Principles of Industrial Engineering
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Lecture - 44
Network Analysis: PERT - II

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering and under the project management we are talking about the network analysis. You know that there are two techniques of the network analysis, one is the critical path method known as CPM and another is program evolution and the review technique.

Under the program evaluation and review technique, we have talked about how to develop the network, how to do the analysis of the network with regard to the latest start latest finish, earliest start earliest finish and the select time calculations, how to identify the project duration and what can be done to determine the critical path. We know that the various time values which are considered in the critical path method determination as well as in the C, in the PERT approach, those are very uncertain.

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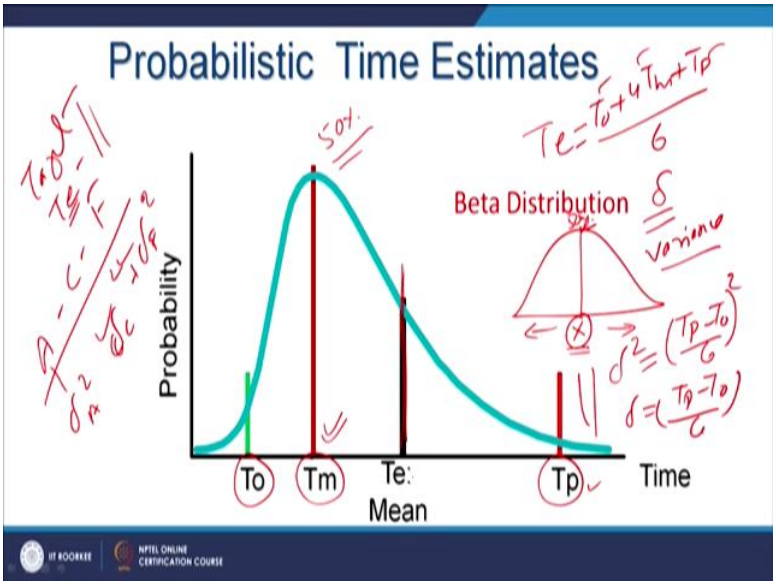

Uncertainty is there like say in terms of, we have the most optimistic time value to complete a particular activity and then there is a pessimistic time for completion of an activity. So, optimistic time, then there is a pessimistic time, the time maximum time, possibility to take the maximum time Tp and then there is a most likely time value. So, these three values which are there in light of the uncertainty which exist, in terms of the time it will take time, it will consume to complete a particular activity.

And based on these time values, optimistic, pessimistic and most likely time values, we calculate the expected time to complete the activity using this simple equation, we have already seen To plus 4 Tm plus Tp divided by 6. So, that we get the Te, that is the expected time to complete. Now, the probability to complete the activity by the most likely time value is 50 percent say, then the time to complete, the probability to complete the activity at other time values will be significantly different on, in case of the normal distribution.

Like say, this is the most expected time to complete the activities, so there will be 50 percent probability for completion of the activity for this time value and the probability to complete the activities for other time values will be changing according to the time by which we are considering that a activity or a project can be completed. And therefore, we come across the standard deviation and the variance kind of the parameters.

Standard deviation for these 3 time values To, Tm and Tp is calculated using like the square of the standard deviation, that the variance calculation is standard of the, square of the standard deviation, which is obtained from the Tp minus T naught divided by 6 whole square. And the standard deviation will be calculated from the Tp minus T naught divided by 6. So, these values we need to calculate for the activities, especially those which are falling in the path, critical path.

So, critical path activities are identified like in our earlier example A, C and F for the critical activities. So, we need to determine the variance for each of these activities, like say variance of A, variance of C and variance of F activity is determined. And that is used to see with respect to the given target time value and the given expected time value Te, what will be the probability to complete the project in a given target time, which may be more or less than the average time value or the expected time value.

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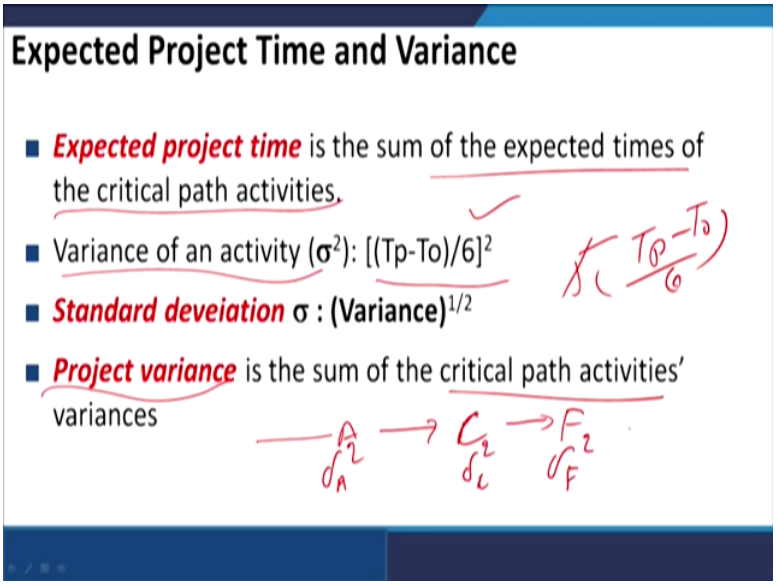

So, considering this expected project time is the sum of the expected times of the activities on the critical path, that is what we have already seen. And variance of the activities, variance of an activity is calculated using this simple equation Tp minus T naught, To divide by 6 whole square and the standard deviation is calculated from the square root of the variance or simple it is, it can be calculated through Tp minus T naught divided by 6, the same.

And then the project variance, since a project will be related with the various activities, especially for those which are falling in the critical path. So, the project variance is important, especially considering the variance of all activities falling in critical path. Say sigma, say activity A, activity C and activity F, so these are the three activities. So, sigma, variance for each of the activity is determined separately to calculate the variance of the project and that becomes the project variance sigma F square.

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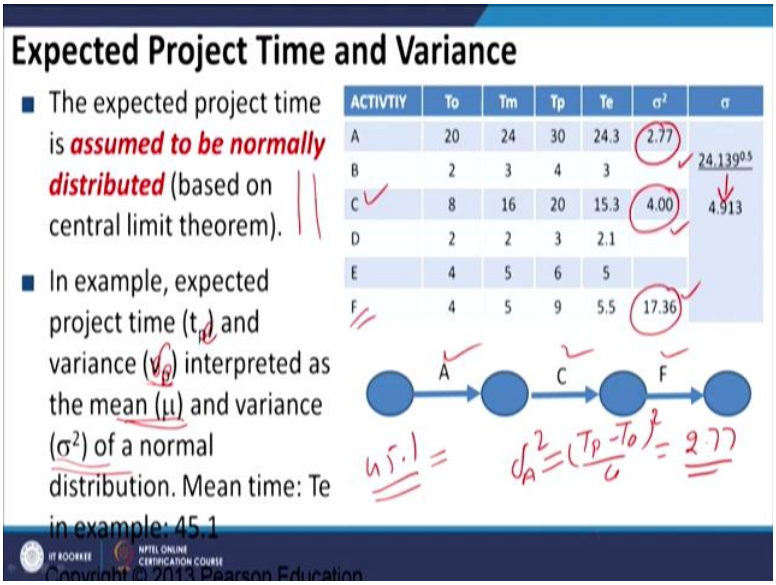

So, considering this only, the expected project time is assumed to be the normally distributed based on the central limit theorem. And if we consider the expected project time, the expected project time is Te, variance is like say sigma e, that is for the activities falling in the critical path and then the mean and variance, mean time for completing the project and the variance of the project activities on the normal distribution. Let us say in our example, it was like say 41.1, that was the mean expected time to complete the project activities A, C, F, which were critical.

So, here, if we see, if we determine the variance, sigma square, for various activities like say sigma A, it was like say Tp minus To divided by 6 whole square, this will be giving us the variance of 2.77, so we have got this value. Similarly, the variance for the activity C on the critical path is calculated coming to the value of 4 and the variance of the activity F is coming 17.6. So, now, we can determine the standard deviation for each of the activities, since the sum of all this variance is like say 24.139, square root of this will be giving the standard deviation of all the activities falling in the critical path.

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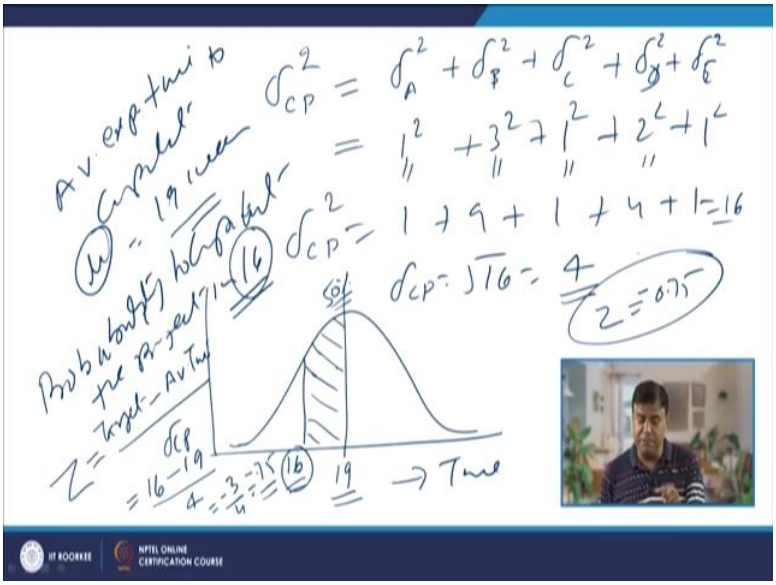

Now, we will see, let us say an example, where in there are say 5 activities having the variance values, having the standard deviation. So, for the 5 activities on the critical path, variance is determined, like say the variance of 1 activity is variance of A, B, C, this is just an example. Here variance of activity D and variance of activity E, say these are the 5 activities on the critical path. And the variance of all these activities is coming, like square of the standard deviation of 1 square plus 3 square plus 1 square plus 2 square plus 1 square.

Say this is the standard deviation, which is coming for various activities, so the corresponding the variance will be like, here it is 1 then 9 plus 1 plus 4 plus 1. So, this total will be 9 plus 4, 13 plus 3, 16. So, the variance for the critical path activity is coming 16 and the standard deviation for critical path will be square root of the 16, that will be 4. So, say for this example, the average expected time to complete the activity is 19 weeks, that is the mu or we can say that is the average expected time to complete is 19 weeks and this is the standard deviation.

Now, if we want to determine that what is the probability to complete the project or all these activities, all the activities in a critical path in 16 weeks, if this is the idea to determine, like what is the probability to complete all the activities on the critical path in 16 weeks, then we have to do some kind of calculations, like say the distribution is say normal and the average time to, average expected time to complete the activity is 19 weeks. And which means there is a 50 percent probability to complete the activities in 19 weeks, here we have time.

So, if we have the less time, if the target to complete the project in lesser time, like say 16 weeks, then the probability to complete the project in 16 weeks certainly will be less than the 50 percent. So, that probability we can determine by determining this the zone, which is like this zone, left side zone is 50 percent and right side zone is 50 percent. And if we are coming, we are interested to determine the probability to complete the activity in 16 weeks, then the probability of having the activities in 16 to 19 weeks is to be determined.

And for that we calculate the Z value. Z value is obtained from the target time minus average time or average expected time divided by the standard deviation that is for the critical path. So, here our target time is 16 weeks, average time is 19 weeks, and the standard deviation for critical path is 4, this will be giving us minus 3 by 4 minus, which is equal to minus 0.75. So now we have got the Z value of minus 0.75.

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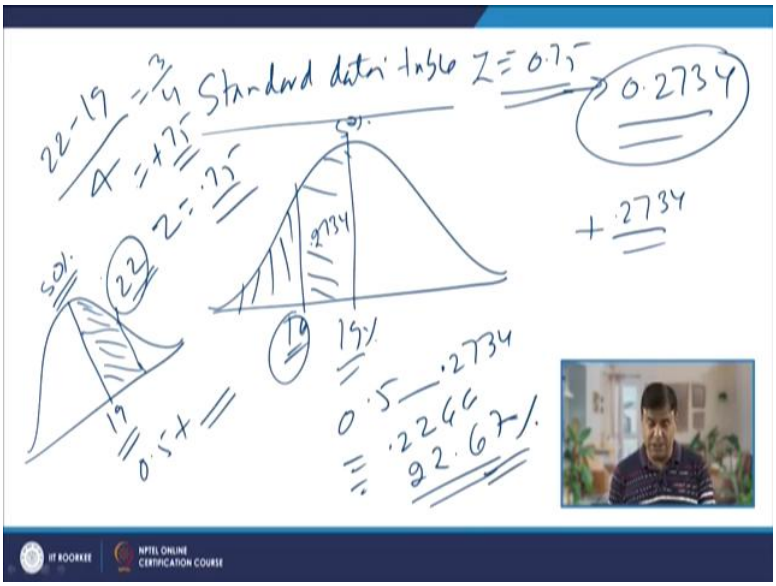

For this Z value we will be determining the probability from the standard data table, using the standard data table, the probability to, probability corresponding to the Z value of minus 0.75 is determined. Say it is coming out to be of the value of 0.2734. So, the probability for the Z value of minus 0.74 determined from the standard table, that gives us a value of 0.2734.

So, since we had the 50 percent probability for completion of the activity in 19 weeks, so if this value is coming out to be, this probability corresponding to this difference of 16 to 19 weeks is coming out to be 0.2734, then this has to be reduced from the 50 percent, that will be giving us a probability to complete the activity in 16 weeks. So, 0.5 minus 0.2734, this

will be giving us the value of 0.2266. So, that is the probability to complete the project in 16 weeks will be like 22.67 percent.

So, that is how we can determine the probability to complete the project in any of the case. Say, if we take the another example, wherein we want to determine the probability to complete the project in 22 weeks. So, average time, where 50 percent probability was there to complete the project in 19 weeks, then certainly the probability to complete the project in 22 weeks will be greater.

So, what will be determining the target time here is 22 minus 19 divided by the 4, that was the standard deviation for the critical path and this will be giving us like 3 by 4. So, again it will be 0.75. So, here the value will be like say, the plus 0.2734. So, basically this value will be greater, will be of some magnitude, which is to be added with 0.5 plus say the value that we will be determining corresponding the probability for completing the project in Z value of 0.75. So, that value will be added here and it will be greater. So, that will be increasing the probability to complete the project in 22 time values.

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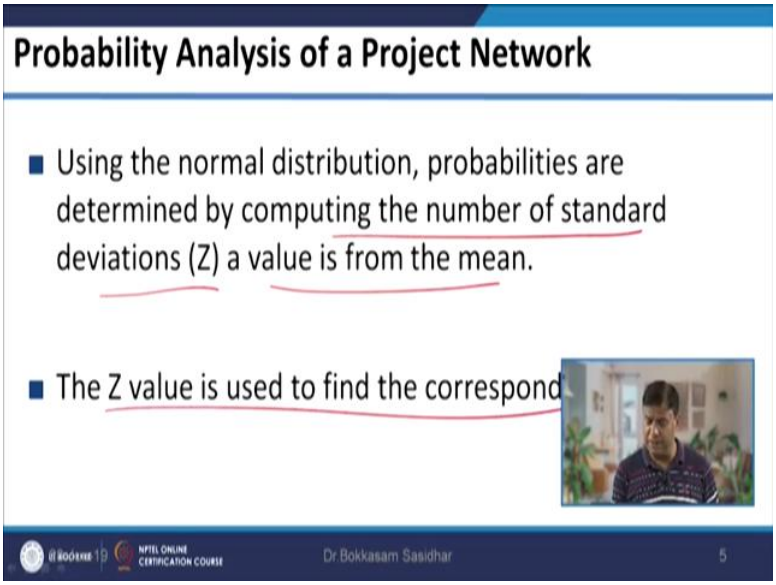

So, now considering this, we will be seeing some of the examples, like say using the normal distribution probabilities are determined by computing the number of standard deviations values from the mean and Z value is used to find the corresponding probability.

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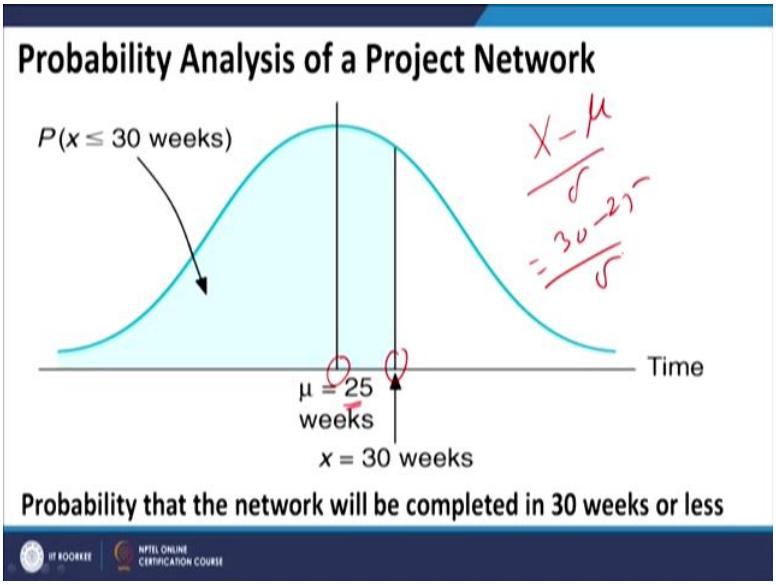

So, say the average time value to complete the project is 25. And if we are interested to determine the probability to complete the project in 30 weeks, then what we have to do is, like the target time minus the average time divided by the standard deviation. So, here 30 minus 25 divided by the standard deviation.

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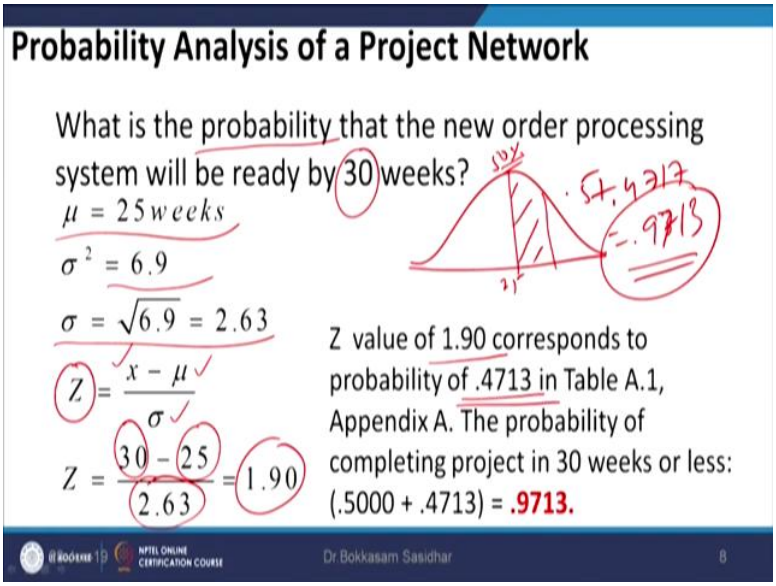

Say, we have to determine the probability for completing the project in 30 days. So, average time to complete the project is say 25 weeks and if you are interested in probability to complete the project in 30 days, then that can be done say standard division for, variance is 6.9, so standard deviation for these activities on the critical path like say 2.63. So, the Z value

will be determined considering the standard deviation, average time to complete and the target time.

So, target here is 30 weeks, average time is 25 weeks and the standard deviation is this much. So, we are getting the Z value 1.9. For this Z value of 1.9 we determine the probability from the standard table, that is coming out to be 0.4713. Since, this value is greater than, average time was 25, which had the probability to complete the 50 percent and the time for 30 week is greater than the average time.

So this value has to be added with like say 0.5 plus 0.4713. So, that will be giving us the probability to complete the project in 30 weeks will be 97.13. So, obviously greater will be the time greater will be the, higher will be the probability to complete the project.

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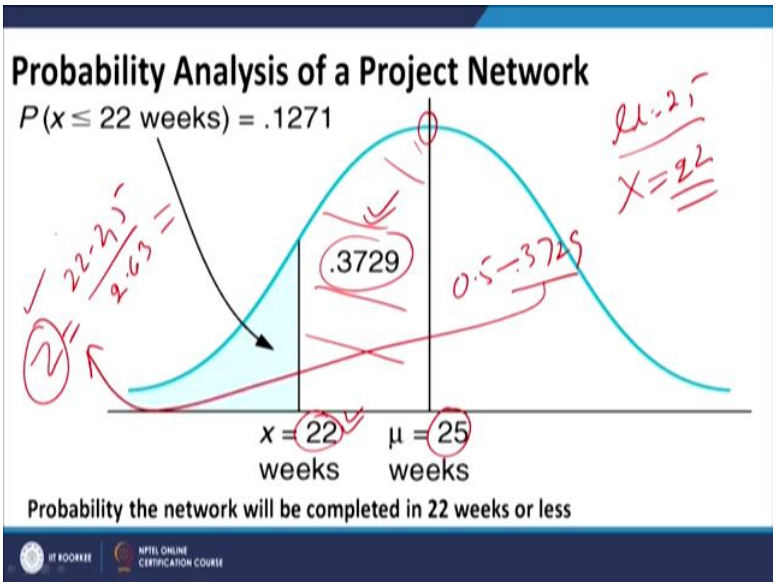

On the other hand, assuming the same average time value of 25 weeks, if we are interested in determining the target time of the 22 weeks, then, again we will be calculating the Z value and the Z value from the 22 minus 25 divided by the same 2.63. So, that will be giving us the some kind of the Z value and using that Z value we will be determining the probability to complete, probability for that Z value.

And since this time is 22 weeks, less than the average time, so the probability to complete the activity or the project in 22 weeks will be certainly less than the 50 percent. So, it has to be subtracted from 0.5 minus say this value, that is 37.29 calculated from, identified from the standard data table using the Z value that we have got from here.

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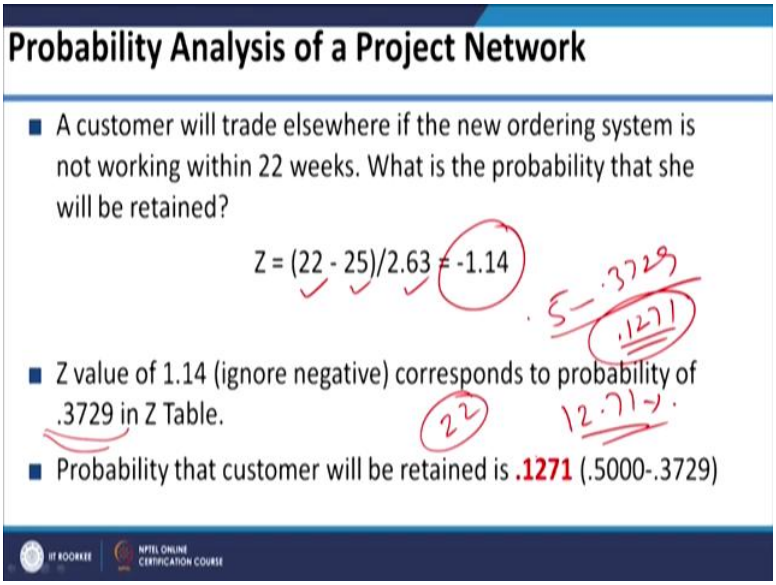

So, that is what we can see here, like 22 minus 25, divided by the standard deviation, giving us minus 1.14, that is the Z value, and for minus 1.14 Z value, we determine the probability from the standard tables, coming out to be 0.3729. And this has to be subtracted from 0.5 to 0.3729, will be giving us 0.1271, which indicates the probability to complete the project in 22 weeks will be just 12.71 percent. So, that will be the value reduced probability.

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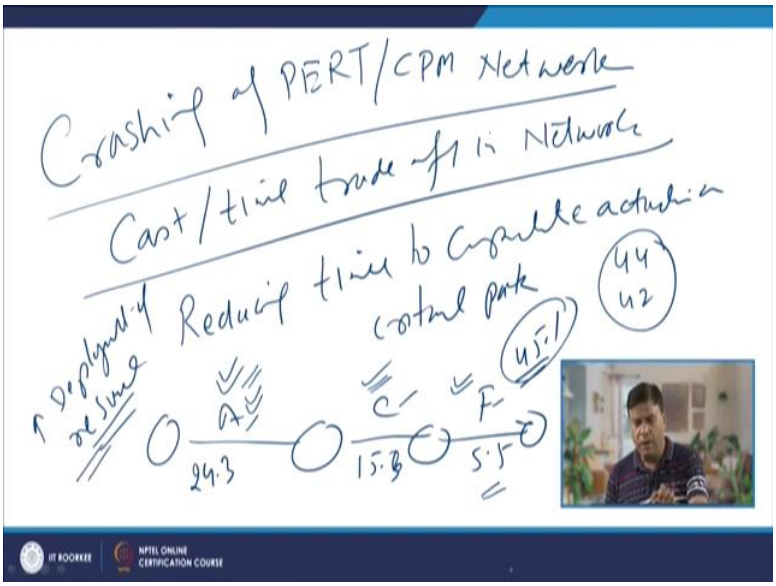

So, now I will introduce the another topic that is about the crashing of network, crashing of the PERT or CPM network. This is also called as cost time trade off in network. The focus of the crashing of the network is on reducing the time to complete the activities on critical path, that is the focus. So, in earlier example, we had identified our 3 activities, like say A, C and

F. And A was taking like say 24.3, C was taking 15.6 and here say F was taken 5.5. And this was leading to the time value of 15.3. So, it was leading to the value of 45.1 project duration.

If our target is to complete the project in 44 or 42 time, then we need to work on cutting down the time required for completing the activities on the critical path. So, activities on the critical path like A, C and F. So, activity F takes the minimum time, activity C takes somewhat larger and further longer time is taken by the activity A. If we deploy more resources in form of manpower, over-time, use of the men and machines, deploying more machines, then this will be simply leading to the increased deployment of resources.

If we deploy more resources, then certainly we can cut down the time required to complete the activities. And, but our focus on deployment of the additional resources to cut down the time will be on the activities, which are falling on the critical path. So, the focus here is on cutting down the time required to complete the activities on the critical path by deploying more resources.

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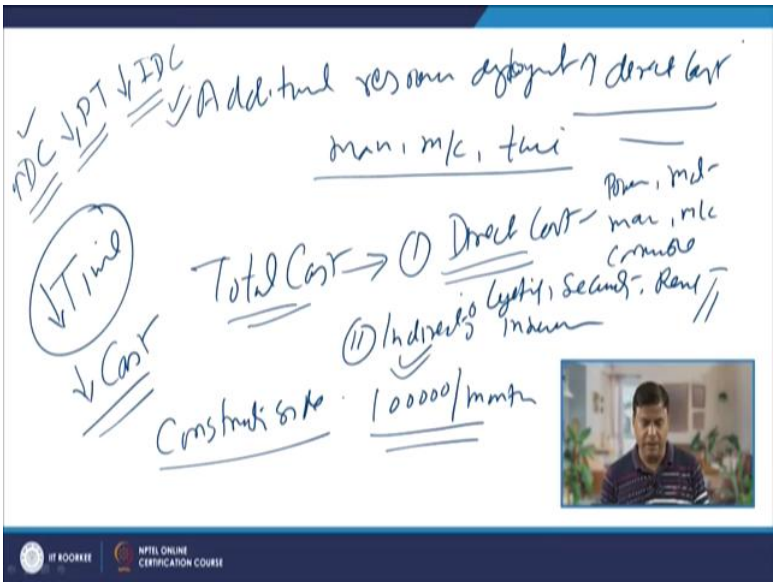

So, when it is feasible when it is not, because additional resource deployment will be increasing the direct cost. So, whether it is beneficial or not to deploy the additional resources in form of man, machine or the time for which the things will be running, because it will be costing. So, as far as the cost consideration is concerned. So, one main objective of crashing the network is reducing that time to complete the project duration.

But at the same time if this deployment of additional resources for crashing the network, means reducing the time for activities on the critical path is done very systematically, then it

can also help in reducing the cost of project, how it is done. Say total of, total cost of any project comprises two components, one is the direct cost involving the, like kind of the power, material, man, machines to be used and all consumables. On the other hand, the second important cost is the indirect cost, which is also known as the overheads.

Now, these overheads, expenditure on these overheads need to be made in any case, whether their production is less or more, in form of lighting, in form of security, in form of rent, in form of insurance. So, these things are to be done in any case, whether the production is more or less. So, say for an example, maintaining the construction side, the indirect cost expenditure on lighting, security, rent, insurance, etc is say 1 lakh per month.

So, if the making additional efforts through the increased direct cost is helping to cut down the time required to complete the project, then we will be saving the indirect cost. So, basically, deployment of the additional resources, increasing the direct cost but it will be reducing the projected time that we will be reducing the indirect cost. So, we will keep on deploying more and more direct resources to cut down the more and more project durations, so that we can have the increased benefit of reducing the indirect cost, in order to reduce the total project cost.

But there is a limit up to which the reduction in time is possible. And there the (reduc), the benefit of increasing the direct cost is realized through the reduction in the indirect cost. So, that will be a limit, that will be deciding the extent up to which a network can be crashed. Thereafter, the crashing of the network will be increasing the cost. So, what is the procedure for crashing the network and what are the things which are kept in mind, when network is crashed, about that I will be talking in the next presentation.

Now, I will summarize this presentation. In this presentation basically I have talked about that how to take care of the uncertainties in the optimistic and the pessimistic time values, related with the completion of any activity. And how to determine the probability for completion of the project in a particular time with respect to the average time values as calculated from the critical path activities. Thank you for your attention.