 day then we need to deploy the resources by putting in extra resources, say it will be costing at the rate of 300 rupees per day.

So, by deploying more resources, say by crashing activity B and C, we have reduced the project duration by 2 days and we spend extra 500 rupees, but the project duration was reduced by 2 days. If you want, if the target is to reduce it by 3 days or 4 days, then again we have to go for activity C or other activities falling in the, activity A and those activities which are falling in the critical path.

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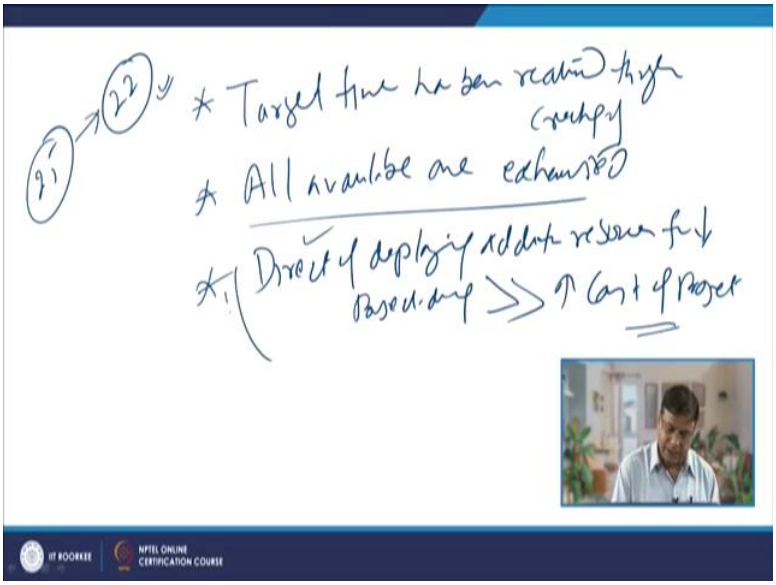

So, and this we will continue until, there is a limit up to which we can continue to crash the network. One situation is that whatever is the target time for which project duration is to be reduced is realized. So, the target time has been realized through the crashing of network. Say, there is a project, in a network the critical path takes 25 days and if we want to cut down it to up to the 22 days, so we will keep on crashing the different activities on the critical path until this 22 days time is realized.

So, this is one way, the second is that all available resources are exhausted. So, we are not left with the resources, all available resources are exhausted or we are not left with the resources for further crashing. And there is a third aspect that also matters a lot, that the direct cost of deploying additional resources for reducing the project duration becomes so high, that increases, it increases the cost of project extraordinarily are exorbitantly. So, significant increase in project cost may not be justifiable.

So, if the significant increase in project cost is taking place in the situation where direct cost for the deploying the additional resources to cut down the project time is so high that it is increasing the cost of projects significantly, then it will not be justifiable.

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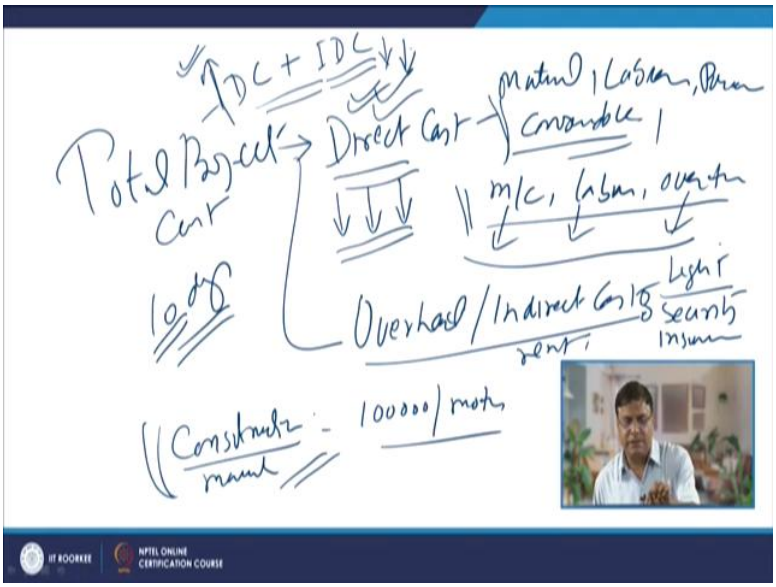

So, to understand this third aspect, we need to understand how the total project cost is influenced, the project total cost comprises basically the two components, one is direct cost which includes the, like say the kind of the material directly being used to make the component, the labor directly involved, the power which is being consumed and all consumables which are directly related with the production.

So, all the resources which are directly involved in making the things is about the direct costs. So, adding more machines or adding more labor or giving more time, like over-time to do or to produce more, all that will be falling in that direct cost case. So, when we have to deploy more we need to pay more for labor, material, machine over-time etc. But there will be a reasonable limit up to which the direct cost can be increased. But at the same time, the another factor which also affects the cost significantly is overhead or it is called indirect cost.

Indirect Cost basically, it is not related with the volume which is being produced or the number of units that we are producing, but we need to make some expenditure always. It may be in form of kind of the amount to be invested in lighting or in security, in insurance, the rent of building or continuing with the facility, just for example, like say the construction site maintaining a construction site per month say cost is 1 lakh rupees per month.

So, if we are, so that is the just to maintain, we can say this is the indirect cost and by putting in the additional resources if a particular construction, the job at the construction site can be finished just in 10 days, then it will be beneficial. So, whatever the time or whatever the resources that we need to cut down the time, so that we can close the facility at the earliest in order to save the indirect cost and that will directly be helping in to reduce the project cost.

Because by putting in more resources in form of direct cost, we are reducing the project duration and reducing the project duration will be helping to reduce the indirect cost as well. So, we need to see really the direct cost plus indirect cost. Initially increased indirect cost will help, may help in reducing the indirect cost. So, until this balance is within the range, it is good to deploy more resources, so that either the project cost is reduced through the reduction in project time.

And this increasing the direct cost or by deploying the more resources to cut down a project period will be justifiable until the indirect cost is more than the direct cost. If a deployment of more resources leading to the direct cost to such a high value, that we do not get much of the benefits in terms of the reduction in indirect cost, then the purpose of crashing with regard to the reducing total cost of project will be defeated. So, there that is the third situation when we stop the crashing of the network.

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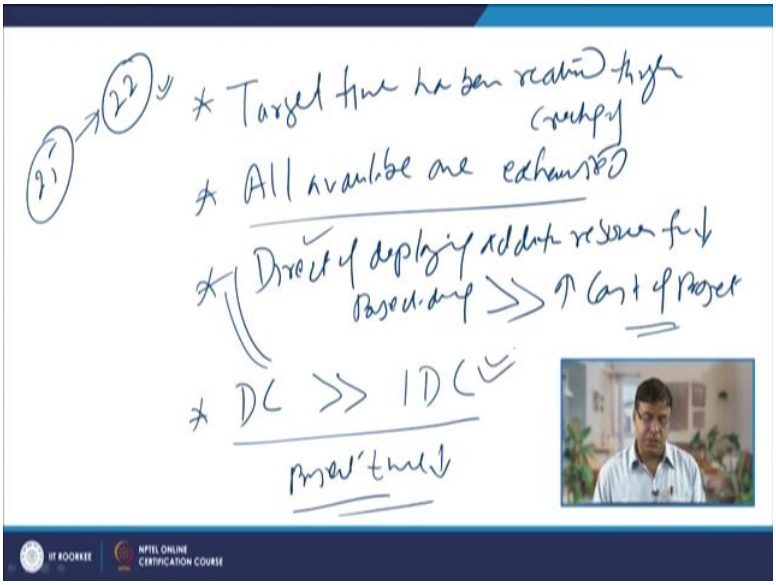

That is when the direct cost starts increasing the indirect, becomes greater than the indirect cost with respect to the time which is being reduced, the project duration which is being

reduced. So, when this situation arise, we normally try to stop the crashing of a network. So now we will see certain basic things as far as the crashing of the network is concerned.

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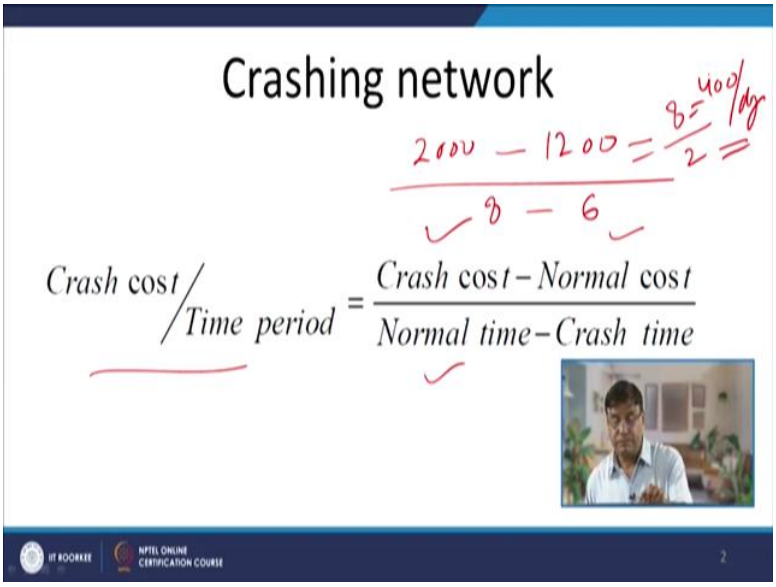

The crash cost per unit time, like how much we have, how much extra we have to invest for crashing the network, that is determined through this simple equation, the crash cost minus the normal cost divided by normal time minus crash time. So, crash cost will always be higher, say it is 2000 rupees, normal cost is say 1200 rupees, say normal time is 8 days and say crash time is 6 days. So, just by solving we can get the crash cost per unit time. So, it is 800 divided by 2, so in this case crash cost per unit time will be coming like rupees 400 per day, this is how it is determined.

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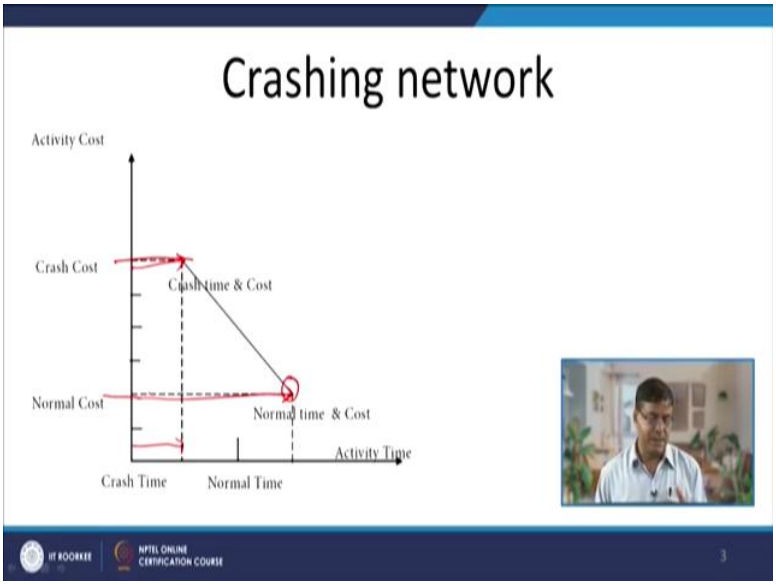

So, now this will be used for calculation purposes and the same has been used schematically. If this is the crash time, then the cost will be this much, if this is the normal time to do the job, then the cost will be certainly the lower. So, crash cost is higher, normal cost is lower. This indicates the kind of the normal cost versus the crash cost relationship will be.

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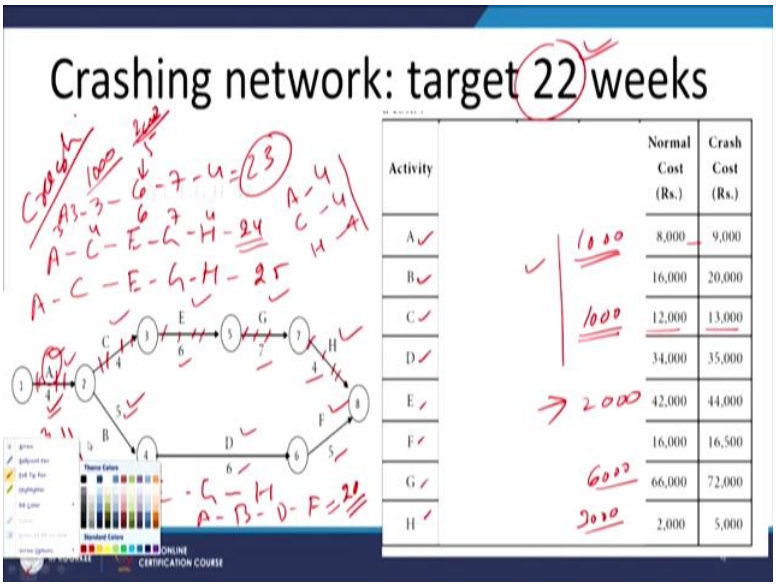

Just for an example, if we consider that there is a network and this network has been developed considering the precedence requirement. So, there are many various activities A, B, C, D, E, F, G, H and considering the precedence requirement, a suitable network has been developed here. So, activity A takes the 4 days, activity C takes the again 4 days, activity E, 6 days and activity G, 7 days, activity H, 4 days. On the other hand, other activities are like activity B, 5 days, activity D, 6 days and activity F, 5 days.

So, we need to see, since there are just 2 paths, path A is like A, C, E, G H, this is one path and another path is like that is simpler path A, B, D and F. So, these are the 2 paths A, B, D and F and A, C, E, G and H. So, these are the 2 paths. So, for both of these paths we will separately calculate the time required to complete the project, that how much time each path takes, and the path which takes the maximum time becomes the critical path.

So if we consider the first path, wherein we have activity A, C, E, G and H. So, on this path like A 4, then C 4, 8, E 6, that is about 14, then 7, 21 and 4, 25. So, A, C, E, G and H, it is the total path time is coming, path duration is coming 25 days. On the other hand, the activity A, B, D and F, so 4 plus 5, 9 plus 6, 15 and plus 5, 20. So, here for the A, B, D, F path, it is 20 days.

So, out of these 2 paths, if we see, even if we finish the activities on path A, B, D, F, the project will not be completed because still we have the many activities left to be completed. So, activity, so the path A, C, E, G and H becomes the critical path. So, this is the critical path and it is taking how much time, 25 days. And if our target is to complete the project is 22 weeks, then we need to see that we have to crash the network, means some activity, time for certain activities on the critical path has to be reduced.

So, if we see which activity, there are many activities like A, C, E, G and H, so we need to see really which activity takes minimum time. So, there are three activities which are taking 4 days A, 4, C, 4 and H, 4. So, the activities that need minimum time they are crashed first, and thereafter we will see out of these three, which activity is costing least. So, if the activity which is costing least will be taken up first for crashing. So, if activity A, let us say the crash cost, normal cost is 8 and crash cost is 9000, so the difference is 1000.

And then for activity B, for the same period like say activity A, for same period crash cost is 9000. For activity C, which also takes the 4 days crash cost data is 12,000 or 13,000 and normal cost is 12,000. So, this also cost extra by 1000. Then we have the another activity that we can consider is H. So, H is like activity H here, it is costing, like H is costing 5000 on crashing and 2000 in normal cost, so it is costing 3000 rupees.

So, now, we will see that which activity we can take first, since the time for. So, out of these two the activity A and C are costing least on the network. Another activity if we see, activity E and G cost of crashing, activity E is costing how much the difference is 2000 and activity G is costing, yeah, there is a difference of like say 6000. So, what we need to do, we need to target first the activity A and C to crash the network.

So, when we deploy more resources for cutting down the time by per day, so here we will say out of A and C, we can take any activity. So, if we crash first activity A, so this will be reduced to 3 days by deploying more resources. In that case, the project duration since rest of the activities are same. So, we have just acted on the activity A, which means activity A now has been instead of 4, now it has become of 3 days. Then activity C still of 4 days.

Activity E still of 6 days, activity G of 7 days, activity H is still have 4 days, but some of these will be leading to the reduction of the critical path by 1 day that becomes 24. So, 6, 4, 10, 13, 20, 24. So, the project duration has been reduced by 1 day by crushing the activity A.

Likewise, now next activity that we can crash at the minimum price of 1000 per day is like C. So, that is the C activity, if we reduce the time required to cut the activity C by 1 day.

So, by putting an extra cost earlier paid 1000 extra for reducing the time of activity A, now we will be paying extra for reducing the time of activity C. So, activity C now becomes of the 3 days. So, 3, for A, 3 for C, then 6, 7, 4, the time for other activities will remain same. So, now it will be reduced by further 1 day, so it will be reduced to 23 days. So, if our target is further, so now, we have paid extra 2000 rupees, 1000 for A and 1000 for C activity, crashing by 1 day each and our project duration has been reduced to 22 days.

Still we are left with 1 day for crashing, so that our target of crashing is realized. So, the another activity which is costing least on the critical path is say E, that is what we can crash. So, activity E, if we crash activity E so that it will be reduced from 6 days to 5 days. So, but it will cost at extra by 2000 rupees. So, by paying more 2000 rupees the project, the activity time of, the time for activity E can be reduced from 6 days to 5 days.

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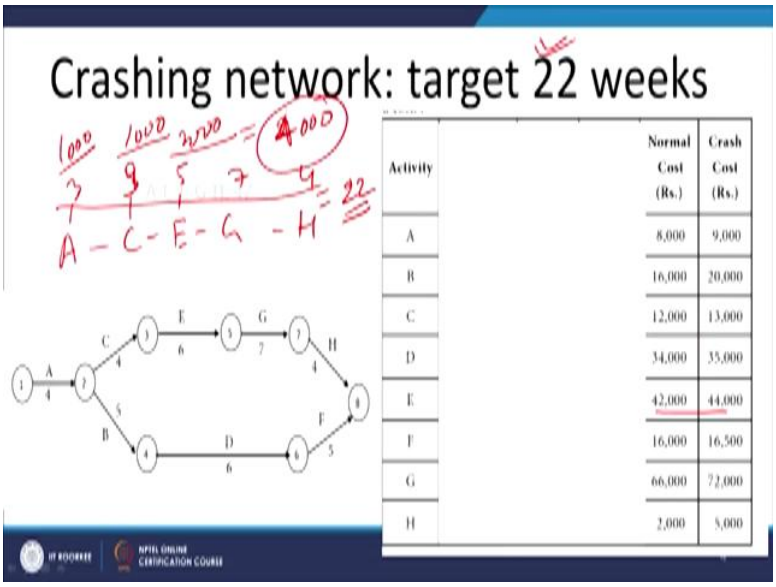

Activity	Normal Cost (Rs.)	Crash Cost (Rs.)
A	8,000	9,000
B	16,000	20,000
C	12,000	13,000
D	34,000	35,000
E	42,000	44,000
F	16,000	16,500
G	66,000	72,000
H	2,000	5,000

So, now we will be having a situation where we have crashed the activities A, we have crashed activity C, next activity being crashed is E, rest of the activities G and H are same. So, the time for G and H are same. So, here the H is still same, G is still same, we have cut down further the time for activity E by 1 day to 5, activity C was crashed by 1 day, so it was reduced to 3, activity A was crashed by 1 day so it was reduced to 3, so activity C was reduced to 3.

So, and what how much we have paid 1000 for crashing activity A, 1000 for crashing activity C and we paid 2000 for crashing of activity E, because there was a difference of 2000 rupees for crashing. So, now total amount paid for crashing the network was 4000 and we realized the target of 22 weeks, like 6 plus 6 plus 5, that is 11 and 7 plus 4, 11. So, that is how it becomes 22. So the target of the 22 weeks is realized by paying the extra cost for crashing the network.

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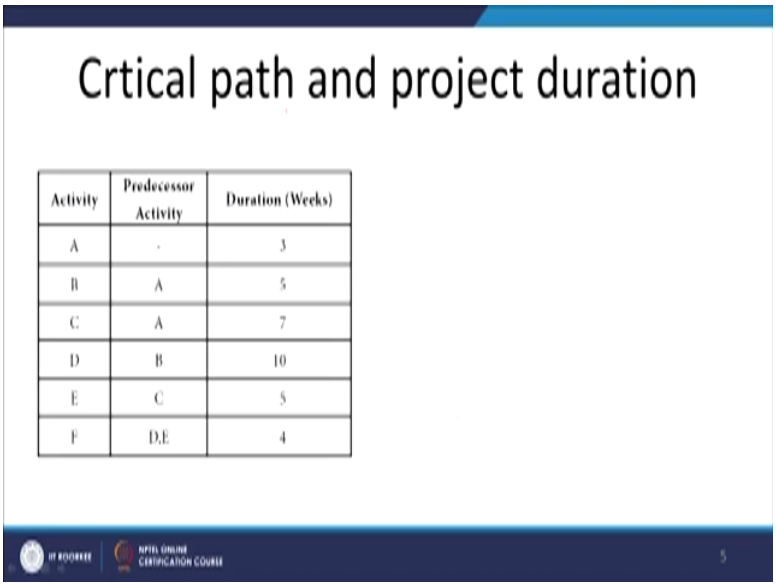

Now, here we will see, this is related with a critical path method. And in case of the critical path method, the only difference is that we get the time to complete the certain activities with certainty, with the great confidence. And the chances for deviation from that time required to complete those activities will be minimum and that is why we do not consider the optimistic, pessimistic or most likely time values. And so, the critical path method is comparatively simpler than the, it is more deterministic in nature and with the greater certainty, it determines the project duration.

And crashing of the network approach can be applied to both the critical path method and the PERT approach. In the next presentation, I will talk in detail about the critical path method. Now, I will summarize this presentation. In this presentation basically I have talked about the importance of the crashing a network, what are the steps related to the crashing of network, and under what conditions we stop the crashing of network, with a suitable example. Thank you for your attention.