Principles of Industrial Engineering
Professor D K Dwivedi
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee
Lecture 39 - Inventory: Wilson Model

Hello! I welcome you all in this presentation related with the subject Principles of Industrial Engineering and you know we are talking about the inventory, we have talked about the various modules related with the inventory and we have noticed that it is useful to make some investments in the inventory so that it can help in continuous and smooth functioning of the production processes and it will help to avoid the stock out situations as well as it will make the things available whenever they are needed and wherever they are needed.

So, that is the primary function of maintaining the inventory. But since the inventory involves investment in form of keeping the items in store, in form of raw material or the semi-finished goods or the finished goods, so how much we should make the investments in the inventory at each and every stage so that the purpose of the inventory is fulfilled? So, the quantity of investment is directly influenced by the kind of quality that we have in inventory, more we order greater with the average inventory size and greater will be the investment accordingly.

So, for optimizing the size of inventory or optimizing the cost incurred on account of the inventory, it is important that the order quantity is identified carefully and thereafter, the point of reorder is also identified in optimum manner so that the total cost of maintaining the inventory is minimized while the purpose of the inventory, purpose of maintaining the inventory is served. So, in this connection, it is required that the proper order quantity is identified and the reorder point is also identified as far as the two important aspects or the management aspects related with the inventory management.

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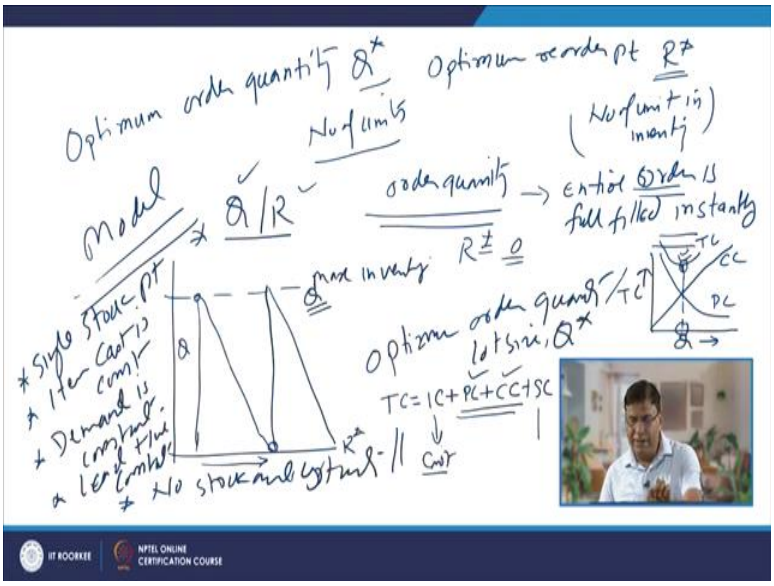

So, determination of the optimum order quantity that is Q star and optimum reorder point. So, that the items are always available whenever they are needed and we do not come across the situation of the out of stock. So, optimum reorder point is identified so R star, so these are the levels, these are the like say the number or the quantity of the units to be ordered and the number of units in inventory when the order is placed again, number of units in inventory when order is placed again, so reorder point is about the number of units in inventory left, thereafter order is placed.

So, determination of these two will heavily depend upon the kind of the model that is being adopted for maintaining the inventory, one the simple Q R model. According to the simple Q R model where whatever order quantity, whatever the order is placed for the given order quantity, the entire order is fulfilled or the order is completed instantly, so the whole of the order quantity is supplied and it is done instantly. So, these are the two important aspects and since the delivery is instant, so in this case we have the situation of reorder point is 0, because we get as soon as the items are finished in stock the order is placed, so that we get the items in stock as per the order.

So, in this Q R model like say the order is placed, so for the item for the quantity Q and it is supplied instantly and thereafter its consumption starts as a function of time. As soon as it is depleted to the 0 level, again order is placed, so this is how the cycle is repeated. So, this is the period when consumption is made and the maximum inventory level here is corresponding to the order quantity, because entire order is being obtained in one go. And the reorder point here is at 0 level, because the delivery is instant on the placement of the order.

So, apart from this Q and R, determination of the optimum order quantity or optimum lot size, few lot size that is Q star, optimum lot size or optimum order quantity, determination is based on the consideration of the various assumptions. Like there is a single stock point, item cost for which inventory is being considered is fixed, item cost per unit item cost is constant, the demand for the item is constant, the lead time for the item is constant. So, all these things are constant.

And the one more thing, no stock out situation, so for these situations how to determine the optimum lot size for minimizing the total cost of maintaining the inventory, last time we have seen the total cost for maintaining the inventory considers the item cost plus procurement cost plus carrying cost, inventory carrying cost and plus stock out cost. So, if we, if our item cost is constant then this factor does not matter, if the stock out situation is absent, then this is, there will be no stock-out cost.

And we are left with the two factors only wherein the total cost is directly influenced by these two factors only, where the order quantity and the total cost varies in a simple manner, where carrying cost increases linearly with the increase of the order quantity because of the increased average inventory size while when we place the lesser order for lower quantity, we need to place many orders and that that is why your our procurement cost is high and procurement cost decreases rapidly with the increase of the order quantity.

And therefore, there is optimum total cost. So, this is procurement cost, this is carrying cost and this is the total cost curve. So, total cost curve gives us the optimum quantity for which maintaining the inventory offers the order quantity for which maintaining the inventory becomes at, this inventory can be maintained at the minimum possible total cost. This is a situation when all these assumptions are considered like item cost is fixed, demand is fixed, lead time is fixed.

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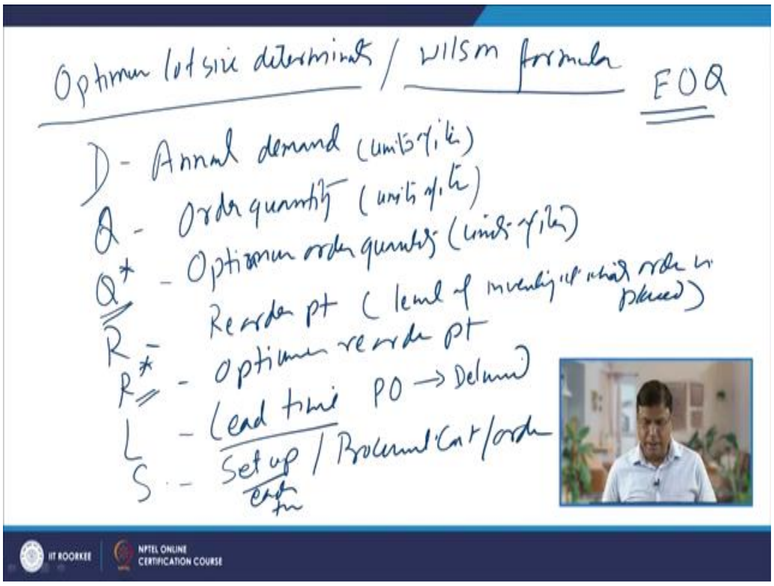

So, now we will see, there are various parameters and which we need to consider in optimum lot size determination. And this theory was developed by Dr. Wilson, so it is also called Wilson formula for optimum order quantity or EOQ economical order quantity. So, various factors are considered in determination of this optimum lot size or the economic order quantity. And these parameters are like the D is the annual demand, which we are assuming is constant.

So, annual demand in inventory we always determine the optimum lot size for annual demand only, even if the demand is quarterly or monthly it is given, we calculate the annual demand for determining the optimum lot size. So, we have to mention the units of item desired annually. Then next is the Q that is about the order quantity, the quantity for which order is placed, again it will have the units of the item like 1000 kg or 1000 units of particular item or 10,000 litres of particular liquid, so like that which is required for maintaining the inventory.

Then we have the Q star, so this order quantity may not be the optimum but Q star is considered as optimum order quantity, order quantity. So again, it will have some units of the item for which order will be placed. But this Q star will be giving us the total, minimum total cost of maintaining the inventory, total minimum cost of maintaining the inventory annually. Then we have already talked about that, this is the reorder point or it talks about the minimum level of the inventory, thereafter the level of the inventory at which the order is placed again for supply of the items.

So, level of inventory at which order is placed again. Then here this may not be the optimum one, so the R star is about the optimum reorder point, optimum reorder point, again it is the same level of the optimum level of the inventory at which the order is replaced. Then we have the capital L which indicates the lead time, lead time is the time gap, which is there between the placement of the order and the item is being delivered.

So, this time gap is known as the lead time, it may be 5 days, it may be 2 days, it may be 0 for instant delivery, it may be 15 days on that has to be taken into account when identifying the optimum order point. Let us take then the S is the set up cost, like in case of manufacturing determining the optimum best size, but in case of the procurement for maintaining the inventory, it is the procurement, a set of cost each time and here procurement cost per order, so that is the S or the set-up cost.

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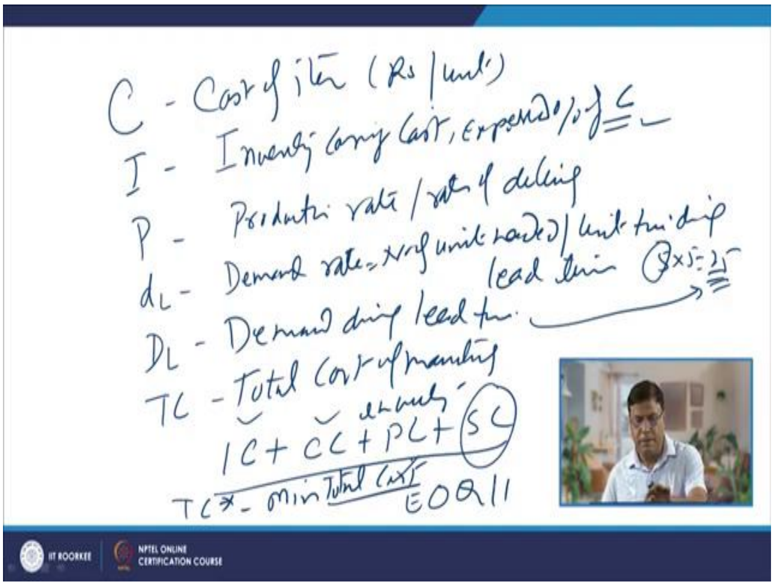

Than we have capital C the cost of item, per unit item, so here it may be in like say the rupees per unit, the price of the item. Then I is the inventory carrying cost, it is expressed as, it is expressed as a percentage of the C, percentage of the carrying cost, so I is basically expressed as a percentage of the item cost or the unit price of the item. In those cases, where the delivery is instant, the production rate does not matter.

But the production rate is about the rate at which we are getting the delivery of item, so capital P can be used to define the condition when the delivery is made in steps and it is the whole of the lot is not supplied in one go, so the delivery is made in parts or the things are produced in steps to fulfil the demand, so the P indicates the position of the indicates the production rate or the rate at which delivery is received, we can say the rate of delivery in case of the and when the delivery is made in stages.

Then we have the small d L that is about the demand rate like number of units consumed per unit time, number of units needed per unit time during the lead time, like order has been placed but the delivery has not been made and delivery will be made after some time, so during the lead time what is the rate of consumption or the demand rate? Then we have the like say the demand rate is 5 units per day and the lead time is 5 days, then the 5 into 5 that is 25 will be the demand during the lead time.

So, this is the total demand during the lead time. So, the demand during the lead time will be 25, if the demand rate number of units per unit time is 5 for a 5 day lead time, the total demand will be 25. So, that is what we can say that capital D L. Then we have another word TC that is about the total cost of maintaining the inventory. So, here we will have the item cost plus carrying cost plus procurement cost plus stock out cost, so all these costs are considered in determining, if this is absent that is fine.

So, in that case 3 will be considered primarily. And then there will be a TC star that is about the minimum total cost and it will be determined on the basis of the optimum lot size or economic order quantity, that is the minimum total cost of maintaining the inventory. So, these are the different kind of the things which are, different parameters which are considered in calculation of the optimum lot size.

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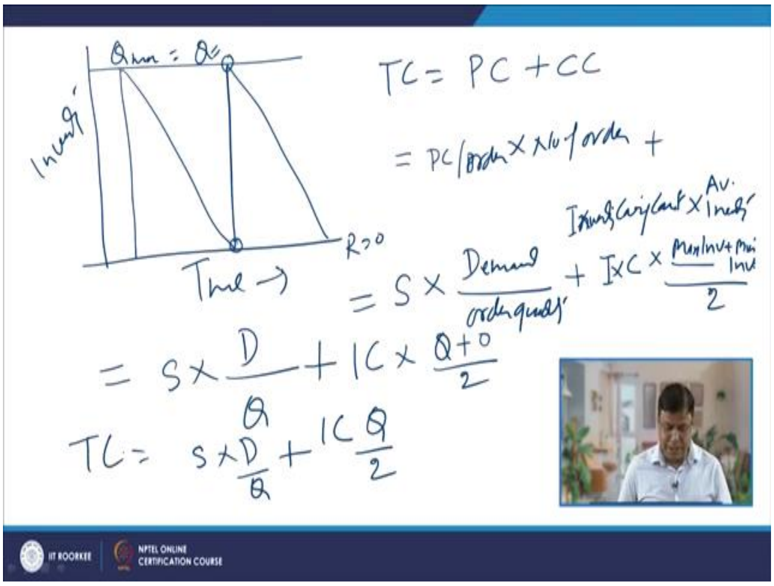

Now, we will see as I have said that for simple Q R model, for simple Q R model, we are considering situation where the order entire order is supplied in one go and it is consumed at a constant rate, so the demand is constant and the lead time is clearly known and that is constant, our reorder point is here 0 and the Q maximum is equal to the order quantity Q, here inventory level and this is the time. So, if we see here, the total cost is a sum of, for the situation where item cost is fixed, no stock out situation and the demand is clearly known.

So, in that situation our total cost equation becomes like procurement cost plus the carrying cost. So, procurement cost is the function of the like the order cost or procurement cost per order into the number of orders being placed, carrying cost is about the inventory carrying cost into the inventory, carrying cost per item into the average inventory which is being maintained. So, if we see if the procurement cost per order is say S then number of orders will depend upon what the total demand which is there, annual demand divided by the order quantity, the quantity for which order is being placed, if the order quantity is less and for a given demand then many orders we need to place.

So, this is one thing, then inventory carrying cost per item, that is I into C, I is the percentage inventory carrying cost and C is the item cost. So, that will be giving us the inventory carrying cost per item and then we have to consider the average inventory. So, average inventory means what is the maximum inventory plus minimum inventory size divided by 2, that will give us the average inventory side.

So, if we take up it, if we take it up further, an S into D is the demand, capital D is the demand and order quantity is Q plus I C, I is the inventory carrying cost in percentage and C is the item cost into the maximum inventory here we have seen since the delivery is instant and entire order is fulfilled in one go, so the Q max or the maximum inventory level is equal to the order quantity.

So, here Q is equal to the maximum inventory level and since the delivery is instant, so we place the order when the order is reduced to the, when the inventory level is reduced to the 0. So, the Q minimum is 0 divided by 2. So, this will be giving us S into D by Q plus I C, Q by 2, so this is the total cost equation, this is not the optimum cost equation.

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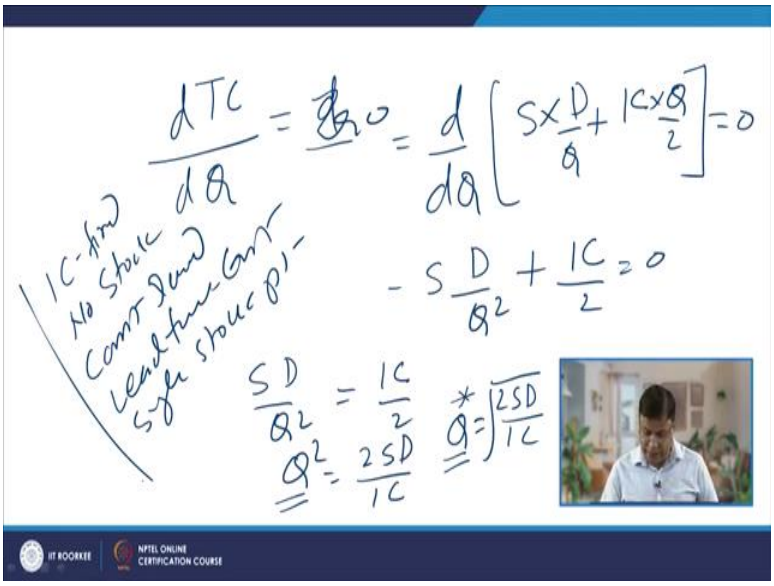

So, what we need to do, according to the calculus method the total cost TC, total cost TC is differentiated TC is differentiated with the, with respect to the order quantity d Q, so and we make it equal to 0 to minimize the total cost of maintaining the inventory. So, that case d by d Q, then we have the total cost equation that is equal to S into D by Q plus I C into Q by 2, when we differentiate this what and this is made equal to 0. So, what we get here, S into D by Q square plus I C by 2 this is equal to 0, so what we get basically I C by 2, this is equal to S D by Q square, so Q square will be equal to the twice of S D by I C.

Since we are interested in the, interested in determining the Q value, that will be giving us the minimum total cost for maintaining the inventory, then this Q becomes equal to the square root of twice of SD divide by I C, this becomes the our economic order quantity or the economic optimum lot size formula for maintaining the inventory. But this is true when the item cost is fixed, no stock out and there is a demand constant demand and the lead time constant and single stock point. So, these are the kind of the assumptions for which this is true. In this case, we have assumed that the item cost is constant.

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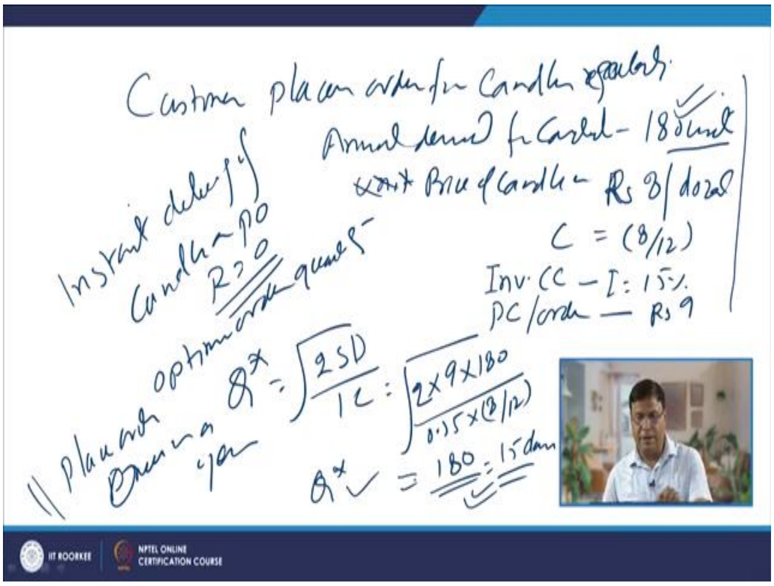

So, considering this as economic lot size we will consider now the two examples for determining the optimum lot size, say there is a customer who places the order for candles, customer places order for candles regularly, for candles regularly and say annual demand for candle is let us say 180 units and the annual demand is constant and that is 180 unit. And unit price or the price of candle is like rupees 8 per dozen. So, now so in that case unit price will be 8 by 12, so that will be the C.

Then inventory carrying cost I, inventory carrying cost is like say I is the say here 15 percent of the C. And then we have the order cost, the procurement cost per order is rupees 9. So, all these values now we have and we have to determine since in this case we get the instant delivery of candles on placement of order. So, obviously, in this case we will be choosing the optimum reorder point at 0 level, because the delivery is instant and entire item is delivered in one go on placement of order. So, the problem is left to the determination of the optimum order quantity.

So, to determine the optimum order quantity, we will be using the same formula twice S D divided by I C, so here what we have? The twice of the S is the set-up cost or the order cost. Now, the procurement cost per order that is 9 and the demand is our 180 and I is the inventory carrying cost that is of 15 percent, so 0.15 and the C, C is the unit price, so it is 8 by 12 that will be the unit price, so on solving this we get the optimum order quantity of 77.9.

So, that is equivalent to 78 number of units required, no here on solving we will be getting, on solving we will be getting the 180 value. So, 180 is the economic order quantity, which comes out to be 15 dozen, annual demand is 180 units and optimum order quantity is also coming 180 units, which means we need to place the order once in a year, once in a year. So, that is the kind of observation, optimum order quantity is matching with the annual demand, so order is to be placed once in a year.

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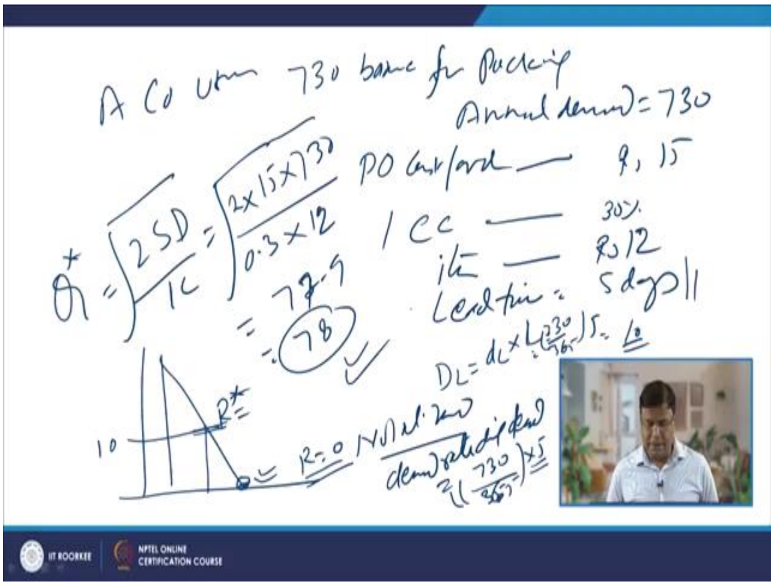

And there is another example where a company uses the packing box to pack the items and say 730 boxes are used for packing the items for customers, so this is the annual demand, annual demand for boxes is 730 units and the procurement cost or the placement or the PO cost per order here is rupees 15 and the carrying cost is, inventory carrying cost I is like say 30 percent and the item cost is rupees 12.

So, in this case, say lead time to deliver the item is 5 days. So, here we have one additional factor of the lead time and in this case the reorder will not be happening at 0 level, but we have to place earlier when there are certain number of items in the inventory. So, as far as the determination of the economic order quantity is concerned, we have to use the same formula twice SD divided by I C, so here twice of S is our 15 and demand is 730 divided by 0.3 is the inventory carrying cost in percentage and C is the item cost 12.

So, this will be giving us the number of units desired like 77.9, so coming is about equal to the 78 number of the boxes to be ordered in per order. So, when this is to be reordered, in earlier cases what we have assumed that order is placed when the items are reduced to the 0, but here since there is some lead time, lead time of the 5 days, so before it reaches to the, so during the lead time some of the items will be consumed. So, in this case, it will not, reorder point will not be 0, it will not at 0, but it will be before that, so what will be that level of the inventory when order is to be placed? To determine that you have to see how many items we need during the lead time.

So, since the demand rate, demand during the, demand rate during the lead time will be obtained from the annual demand divided by the number of days, annual demand is how much here? 730 and the number of days in a year say 365, so this will give us the demand rate during the lead time. So, here we have the lead time of the 5 days. So, when we multiply this 730 divided by 65 into the 5 days is the lead time, this will give us the DL, so DL is the small l d L small d L into the lead time, it is the L.

So, here 730 divided by 365 into 5, this will give us the lead time say it is 10 units. So, order will not be placed at the 0 level, but when the number of items left in the inventory are 10, then the that will be the optimum reorder point. So, reorder point here will be the 10 number of units, inventory and at the time the order will be placed again for 78 units. So, this is how we can understand the case is related with the lead time and there is a clear lead time, clear demand and the every detail is very clearly given. So, now I will summarize here this presentation.

This presentation basically, I have talked about how to establish the optimum lot size formula and how to take into account the lead time in determining the reorder point. Thank you for your attention.