Principles of Industrial Engineering
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Lecture 40 - Inventory: Gradual Replenishment Model

Hello! I welcome you all in this presentation related with the subject Principles of Industrial Engineering and you know that we are talking about the inventory. It is important and good for the organizations to maintain the inventory of the items which are consumed regularly, so that the items can be made available whenever and wherever they are needed, so that the functioning of the organizations, productions and services, production of goods and services can continue to happen smoothly.

But at the same time, to at the same time when the inventory is maintained, efforts are also made to maintain inventory in such a way that the cost of maintaining the inventory is minimum and for that the quantity which is reordered for having the items in the inventory that is identified in such a way that the total cost of maintaining the inventory is minimized. One of the simplest case that we have considered earlier was like the item was delivered in one go against the order quantity and the delivery was instant.

But in the real life, there may be various situations where the delivery is not instant and the delivery is made in stages and in those cases how the inventory model should be developed or how the economic order quantity should be identified or optimum lot size should be identified, that is the case we will be considering in this presentation. So, the inventory through the gradual replenishment model wherein we will try to determine the optimum reorder quantity and the optimum reorder point.

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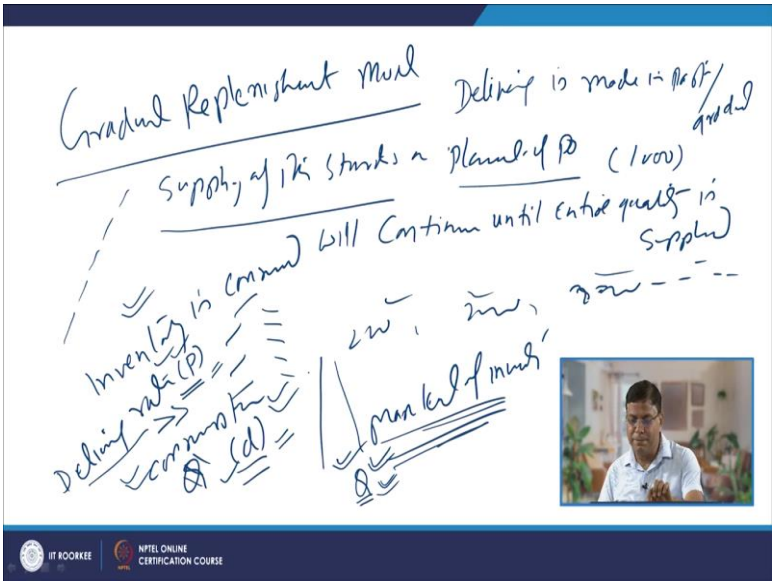

So, in case of the gradual replenishment model, Gradual Replenishment Model as it reflects from its name, in this case the delivery is made in parts, it is gradual not instant, entire lot is not delivered in instant. So, this case basically, the supply of the items starts on placement of PO but the supply is not a complete in one go against the quantity for which order was placed, so this supply of the items will start with the placement of the order and will continue until the entire order quantity is supplied.

So, maybe like say the order quantity was of 1,000 units, so initially it will supply 200 then again 200 then again 200 like this, so this supply in stages will continue until the entire order is fulfilled. But in this situation, when the items are supplied in stages, the inventory is consumed also, inventory is or items in inventory those are consumed. So, when there is addition of the items, at the same time there is a consumption as well. So, this is the situation, where we are adding the items and consuming the items at the same time.

And this will make the determination of the maximum inventory level difficult, maximum level of inventory that becomes difficult to determine, because when the items are being added at the same time, they are being consumed also. So here if we assume that the replenishment rate or the delivery rate is greater than the consumption rate or the rate of depletion of the inventory, so if let us say the delivery rate or the rate at which the inventory is filled in against the order is P, it is the production rate or the rate at which it is being replenished.

And the rate at which it is consumed is say small d. So, the d is the, small d is the consumption rate and that the capital P is the rate of the delivery. So, if the rate of delivery or the replenishment rate is greater than the consumption rate then certainly, the inventory will keep on increasing during the delivery period, when the order is being fulfilled. But the rate of the rise will depend upon the difference between the rate of delivery and rate of consumption. So, this is a very typical situation, where we are getting the delivery and at the same time, we are getting, we are consuming the same and that is why the maximum inventory level is never equal to the order for which order was placed.

Let us say order was placed for Q quantity, but the maximum inventory level in this situation will never be equal to the, Q max will never be equal to the quantity for which order was placed, that is Q. Because we are consuming at the same time when it is being delivered, for this typical situation when the delivery rate or the replenishment rate is greater than the consumption rate, the inventory will definitely be rising.

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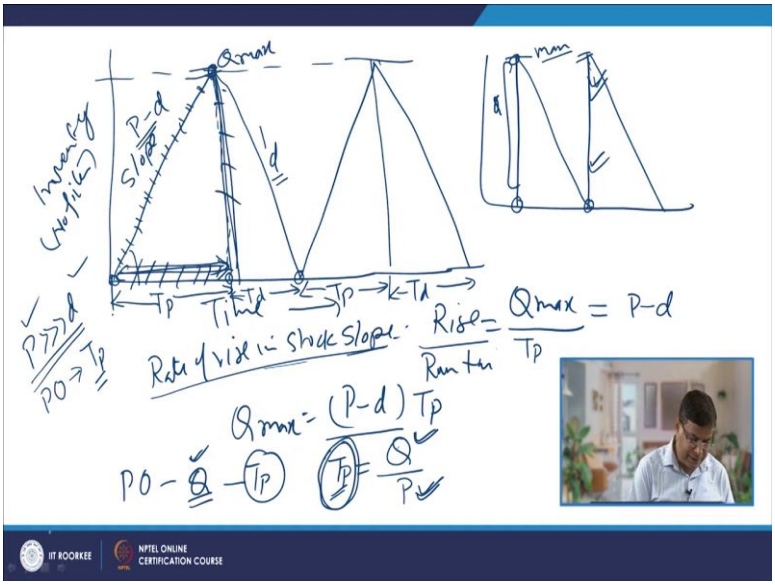

But at what rate the inventory level will be rising that will depend upon the rate of replenishment and the rate of consumption. So, to see this typical situation, we will make, since in this case delivery is gradual against the order for order quantity Q, so there will never be, earlier what we have seen that as soon as order is placed, the delivery is instant for the items Q and the inventory is fulfilled to the maximum level and then consumption starts. Again when the order is placed then the delivery is instant.

So, there is a vertical line which is showing that the entire stock is completed in one go, the entire the PO and or the entire supply against the PO is made in one go and inventory reaches to the maximum level and thereafter its consumption starts, so we use a vertical line because of the instant delivery situation. But in this case, the moment order is placed, the delivery starts, so there will be a certain rate at which the inventory level will be increasing, this is the rate at which it is increasing.

And then it will reach to some maximum level that is the Q max. So, here we have inventory that is about the number of items in inventory and here we have the time as a function of which the inventory, variation in the inventory level will be observed. So, the initial stage when the order is placed, so we start getting the delivery against the order and in this case since we have assumed that the replenishment rate is greater than the consumption rate D, so there will be gradual rise in the level of the inventory like this.

And say it takes time after placement of the PO, the delivery is completed in certain time, so that time is say P, the time for which the order is completed or the items are produced to complete the order, so this we will be considering as a TP the replenishment period during which the items are delivered in the inventory against the PO. And thereafter, so this is the Q max level, say the moment the delivery is starts against the PO the level of the inventory in the stock will keep on increasing like this.

And as soon as the items delivery against the PO is completed, we will see that the inventory has reached to the maximum level and thereafter since the consumption is continuous, so the inventory level will again start decreasing like this. So, the time during which the inventory is consumed say T d, again on reaching to the 0 level of the inventory, say order is placed because we start getting the delivery immediately, but the delivery is not compete against the PO but it is in stages.

So, again, the same cycle will be repeated like this and then it will be consumed. So, here again, we will have the like TP the period which items are delivered and the T d is the period during which items are consumed. So, this the slope this slope is positive which is indicating that there is a rise in the level of the inventory in the stock and this rate of the rise will be governed by the difference of the replenishment rate and the consumption rate. So, this slope basically, slope of this line will be indicating the rate of rise in the inventory. So, the rate of rise in stock will be indicated by the slope.

And how can we get slope? Slope is basically this level Q max and this run, so the rise due to the addition of the stock and the time in which this stock rise takes place, so this vertical length that is the height Q max and the run time run is this during which the delivery is being made that is the run time, so the rise is maximum of the Q max that is this height and that the this run that is the time during which the delivery is made against the order TP, so the runtime is here TP, there is the time during which the inventory is replenished or the items are produced.

So, this rate of rise is directly being influenced by what? The difference of production and the consumption rate P minus d. On the other hand, on sloping down it is primarily the d negative the P minus d, that is the items being consumed. So, if we see the rate of rise in the stock that is indicated by the slope, this can be given by P minus d, so the Q max here will be equal to P minus d into the TP, so P minus d is the rate of rate at which the inventory is rising during the replenishment period and TP is the replenishment period.

Now, since we place the order for the quantity Q, P was placed for order for supply of the items in Q quantity and it took the time of TP time to replenish that to fulfil that order, so the TP we can determine the time to produce quantity P at certain rate, those time to produce the quantity Q at the rate of the P, P is the production rate the rate at which the items are being delivered and the total quantity which is to be delivered here is Q.

So, the time to deliver the quantity Q when it is being produced at the rate P, that can be determined from the TP is equal to Q by P, Q is the quantity which is to be supplied and the rate of production is P or rate at which it is being delivered is P, so the time to fulfil the order or to produce the order Q, that TP will be equal to Q by P.

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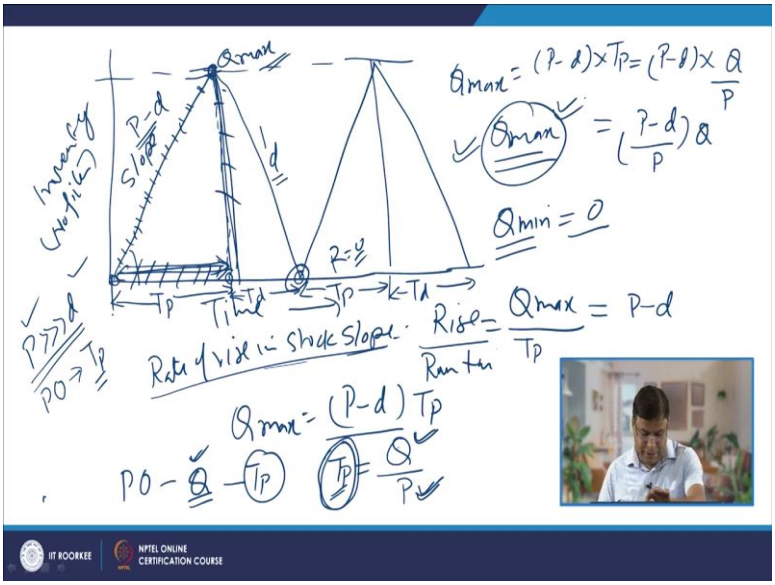

So, here if we simplify this further then what we will notice that this can be used to calculate the Q max. So, the Q max or maximum inventory level which is P minus d into TP this we can write as P minus d into TP is equal to how much? Q by P. So, it becomes equal to P minus d by P into Q, Q is the order quantity, P is the production rate at which the rate at which the items are being replenished and this is consumption rate, so that is what will be giving us the Q max.

Now, this equation we can use, now we have got the Q max for this difficult situation, where the items are not delivered in one go against the order quantity but items are delivered in stages and therefore, the Q max is not equal to the order quantity but Q max is different from the order quantity for which order was placed. Because inventory is being replenished at the same time it is being depleted or consumed.

So, the Q max here now we have determined, since in this case the order is being placed and the delivery starts, but the entire order is not fulfilled in one go, so reorder point here is 0. So, if we see Q minimum that is the inventory level minimum, here is 0 and Q maximum is the P minus d divided by P into Q. So, that is what will be giving us the maximum and minimum values.

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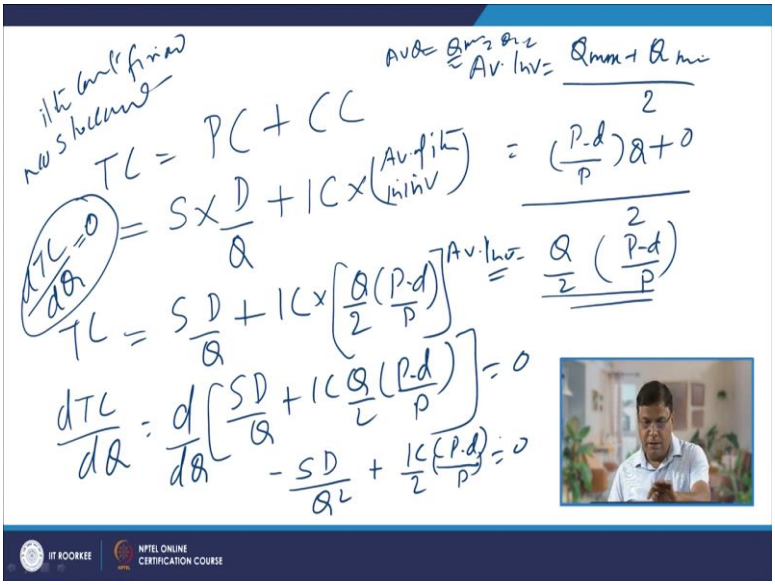

Now, if we have to determine the average inventory, then that will be the Q max plus Q minimum divided by 2, so here P minus d divided by P into Q plus 0 divided by 2, so this is what we will be giving as Q by 2 into P minus d divided by P, this will be the average inventory which we need to use in the total cost calculation purpose. For determining what should be the quantity for which order should be placed in the gradual replenishment situation for determining the economic order quantity.

So, like earlier case assuming that the item cost fixed and there is no stock out situation, our the total problem is reduced to the, means the total cost of maintaining the inventory is reduced to the procurement cost and carrying cost. So, procurement cost like earlier case we can calculate as that is the per order cost divided by the demand into the order quantity for which the quantity for which order is to be placed. And then the carrying cost will also determined in the same manner, the inventory carrying cost into the average number of items in inventory, average number of items in inventory.

So, the same will be applicable here also except the difference is earlier average inventory was like Q max by 2 or Q by 2 because the Q minimum was 0 and here this value is different because order is to not being fulfilled in one go or entire order is not supplied in one go, but it is done in stages. So, now S D by Q plus I C into the average inventory equation here for the gradual replenishment model is Q by 2 into P minus d by P, so that is what will be giving us the total cost.

Now, to minimize the total cost again we will be using the calculus method, where we will be differentiating the TC with respect to the order quantity d Q and making it equal to 0 to minimize the cost. So, d TC divided by d Q, so this will be differentiating with respect to the d Q this entire expression S D by Q, I C into Q by 2 into P minus d by P, this will be made equal to 0. So, again, we will be getting the same thing, S D by Q square plus I C into P minus d by 2 and P like this, I C by 2 into P minus d by P.

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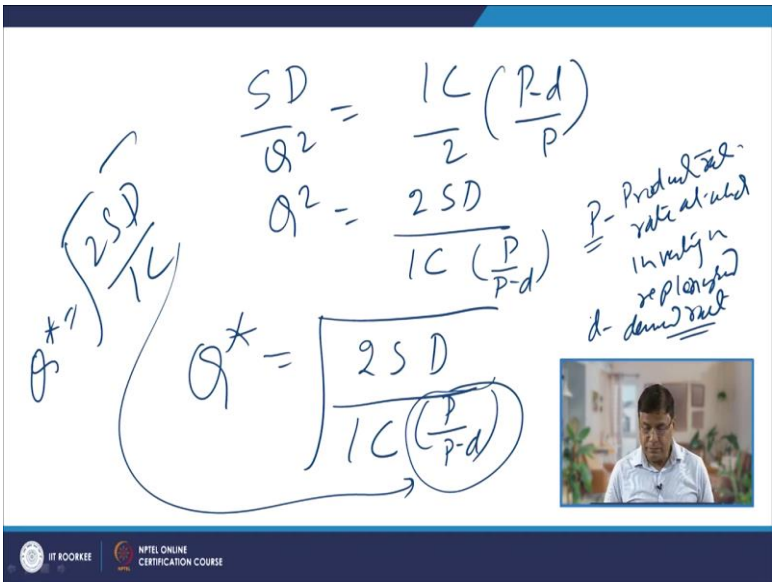

On solving this equation further, what we will be getting is like S D by Q square will be equal to I C by 2 into P minus d by P, on solving this further Q square will be equal to twice of S D divided by I C into P minus P divided by P minus d. So, the equation for the economic order quantity becomes square root of the twice S D divided I C into P minus P by d. So, if we see, if we just forget about, in this case this is just additional factor as compared to the earlier one. Earlier for the simple lot size formula when the delivery was instant for the entire order, we had the economic order quantity equation was like twice of S D divided by I C.

Only the additional factor to take into account, the gradual replacement, replenishment is like this, P by P minus d this where P is the production rate or the rate at which the inventory is replenished, while the d is the demand rate or the consumption rate when the or the rate at which the inventory is depleted. So, in this situation, we have assumed that the P is greater than this d, otherwise it will lead to the situation of the stock out.

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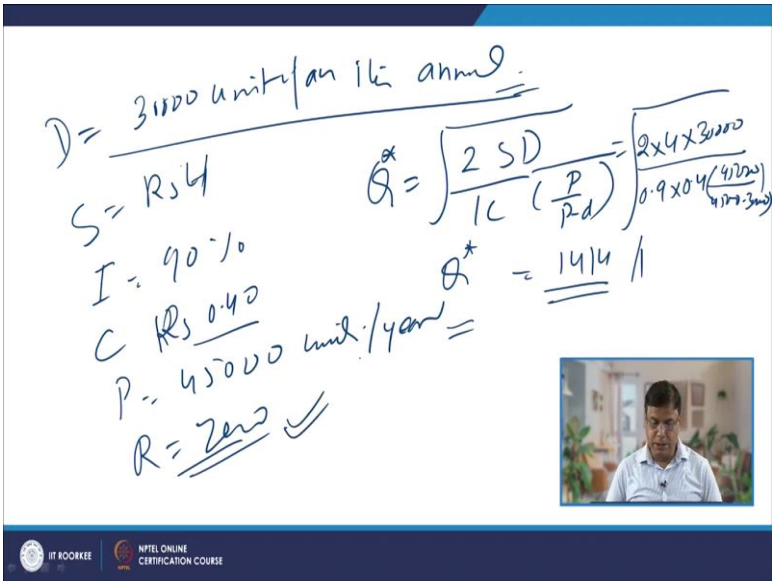

Now, we will consider one example, say one company or an organization which uses the 30,000 units of an item annually, so annual demand is 30,000 units, that is the D for the item, particular item being used by a company. And say the S like the per order cost is rupees 4 per order, that is 4 per order and the nature of item is such that there is a lot of wastage when the items are in inventory, so the carrying cost is extremely high, I say 90 percent because many perishable goods are such that it is difficult to maintain them in the inventory and there will be lot of scrap and wastage or damages to the items.

So, I is very high and the C is the item cost say rupees 0.4, cost is less but the inventory carrying cost is extremely high and if like say the production rate is 45,000 units per year, that is the P and we have already talked about the demand, annual demand is this, annual production rate this, then what we will have here is the determination of the order quantity R is reorder point here will be 0 because we start getting the delivery of the items immediately on placement of order.

So, if we have to determine the economic order quantity using the gradual replenishment model then S, twice of S D by I C into P divided by P minus d. So, here all the quantities are known to us, twice of S is our 4, d is the 30,000 units annual demand, I C, I is 0.9, C is 0.4 and the production rate is 45,000 divided by 45,000 minus 30,000. So, on solving this, what we get? The economic order quantity 1414, so this becomes our economic order quantity for this situation.

When the items are being consumed, at the same time when the delivery is taking place and the reorder point here is 0. So, there is another situation where they, where we get that discount in the price and the price does not remain constant, but it changes as per the order quantity. So, in those cases, determination of the total cost and for maintaining the inventory the procedure becomes different and those cases we will be seeing now.

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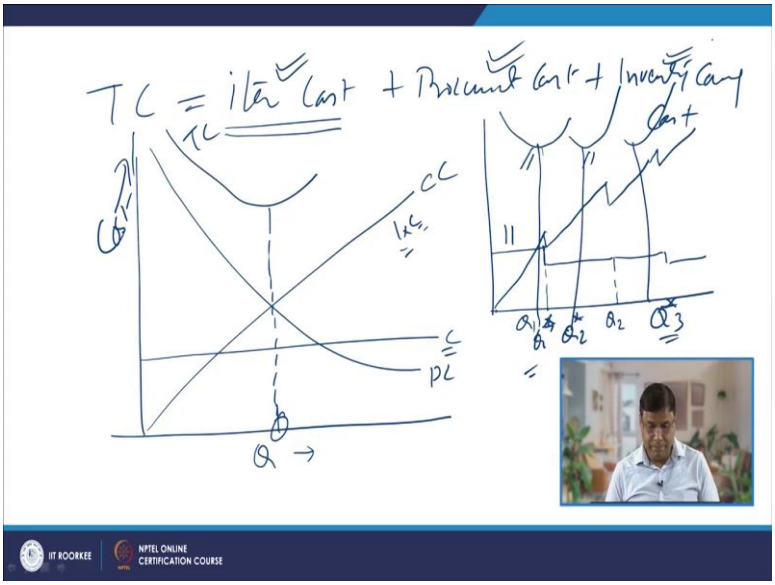

Here, if we see our the total cost consideration for maintaining the inventory, we have considered the item cost, procurement cost and inventory carrying cost, while assuming that there is no stock out situation. So in this case, whatever we have calculation, whatever calculations we have made in all those cases, we have considered that item cost is constant, but if we get the benefit of maintaining the if we get the benefit of the lower price, if we place the order of the larger quantity, then the economic order quantity will also change, that is what we can realize from this simple plot.

Let us say some of the like in the event when there is no change price, so whatever quantity for which order is placed, price remains constant. So, the total cost of the item will remain constant as per the quantity for which order is placed. Of course, it will change with the number of units, but the item price will remain same. And since the C is fixed, so I C also remains that is the carrying cost also changes linearly with the order quantity. And we get the procurement cost variation as a function of time.

So, here we have the total cost or you can say the cost aspect only, cost and here this shows the carrying cost, then this shows the procurement cost and this is the item cost which is

constant. So, in that case, we simply get the total cost curve and which corresponds to the minimum the, which easily gives us the minimum order quantity at a particular level. So, but if we see that there are price breaks, like when the order quantity is less, the price is high. As the moment our price or the order quantity go, this is say for Q1, if the order quantity is greater than Q1 then we get the price break and the price is reduced.

So, our the item cost will be reduced then after again after sometime say beyond a Q2 level, again we get the another price break, we get the items at the lower price. Since the carrying cost is directly linked with the I into C, C is getting reduced, so even for the given value of the inventory carrying cost in terms of the percentage, I value, since the C is reducing so we will be getting the breaks in like this, I C will also be reduced like this.

Since these factors are reducing are changing as a function of, as function of the cost like this, so these will be leading to the shift in the economic order quantity. Like earlier if it was here, then it will be somewhere here, then it will be somewhere here like this. So, our economic order quantity will also be changing, Q star will also be changing say Q2 star or Q1 star. So, the point here is for one item cost, we get the one economic order quantity, for another item cost we get the economic, another economic order quantity.

Since, there is a reduction in price with the number of units, so item cost also need to be considered apart from the procurement cost and the inventory cost and we can make the simple calculations for determining the cost at which the minimum total cost for maintaining the inventory is coming.

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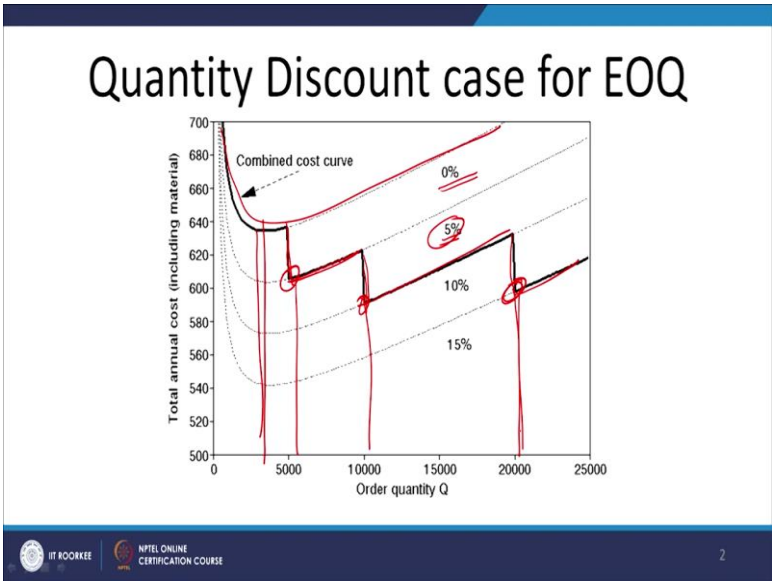

So, now we will see that these things from these curves like when the, when there is no price or discount, our we get one minimum order quantity that is this, the moment we get the price break of 5 percent price discount of the 5 percent, again the I C is reduced, item cost is reduced, so we get another minimum order quantity that is here. So, as per the number of items for which order is being placed, we will be getting the different order quantity as per the level at which the benefits are being realized.

So, in this case if the 5 percent discount is here and this is the minimum order quantity, otherwise it was here, then with the 10 percent discount this is the minimum order quantity likewise, we so with the price change there will be the change in the economic order quantity and that has to be considered.

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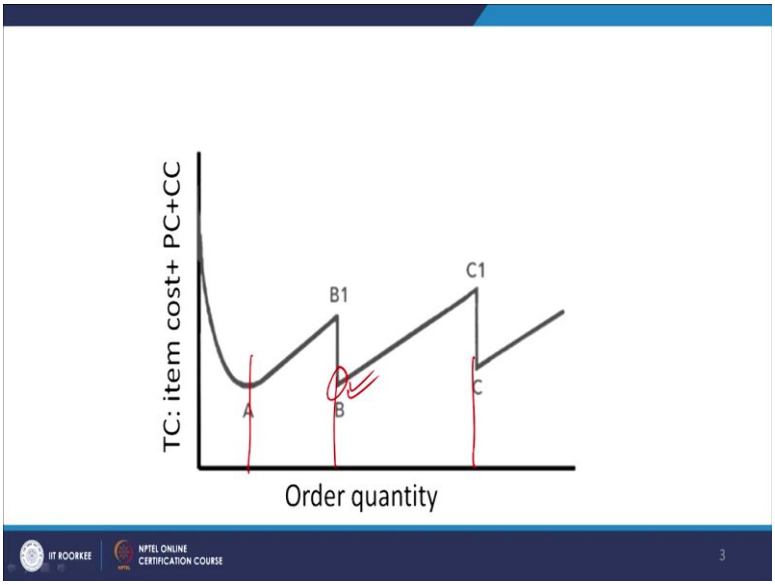

So, this is the same thing which has been shown like when there is no price break, this is what we get. When there is a price reduction with on placing the order beyond a certain limit, then we get the price reduction and then economic order quantity is reduced accordingly. So, now I will summarize this presentation. In this presentation, basically, I have talked about the gradual replenishment model, how to determine the economic/ order quantity for gradual replenishment model and if there is the benefit of the quantity discount then the economic order quantity and the total cost for maintaining the inventory should be calculated differently. Thank you for your attention.