Principles of Industrial Engineering

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Lecture - 43

Network Analysis: PERT

Hello. I welcome you all in this presentation related with the subject Principles of Industrial Engineering. And we are talking about the network analysis techniques like the Program Evaluation and Review Technique that is PERT and Critical Path Method that is the CPM.

So, we have seen that these techniques can be effectively used for making the plans, scheduling the activities and making the suitable controls so that the project can be completed in stipulated period as well as efforts can be made in such a way that the total project duration can also be reduced, which in turn will help in reducing the delays and sometimes even reducing the cost of the project.

In this presentation, basically I will be talking about one typical example related with the Program Evaluation and Review Technique. Wherein, we will see that how to calculate the various time values like earliest start time, earliest finish time, latest start time, latest finish time, slack time, how to identify the critical path and how to identify the total project duration.

So, those things will be determined. At the same time, I will also be talking about the how to crash a network so that the project duration can be reduced while taking the advantage of reduction in cost of the project as well. So, these are the two things about which I will be talking.

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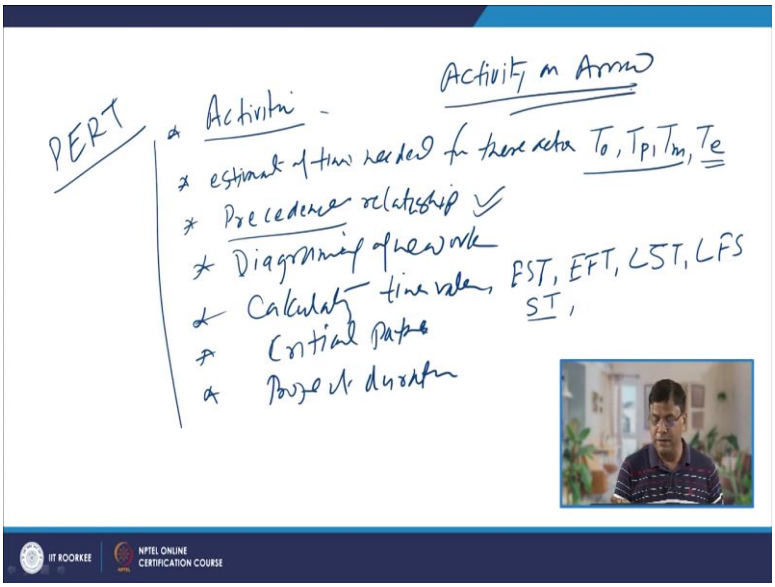

So, in the PERT technique, it is important that for in Program Evaluation and Review Technique, it is important that we understand the various activities that need to be performed. So, activities with the details need to be identified and then we need to have the estimate of the time values. Estimate of time needed for performing these activities which we will be in form of like optimistic time, pessimist time and say, most likely time. And based on this, we determine the expected time Te. And then after estimating these time values, we will try to determine the precedence relationship if it is there.

So, that precedence relationship for various activities is identified and based on that the diagramming of network or which is like developing a suitable diagram showing the various activities with the precedence relationship. So, that is about the diagramming the network considering the precedence relationship. And then, determining or like say by calculating the various the time values for the network like earliest start time, earliest finish time, latest start time, latest finish time, and the slack time.

And based on this, we will be identifying the critical path and this critical path will be giving us the project duration, the time a project will take to complete and then there are various techniques to identify the probability to complete the project in a particular time value. And so, this is how the network is developed. While developing the network in light of the precedence relationship, there can be various situations. So, how to deal with those various situations when developing a network based on the activity on arrow kind of the approach.

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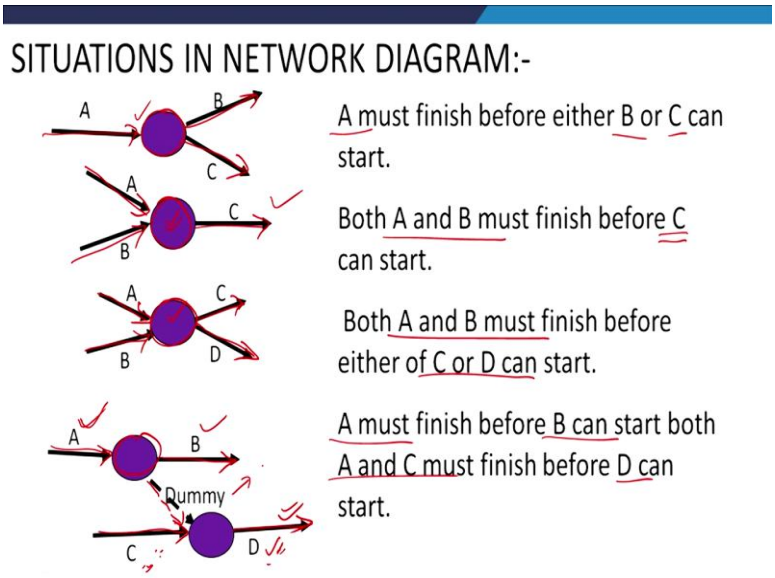

So, here, we will see certain the situations and that will help in like developing the suitable network. Like say, activity A must be finished before either B or C can be taken up. To show this, first of all activity A will be plotted and then there will be a node to indicate that activity A has been completed and thereafter activity B and activity C can be started. Likewise, if the situation is that where A and B activities must be finished before we can start the activity C.

So, show this situation, activity A and activity B both need to be completed and which will be shown with the, with this node which is showing that the end of the activity A and B and thereafter the commencement of the activity C can take place. And if a situation where A and B both need to be finished before other two activities like C and D can be started, so to show the situation, the 1 node wherein we have the two activities A and B are ending at this node and thereafter activity C and D are starting.

And if we want to show that some kind of the dependency of certain activities on other activities before commencement is needed then, that can be shown with the help of the dummy activities. Like say, A must be finished before B can start and both A must be finished before D can take place. So, this is one situation where B can be started only after the completion of the A.

So, that is been shown by this node and thereafter we are starting activity B. But here likewise, the activity D can be started, activity D can be started when the activity C is completed, at the same time activity A is also completed. So, dependency of the (active) start of the activity D on the C is shown by this simple schematic of the arrow and node.

But to show the dependency of the start of that, of starting the D on A, need to be shown with the help of this dotted line which will be indicating a dummy activity that this primarily we used to indicate the precedence relationship. It does not consume any time and resources in its, in showing this the dummy activity.

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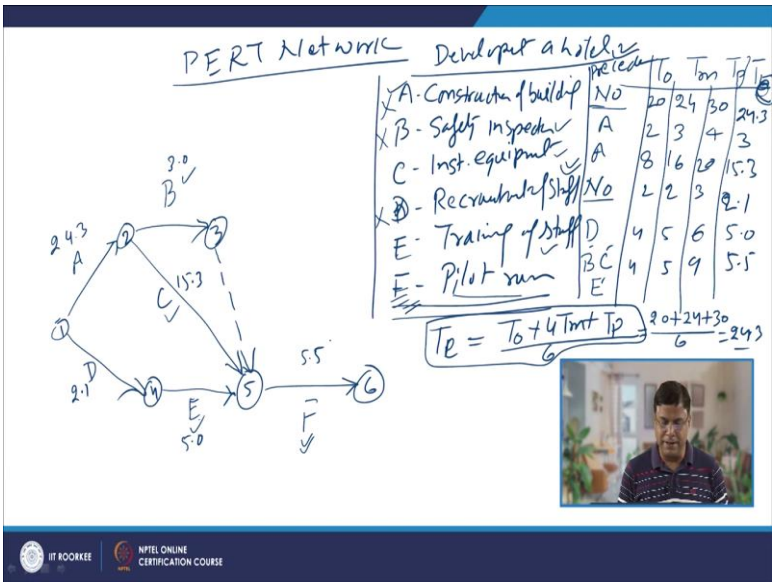

Now, we will be taking up one case where will see, how to develop a PERT network and how to calculate the various time values. Say, example is here, the development of a hotel involving various activities. Say, the construction of the building, purchase of the equipment, hiring the people or the manpower, then training of the manpower, then inspection and testing by the government officials of the building as well as equipment and then running a pilot project or running the things at a preliminary level just to test the things. So, say if these are the various activities that are, that need to be performed for development or establishing a hotel facility.

So, say activity A starts with, there are various activities, I am just listing the various activities like construction of building. Then say the activity B is the safety inspection. It does not talk about the kind of the precedence relationship, but we are just listing the various activities that need to be performed. Then, C is about the installing the equipment or the machines, then activity D is the, D is the recruitment of staff. Then we have the training of staff and say, F is the Pilot run of the things.

So, if these are the various activities that need to be performed. So, but what will be the sequence and if there is any independency of one activity with other. Like a building and can be inspected only if it is constructed. So, the so here, we cannot start with the safety

inspection but first of all we need to see that the building is constructed. Likewise, if we see until the building is constructed, there cannot be the installation of the equipments. So, we cannot install the equipment unless building is there.

Likewise, we cannot train the people unless the manpower is recruited, or the staff is hired. So, like that E can be tak, E cannot be taken up unless B is completed or B cannot be completed unless until A is done. And similarly, F cannot be done like running a pilot test cannot be done unless safety inspection is over or the staff has been trained or the equipments have been installed. So, unless all these things are done, we cannot run the show.

So, here if we have to write about the precedence relationship based on the general experience, then what we will see that the for activity A there is no precedence activity so means we can start at any time. Likewise, means as far as the sequence of activity is concerned, and likewise the recruitment of the manpower is also possible in parallel. So, there is no precedence relationship. Likewise, inspection is possible only if the building A is completed.

So, here the precedence requirement for activity B that is the safety inspection is that, A should be completed. And likewise, what we have the equipments can be installed only if the building is completed. So, that the equipment can be housed. Likewise, once the manpower has been recruited then, its training can start. So, for precedence relationship for activity E is the like say D, D is the precedence relationship and once all these things are there, then pilot run can be done only if our safety the part of the building has been done, equipments has been installed and the manpower has been trained.

So, B, C and E are the like precedence requirements for the activity F that is the running a pilot test of the hotel itself. So, now if we mention the various times values for performing these activities in terms of like optimistic time, then we have the most likely time and pessimistic time Tp. And based on this we have to determine the Te as well. So, first we will start with the Te; To, Tm, Tp and then Te at the last.

So, if say, construction A, like say the activity A takes the time of the 20 days that is the most optimistic time and 24 days is the most likely time and 30 days is the most pessimistic time like maximum it can take 30 days. Thereafter, we have the inspection which in normal case, in optimistic manner it will take 2 days, then in most likely time is 3 days and the pessimistic time for inspection is 4 days.

Likewise, for the installing the equipment, hiring, procurement and installing the equipment is like say, 8 days most optimistic time, 16 days is the most likely time and the pessimistic time is 20 days, then recruitment the staff 2 days for most optimistic time, most likely time is also 2 days and the pessimistic time is say 3 days. Then for activity E, training of the manpower is like say, 4 days most optimistic time, sorry optimistic time 4 days, most likely time is 5 days and the pessimistic time is 6 days.

And then we have the last one, pilot run needs the 4 days in most optimistic time, then 5 days and then 9 days is the pessimistic time. So, now we have seen that, for determining the expected time to complete any activity that is, Te is calculated using the equation To plus 4 Tm plus Tp divide by 6.

So, by putting these values in this equation by one by one, we can determine the expected time for each activity. Say, if we put the time of the first, the optimistic time for the first activity 20 then most likely time is 24 and then pessimistic time is 30 divide by 6. So, say this time is coming 24.3 so, we will be writing the expected time for the construction of the building is 24.3.

Likewise, we will be calculating the expected time values 3 days for activity B and then for activity C, it will be like say, the 15.3 and then for activity D it is 2.1, then activity E, the 5 days and 5.5 days for the activity F. So, these calculations for that expected time values are made using this simple equation.

And once if we have got these the time values, now we have to develop the suitable network diagram considering these precedence relationship. So, if you see there is no precedence requirement for activity A and activity D, so both can be started in parallel. So, if we see the activity, this is the node for start activity A and the activity D can be started in parallel because there is no precedence require, relationship between, precedence requirement for these two.

So, these are the concurrent activities construction of building and the recruitment of the staff. So, once these has been done, thereafter we have the another activity D, so that is about so, the next activity is the construction of the building. Activity A is done then we can get the equipments like this.

Thereafter, once the hiring is over then we can do the training of the manpower, training of the manpower is activity E. So, what we have done? We have done with A and we have done

with D, we have done with activity B which was about the inspection and testing of the equipments, then inspection of the building basically after D we can have the activity E. This is how the manpower training ends and then after A, we can also have the another activity the installing the equipments.

So, this was the inspection and installing the equipment is also possible here. So, C activity is coming like this. And then last if you see the pilot test, or the pilot run is possible. Activity F is possible only if the E, C and the B are completed. B, C and E are completed. So, if you see both E and C are converging here, so of course we can take up the activity, activity F here.

But there is a problem like, the activity B need to be, it is to be shown that there is a dependency of the start of the activity F on the completion of the activity B that is the inspection, safety inspection of the building. So, to show this precedence relationship, we need to use the dummy activity sequence like this.

So, here, this will be the dummy activity just to show the sequential or the precedence requirement between the B, C, E for start of the activity F. So, that is how the network is completed. Now, we will mark the arrow just to properly indicate the activity A then activity B, then activity C, activity D, activity E and activity F. This is how the diagramming or development of the network is done considering the precedence requirement.

Just for simplicity, we can do the naming also of these various nodes. 1, 2, 3, then we have a like say 4, 5 and 6. These are the 6 nodes indicating the start and end of the activities. Now, we will be mentioning the most expected time on these activities. Like say, for activity the expected time for completion is 24.3.

Now, likewise we will be mentioning the time, expected time for activity B that is, 3.0. Then expected time for activity we have C that is 15.3. Then the expected time for completion of the D is 2.1. Then, expected time for completion Te for activity E is 5 and then, expected time for completion of the activity F is 5.5.

So, now we have got the time values that are expected to take for each of the activity. So, based on this, now we will be determining the start and end of the activities. So, we know that the Program Evaluation and Review technique is based on the probabilistic approach where we do not know exactly the time required for completing the activities. So, there is no certainty.

So, to take care of that, the probability aspect, we need to see that how the probability to complete the project in a particular time can be calculated.

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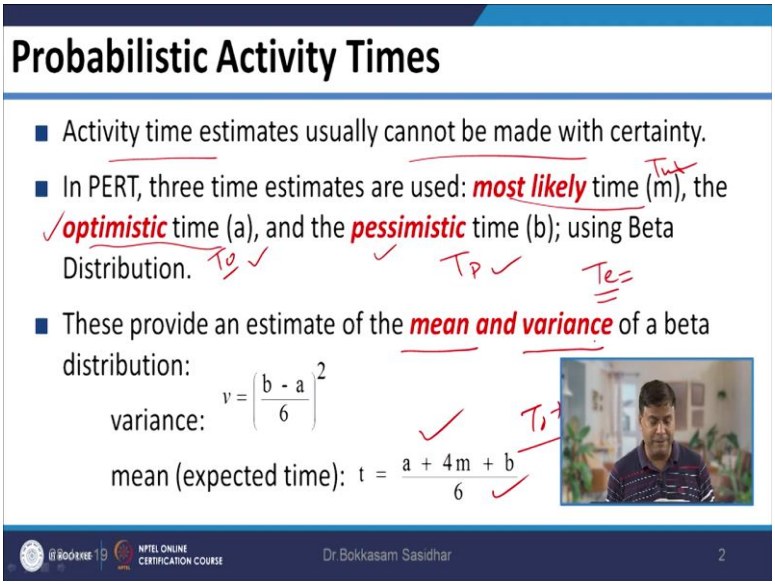

So, in light of this we will see that the activity time estimates usually impart cannot be made with certainity. And that is why, it considers the 3 time, 3 types of the time values like the optimistic time, most likely time that we write as Tm as well and the optimistic time To and the pessimistic time as a Tp. So, there are different symbols and designations can be used to show this.

Most likely time is the best guess about the kind of time that it will take and under the best possible conditions, the minimum time that it will take is the optimistic time and the pessimistic time is the kind of worst situation that will be leading to the maximum time required to complete the activity.

So, since there is a minimum time value, maximum time value and the most likely the time value, we need to calculate the time expected Te, which is obtained from like say, the equation like this To plus 4 Tm plus Tp divide by 6. So, this gives us the most expected time value, and considering this a, b and m, we can calculate the expected time or the mean time using this, A plus 4m plus so, both these equations are same.

If we have to calculate the variance of the time required for completing a particular activity considering the pessimistic and optimistic time, then equation for the variance comes out to be like the Tp minus T naught divide by 6 whole square. So, these are the like equations which are used for calculating the mean time or the most expect, expected time and the

variance of the time required when there is a beta distribution with regard to these time values.

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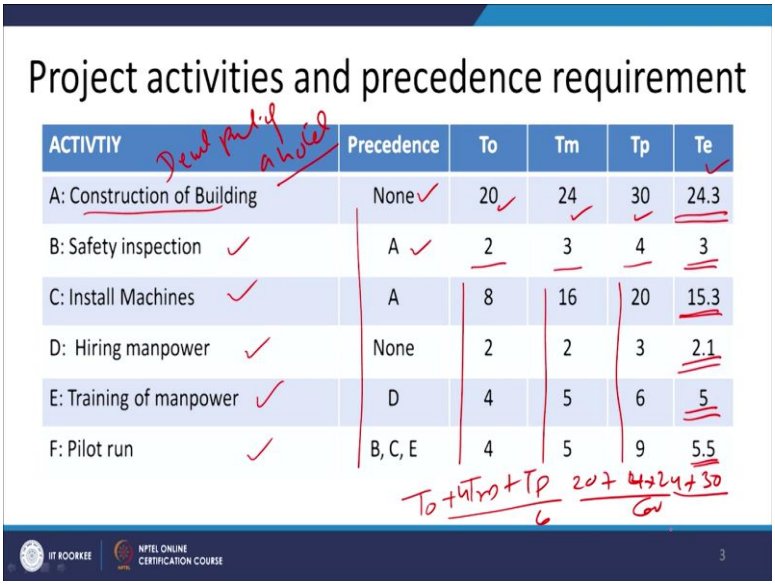

Considering the example about which I have talked in more systematic way of presentation like say, for development of a hotel. It is required to perform these 6 activities, construction of building where there is no precedence requirement, optimistic time, most likely time and pessimistic time values are here and like say another activity, safety inspection has the precedence requirement for A and the optimistic time, most likely time and pessimistic time values are here.

Then, install the machines, hiring the manpower, training of the manpower and the pilot run, these are 6 activities with the corresponding the precedence requirement and their the optimistic time, most likely time and the pessimistic time values. And based on these values, using these equations of T naught plus or To plus 4 Tm plus Tp divide by 6.

This equation, the expected time values are calculated for each of the activity. Say, for the construction of the building, it is coming out to be 24.3. By putting in same 20 plus 4 into 24 plus 30 divide by 6. So, likewise we can put the values for all activities. Then the 3 is for the activity B, that the expected time 3 for activity B.

Then 15.3 for activity C. 2.1 for activity D and for E and F it is 5 and 5.5 respectively. So, we calculate basically these expected time values and now we have to develop the network diagram based on these.

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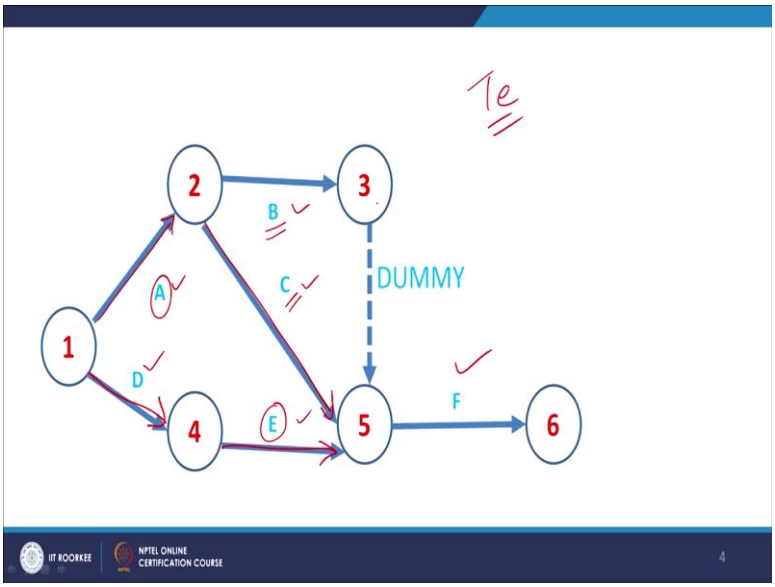

So, considering the precedence requirement where A and D had no precedence requirement so, both can be started at the same time like this. And then for B, B can be started only after the completion of A so the B is started after the A activity is completed. Likewise, precedence requirement for C is A so, this is plotted.

So, B and C both have the precedence requirement of completion of the A. Similarly, for manpower training, the hiring is the precedence requirement so, here we have the activity E and precedence requirement for start of the F is like completion of the B, C and E, all 3. So, thereafter, we will be showing the activity F and to show this precedence requirement of the F on B, we are using this dummy activity.

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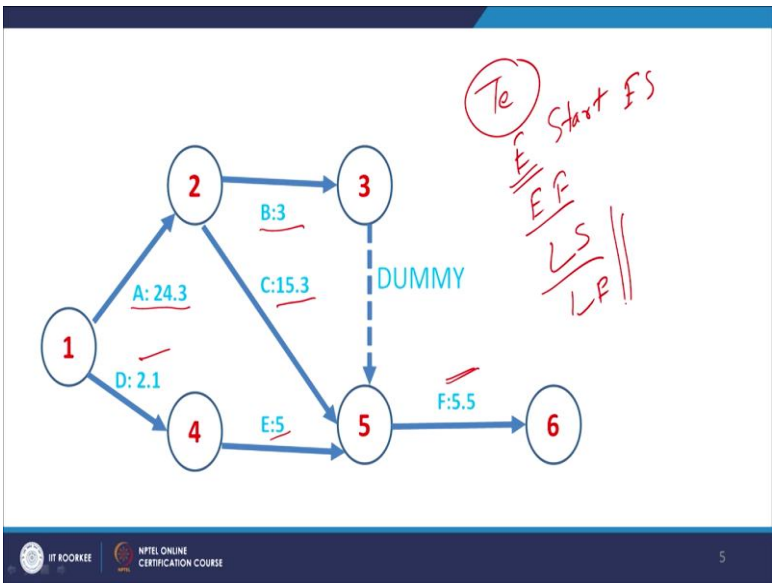

Now, we have already calculated the expected time values for various activities so those expected time values will be shown here for each of the activity like, what we have calculated, the expected time value for A activity is 24, for B it is 3, for C is 15.3, for D 2.1, and for E it is 5 and F for 5.5.

So, these are the simple expected time values that we have calculated in the previous tabel. That we have simply written. Now, the next job is to calculate the various time which are related with the PERT and the CPM techniques. Like, the earliest start. What is the earliest possible time a particular activity can be started?

So, it is represented like ES, then the earliest finish time EF. Then, latest start, the maximum time by which a activity can be delayed without effecting the project duration. It is shown by the LS and the LF for the latest finish time. So, one by one we will be seeing these various the time values and the way by which these are calculated.

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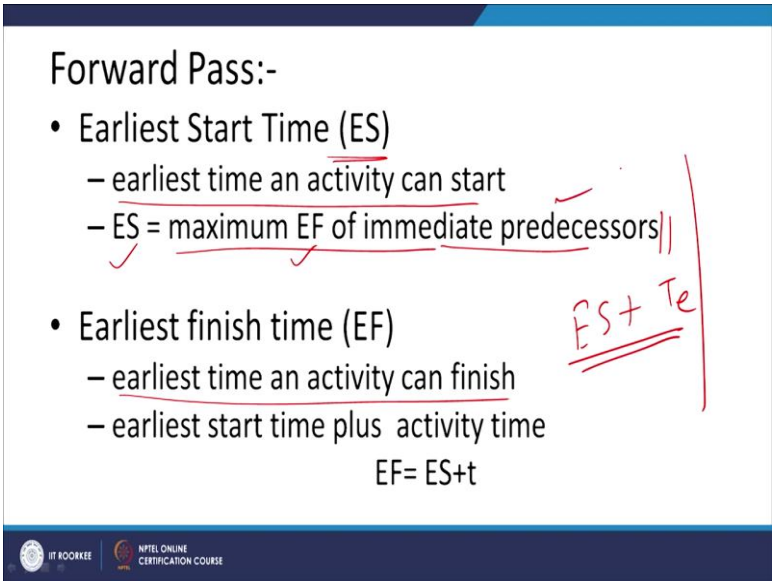

So, when we are calculating the earliest start and the earliest finish time, this is how it is done. Earliest start time ES, earliest time by which activity can be started and normally we take the maximum of the time value by which immediate predecessor can be finished. So, maximum of the earliest finish time for the immediate predecessor that will be the earliest time for start of the next activity.

Similarly, earliest finish time is the earliest time by which activity can be finished and it is obtained simply from the ES the start time plus the time to complete the activity that is the Te value, that is the time, expected time for completion of particular activity. Considering this

thing so earliest start is the maximum of the earliest finish time for the predecessor activity. This is what we have to keep in mind because we may have different situations.

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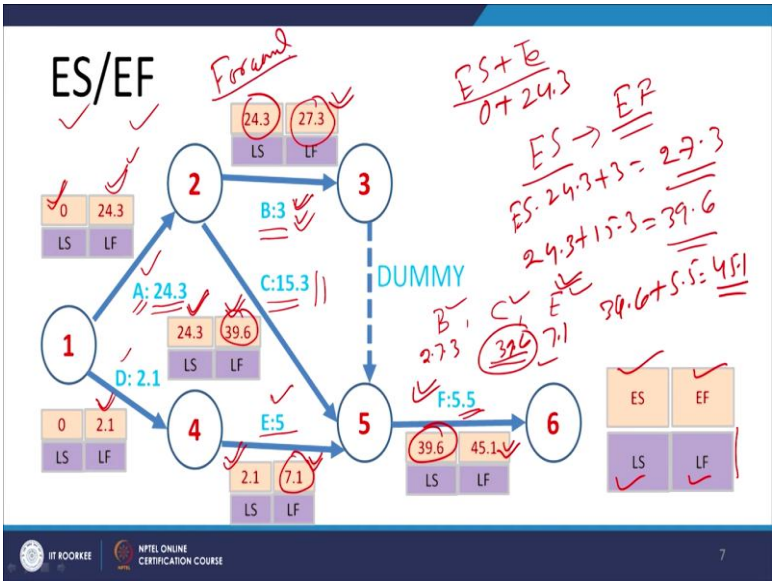

So, if we see here, for activity A and activity D, there is no precedence requirement, both can be started at the 0 time. So, if we see here, if we see here, the earliest start so here this is the symbol, the earliest start that we are using, earliest start and earliest finish shown by this colour.

And the (earlier) latest start and latest finish is shown by the different colour. So, if we have to see the earliest time for, earliest start time for the activity A. Since the activity A has the 0 precedence requirement or no precedence requirement so it can be started immediately. So, earliest time is 0. There is no waiting. But since the expected time to complete this activity is 24.3, so the earliest finish time possible is the earliest start plus the activity time time expected time.

So, that will be giving us the earliest finish time. It will be 24, so here 0 plus 24.3 so, 24.3. So, now this is the earliest start time. Since the precedence requirement for B is the completion of A and the B can be started only if the activity A can be completed. So, we know that the earliest start time becomes the earliest finish time of the predecessor activity.

So, earliest finish time for the predecessor activity A for act B is how much (())(30:34) activity B when activity B can be started? Activity B can be started only after the completion of activity A for which the earliest finish time is 24.3. So, earliest start time for activity B becomes the 24.3, so ES is 24.3 for activity B and since it takes the 3 units of the time to

complete it, the expected time to complete the activity B is 3. So, plus 3 this will be giving us 27.3, that is the earliest finish time for activity B.

So, ES now is coming 27.3. Similarly, for activity C, for activity C the predesence requirement, precedence requirement is of the completion of A. So, so, earliest start time for activity C will be the earliest finish time of the activity A and activity C so can be started when the activity A is completing the earliest possible. Since the activity A is being finished after 24.3, so earliest activity C will be the 24.3 only.

We are writing 24.3 earliest start activity. And then earliest, since the earliest start activity for C is 24.3 and the expected time to complete the activity C is 15.3 so the time to complete the activity C, earliest completion time will be like 39.6.

So, the here we have earliest finish time for activity C is 39.6. Now, on the other end we can see, activity D where the earliest start time we can start immediately so this 0 start time and the 0 plus 2.1 is the expected time to complete. So, the earliest finish time is 2.1. Similarly, since the precedence requirement for activity E is the activity D, so activity D is the finishing 2.1 time. So, this will be the earliest start time for activity E. That is the 2.1.

And this is the time to complete the activity E 5, so 2.1 plus 5 will be giving us the earliest finish time for activity E, 7.1. Now, here we have a situation wherein there are 3 earlier finish time. One is activity E having the 7.1 finish time. Another activity C having the earliest finish time is 39.6 and on the other hand, activity B is also the precedence requirement for the start of F.

So, the earliest finish time is 27.3. So, there are 3 time values of the earliest finish time which are to be completed before commencement of the activity F. So like say, activity B, activity C and activity E, the activity B is completing after 27.3. Activity F, C is completing after 39.6 and activity E is completing after 7.1. So, since when all these completed, then only F can be started.

Since the activity C takes the maximum time and before that we will be complete in any case the E and B. So, herein the maximum of all these three earliest finish time values as far as the predecessor requirement is concerned, we will be dictating the earliest start time for the successor activity or the next activity that is F. So, maximum of these three is 39.6, so this becomes the earliest start time for the activity F. Since it takes 5.5 time for its completion. So, 39.6 plus 5.5 this will be giving us 45.1, the time to complete.

So, earliest finish time for completion of the activity F is the 45.1. So, this is how we can say, we can determine the earliest start and earliest finish time. Now, we will see how to calculate, this was the forward calculation. Now, we will see how to calculate the latest start and the latest finish time values.

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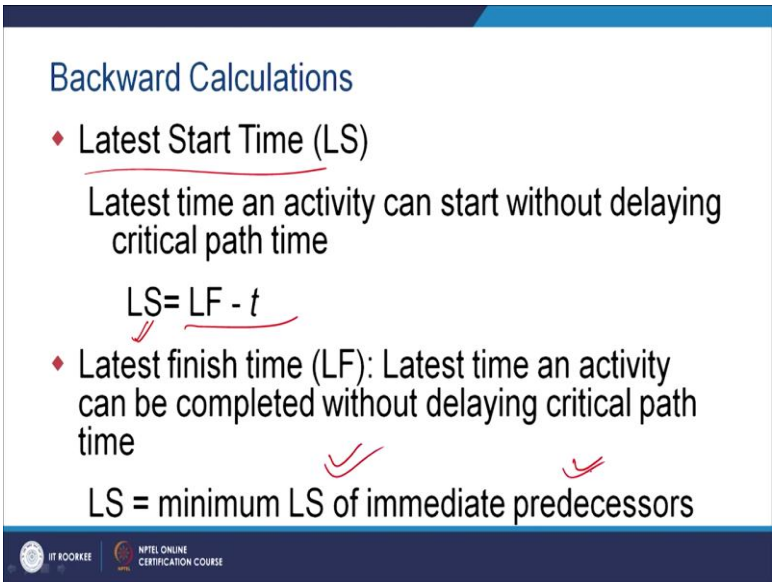

So, to start the, in the backward calculations latest start time is calculated from the latest finish time minus the time required for completing the activity that is LS and the latest finish time is the time, latest time by an activity can be completed without delaying the critical, without delaying the activity on the critical paths. So, LS is the minimum time of the LS of the immediate predecessor.

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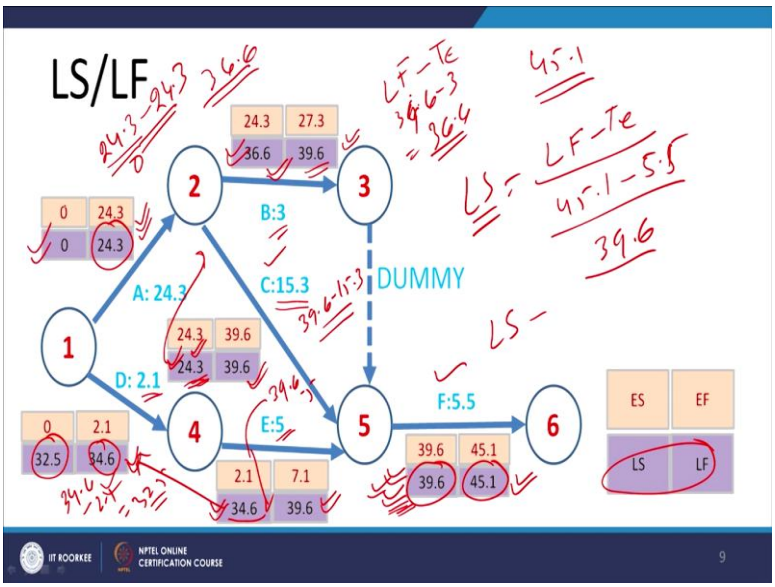

Now, we have to see, we will be calculating the latest finish and the latest start time values. So, first of all we will seeing the latest finish time by which it has to be completed. So, if we start like say, if the project is to be completed in 45.1 unit of the time then by what magnitude it delayed the most, means the activity F can be delayed maximum to which value, so that is we can calculate from the LF minus the expected time to complete.

And LF here is 4, 45.1 minus the time to complete this activity is 5.5. So, in any case, this will be giving us the latest start time. So, this will be again giving us the 39.6 value. So, in any case we need to start the activity, the latest start for the activity F is 39.6 means, we cannot delay this activity by 39.6 days if 39.6 time values if we have to complete the project in 45.1 time value.

Now, this is the latest start time. Now, if we have to start the activity at 39.6 then, what will be the time by which the activity B, C and the E should be started? So, again we will be calculating the, the latest start value of the successor. So, here say, latest start for the activity B. So, we need to see, latest start the activity of the B is how much by the time by which it must be finished, the latest time by which it should be finished.

Since the precedence requirement for start of the, the latest start for the activity F is 39.6. So, we can delay the completion of the activity up to such an extent that it is completed by this much time 39.6. So, activity, the latest can be completed by 39.6 since so this becomes the LF.

So, if the 39.6 is the kind of the LF value then the latest start possible for activity B will be like LF minus the time activity, 39.6 minus 3 so it becomes 36.6. So, this becomes the latest start. Likewise, here if we have to start the activity F at 39.6 then activity C also is to be completed by that time.

So, here what we will have? 39.6 minus 15.3 that is the activity time required to complete the activity C. So, 39.6 minus 15.3 will be giving us the latest start time. That will be coming like 24.3. So, this becomes the latest start time and this becomes the latest finish time by which it should be completed so that the activity F can be started.

Now, it will be important to see that by what time the activity E is to be started. In any case to start the activity F at 39.6 time, it is necessary that activity F is completed by that time. So, 39.6 becomes the latest finish time for the activity E and here if we see the time to complete the activity E is how much? 5. So, 39.6 minus 5 this will giving us the latest start time for

activity E. So, it is 34.6. 34.6 is the latest finish, latest start time. So, to start the activity at 34.6, activity B is also can need to be completed before that.

So, the start time, latest the start time for the successor activity becomes the latest finish time for the predecessor activity. So, here 34.6 is the latest finish time for activity D and since the time to complete this is 2.1 so 34.6 minus 2.1. This will be giving us 32.5. So, this will be giving us the latest start time. And this is the latest finish time. So, that is how we are able to complete the latest start and latest time values.

And here one more thing, here for activity A the latest start and latest finish time since this is the time when the activity B is to be started 36.6 and the time by which the activity C is to be started is 34.; 24.3. That is the latest start time. So, to start the activity at 24.3, the activity A must be completed after 24.3. So, since the latest finish time for the activity A is the latest start time of the activity C. So, this will be giving us the minimum value of the latest start time of the successor activity.

On the other hand if we consider this the activity B, that is 36.6 is the latest start time and if this becomes the latest finish time then activity C will be delayed and that is why we will not be considering the latest start time for the activity B. But we will be considering the latest start time of the activity C which is 24.3. So, 24.3 the latest finish, start time for the activity C becomes the latest finish time for the activity A.

So, when the latest finish time of activity C that is 24.3 is subtracted with the time required so this is becomes the latest finish time for activity A and when it is, when it is subtracted with the time required to complete the activity A that is 24.3 so the difference becomes the 0. So, the latest start is the 0. So, that is how we are able to complete the latest start and latest finish time for both these activities.

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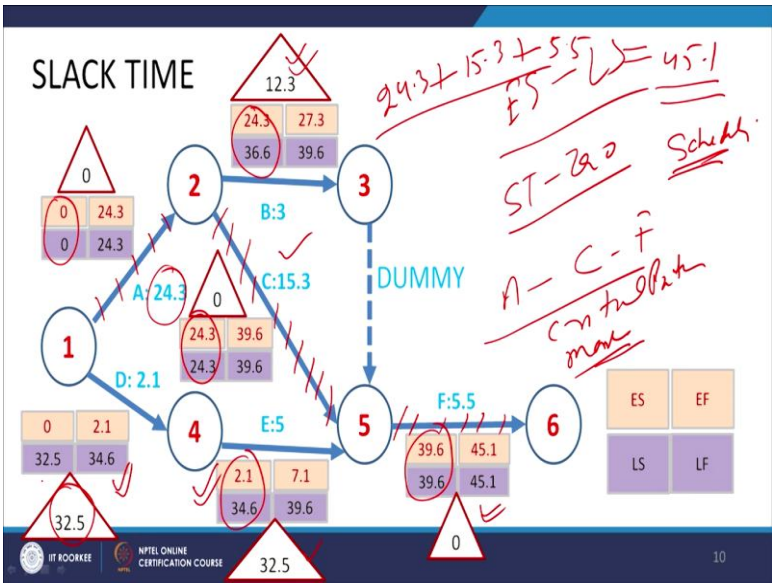

Now, the slack time is about the difference in the earliest start and the latest start time.

So, here if we see the earliest start and latest start time is same so the difference will be 0. Here the latest start and the for activity E the latest start and the earliest start time difference is 32.5.

For activity, this is the dummy activity, for activity B the latest start and earliest start time difference time is of 12.3. And the difference of the latest start and earliest start time values of activity C is 0. And then we have the activity D, activity D having the difference between latest start and latest finish time of the 30, latest start and earliest, earliest start time is 32.5. And similarly, the difference of the earliest start and the latest start time values for activity A is 0.

So, all the activities having the slack time 0 means these cannot be delayed in any of the ways without delaying the project duration. So, these in that case becomes the critical activity, activity A, activity C having the 0 slack time and activity F. So, the path A C F becomes the critical path and it will be governing the time required to complete the project, because it will be taking the maximum time to complete the project.

How much? Let us say 24.3 plus 15.3 plus 5.5 this will be taking the, this is the longest root and it will be taking the maximum time of 45.1. That is the, that will giving us the project duration. Now, these time values can be used to see how to schedule various activities and where there is possibility to accommodate some kind of the delay in performance of activity like this activity can delayed hugely by 32 days, 32.5. Similarly, this activity can also be

delayed significantly. So, there is lot of slack time with few activities were some kind of the (())(46:01) can be taken.

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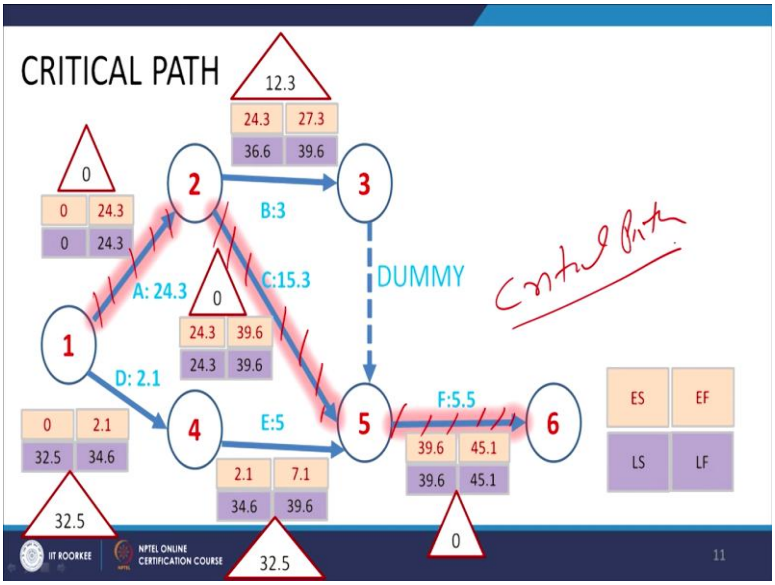

So, considering this the 0 slack time as well as the maximum time required to complete certain activities. The path is identified which is shown here like A, C and F is the critical path.

So, now I will summarise this presentation, in this presentation basically I have talked about how to take care of the probability aspect or uncertainty aspect related with the time values for completing the particular activity and how to develop the network and how to calculate the latest start, latest finish, earliest start, earliest finish time, how to calculate the project duration and how to identify the critical path. Thank you for your attention.