Acoustic Materials and Metamaterials
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Lecture - 06
Sound Propagation at Medium Boundaries-II

Welcome and today this is the lecture 6th on our course on Acoustic Materials and Metamaterials. I am Dr. Sneha Singh of the Department of Mechanical and Industrial Engineering at IIT Roorkee. And we were discussing about Sound Propagation at Medium Boundaries. So, in the last class we studied what happens when sound is incident normally on the medium boundary which is a planar boundary. And we derived certain equations for the reflection coefficient and transmission coefficient.

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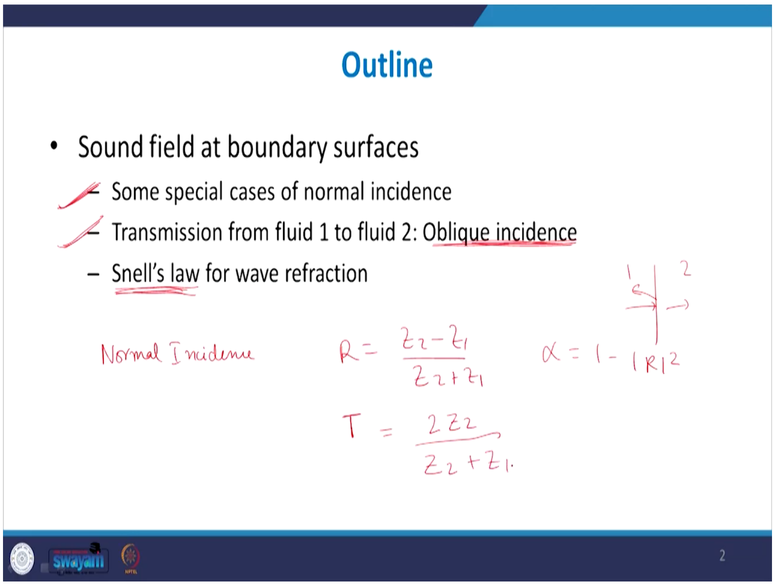

Now, in this class we will study about some special cases of normal incidence and then we will proceed into our discussion on the transmission from fluid 1 to fluid 2 in the case of oblique incidence. And while we study oblique incidence we will encounter a very important law which is called as the Snell's law for wave refraction.

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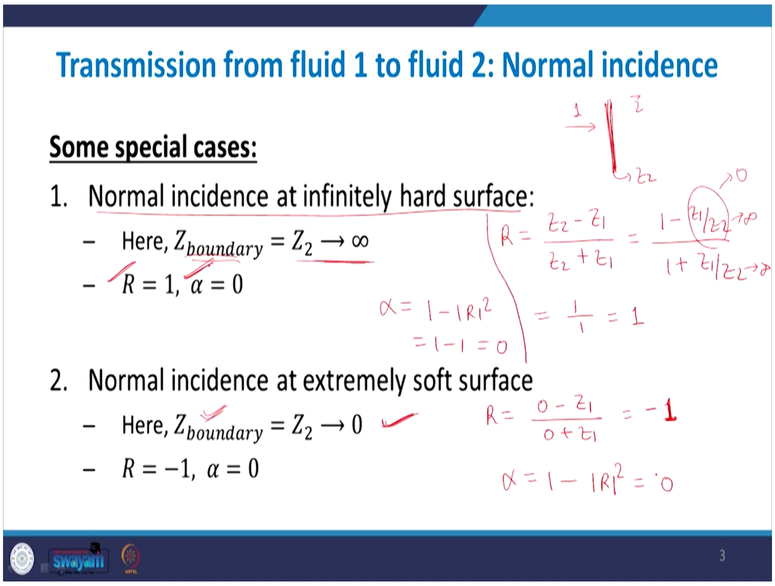

So, you already know that in the case of normal incidence, so normal incidence here R is given as Z 2 minus Z 1 by Z 2 plus Z 1 for transmission from 1 to 2. And alpha is 1 minus mod R square and transmission coefficient is the pressure transmission coefficient becomes 2 times of Z 2 divided by Z 2 plus Z 1.

So, what happens? Let us say when there is normal incidence at infinitely hard surface, so example of infinitely hard surface can be any hard walls, so any hard extremely polished reflecting walls. So, when the normal incidence is infinitely if we if it is normal incidence on an infinitely hard surface which means that the surface is acoustically very hard, so it does not allow the sound waves to pass through.

So, the resistance to sound flow is almost infinite, so that means that the Z of boundary will be infinite. So, when you use this Z 2 as infinity then R which is Z 2 minus Z 1 divided by Z 2 plus Z 1 you can write this as 1 minus Z 1 by Z 2 1 plus Z 1 by Z 2 and this Z 2 tends to infinity right.

So, this overall quantity will tend to 0, so what you get is 1 by 1 which is equal to 1. So, whenever a surface is infinitely hard or it has infinite impedance the entire wave gets reflected back there is no transmission, so it completely blocks the sound. So, that is meant by a infinite impedance that the resistance to flow is infinite, so no sound passes through the boundary surface.

And alpha in that case becomes 1 minus mod of R square which is 1 minus 1 which is 0, so you get full reflection, 0 absorption, no transmission. The second case let us say we have an infinitely soft surface which means that it offers almost no resistance to the flow of sound waves, what will be the case then? In that case the Z 2 or the Z of boundary, so, here the sound waves are propagating from medium 1 they are going into medium 2 and this boundary Z 2.

So, the Z of boundary will be same as the boundary due to the medium 2 which is going to be approximately tending to 0 for an infinitely soft surface. In that case R will be 0 minus Z 1 divided by 0 plus Z 1, Z 2 is 0. So, it will be minus 1 and alpha will be 1 minus mod R square which is going to be again 0.

So, in this case what you get is that whatever waves you are sending back you are getting the reflected wave, but the reflected wave amplitude is reverted and the absorption is again 0. So, both for infinitely hard surface or for infinitely soft surface, the absorption is 0. And in the first case the wave gets entirely reflected by in the second case it is like the wave is getting reflected with a reverted wave front.

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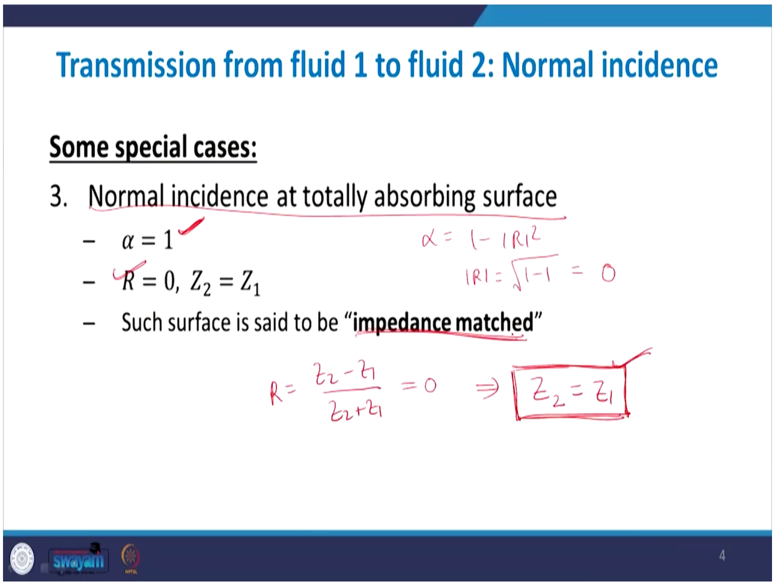

The third special case is what if we have a totally absorbing surface. So, let us say we have this particular building or a classroom and all the walls here were hard surfaces they were completely reflecting. So, in that case there is no transmission, alpha 0 and R is equal to 1.

Now, let us we if we line this surface with some acoustic material and that acoustic material is a very good absorber, and it absorbs whatever sound is incident on them. So, what will be the case in that? So, when there is normal incidence at totally absorbing surface which means that the alpha of that surface is 1 because it is total absorption.

So, alpha will become 1, no sound will be reflected back, because alpha is equal to 1 minus mod R square. So, mod R will be 1 minus 1 under root which is 0. So, overall there will be no reflection, because by definition itself alpha means that it gives you what fraction of energy that is being lost in reflection.

And if alpha is 1 which means that all the energy is being absorbed or transmitted, no energy is coming back all the energy is lost. So, R will be 0 and what can be such a case and a very good example of such a case is R is equal to Z 2 minus Z 1 by Z 2 plus Z 1 and this comes out to be 0.

So, this can be possible when Z 2 is equal to Z 1. So, when we study about acoustic materials and meta-materials in our subsequent lectures this is an important concept. So, whenever two mediums they have same impedance which means that it is virtually or effectively for a sound wave the two mediums are same, because they rho c is same.

So, in that case the reflection is 0, alpha is 1, so this is called as an impedance matched medium. One example of this can be if we take the same classroom we had some referred reflecting walls then we had some absorbing walls which was lined with absorbers, now we have a small window in that classroom.

So, the medium inside the classroom is air at room temperature and the medium just outside the classroom through the window. So, window is a boundary here and the medium at the boundary of the window is also air. So, this window is actually a boundary between air at room temperature and a boundary between air at room temperature, so both medium are same.

So, the impedance inside and outside of the window is the same, so the window here is fully absorbing. So, effectively when the medium is a full absorber usually it means that it is treated as a same medium and the sound wave just propagates through no reflection takes place.

Now, it is these special cases and the understanding of what is meant actually by reflection, absorption, and transmission. Now, we will proceed into the second case which is a slightly more difficult case which is when a wave front is incident at a particular angle not normally, but obliquely on the boundary surface.

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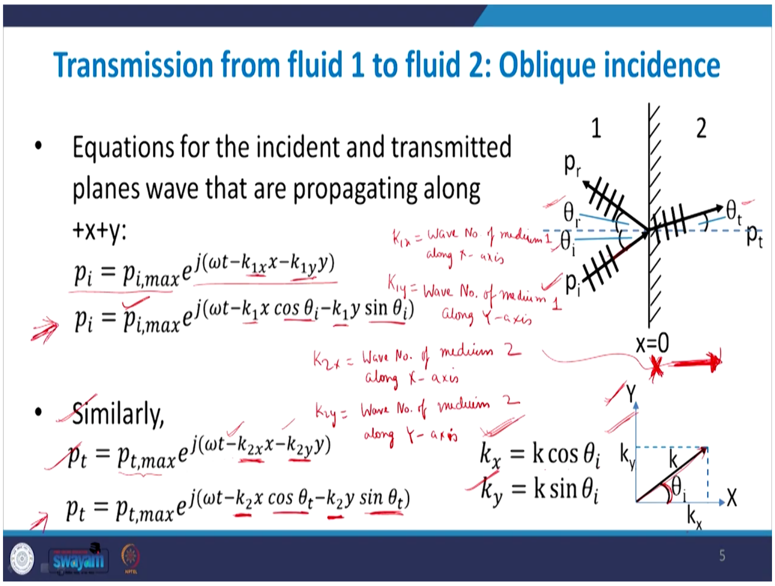

So, here I am giving you the schematic X equals to this is the positive direction of X just like in the previous class. So, this is the schematic and p i is being incident at some angle theta i. So, let us say transmitted wave has some angle theta t and p r has some angle theta r all these angles are the measurement all these angles are angle between the direction of the wave front so, in the direction of the wave propagation or the direction of k vector and the normal to the boundary surface.

So, all of these angles are measured with respect to the normal to the boundary surface and the direction of that wave propagation. So, here as you see this incident wave is now no longer a wave that is propagating along only the x direction.

It is a harmonic wave that is propagating along the X Y plane. So, here I have drawn a top view of this X Y plane and let us say this is the incident wave that is propagating along this X Y direction.

So, the general equation can be written in both x and y terms as p i max which is the amplitude into e to the power j omega t minus k 1 x x minus k 1 y y. So, here k 1 x is the wave number of medium 1 along X axis and similarly k 1 y is the wave number of the medium 1 along the Y axis so on.

So, this is the general form that you can write. Now, in when we were discussing about propagation vector a few classes before we said that the propagation vector is simply the equivalent vector due to the components along X, Y and Z axis. So, if we have different we have different wave numbers along X, Y, Z axis then their equivalent vector will be the k vector.

So, in this case you have this k and this is the angle theta i which is the angle between this normal and the wave direction or the direction of propagation. So, this is the angle theta i, so the component of k along this X axis is what? It is k cos theta i. So, this k x is k cos theta i and the component of k along Y axis becomes k sin theta i, so k sin theta i is the component along Y axis.

So, if we replace this values if we substitute these values of k x and k y here, so what we get is p i is p i max e to the power j omega t. Now, we have replaced this by k 1 cos theta i, so this becomes k 1 cos theta i times x, and this becomes k 1 sin theta i times of y.

So, we have replaced it with this by we have decomposed it with respect to the equivalent propagation vector. Similarly, p t is also a propagating wave along positive X and positive Y direction; p t also has this same direction as this same equation applies there the only difference will be then there the angle will be theta t this will be the theta t.

So, p t just like in p i we can write it as the amplitude into e to the power j omega t minus k 2 x x minus k 2 y y. And again k 2 x is the wave number of medium 2 along X axis and k 2 y is the wave number of medium to along the Y axis.

So, we have written it in the same form, it is a forward propagating wave, so we have the minus sign in both cases, so we can decompose it in the same way. So, k 2 x can then be replaced as the net equivalent propagation vector into cos theta t. And k 2 y can be replaced as the net equivalent propagation vector that is k 2 into sin theta t, so this is the expression we get when we substitute in the same way.

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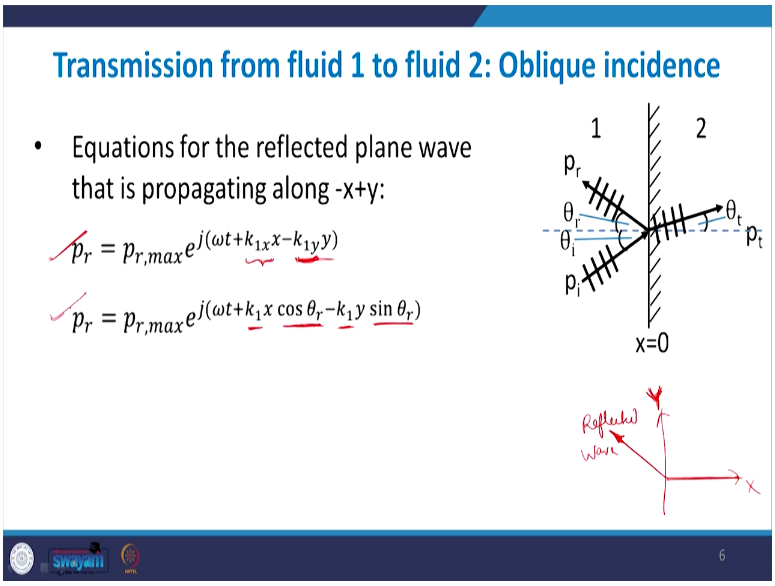

So, we are getting the equation for p i and p t; similarly we can get the equation for the reflected wave. So, reflected wave it has the directions as you can see if this is X and this is

Y, so this wave is transmitting somewhere here this is the reflected wave. So, which means it is traveling along the negative X, but positive Y.

So, here with X you have plus sign with Y you have negative sign because it is negative wave negative propagation along the X axis and positive propagation along the Y axis. So, using the same way we can decompose it. So when we decompose it, so, we if we write it in terms of the equivalent vector then this simply becomes this we replace it as k 1 x as we got the value of k 1 x before.

So, here k 1 x will be k 1 times of cos theta r and k 1 y will be k 1 times of sin theta r. So, the y component is simply the equivalent into say it is a sin theta its component and this is the cos cosine component. So, we have replaced it with these values and this is the final value we are getting for p r.

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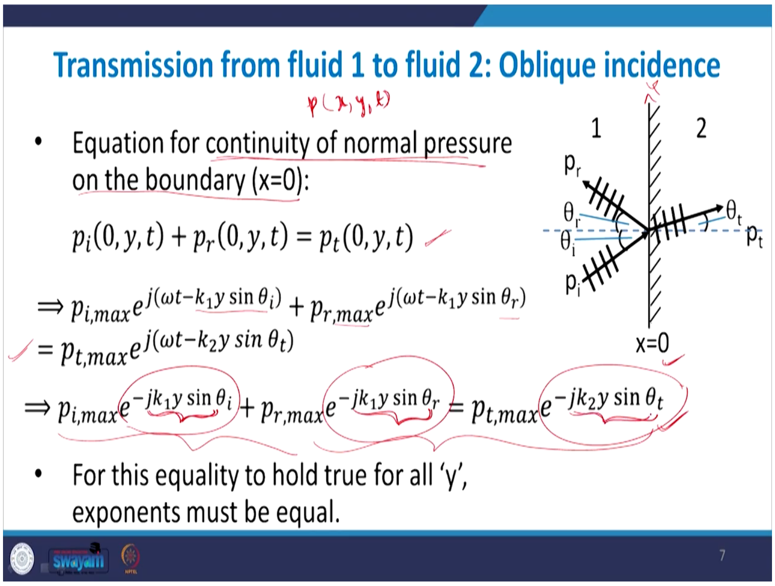

Now, applying the continuity of normal pressure at the boundary, what we get is p i at the boundary is X at 0. So, which means that p I, here p was a function of x, y and t this was a because it is a function of these 2 and t. So, x is put as0, so p i at 0 y t plus p are at 0 y t should be equal to p t at 0 y t.

So, you can replace, so whatever values we have found here. We found the values for this was the value for p i this one values of p t and the values of p r. So, we replace it with these expressions, so what we get is this is the expression we are getting. So, the k component has vanished because x is taken as 0 here, so at the boundary x is 0.

So, all these components vanish this component vanishes, this component here vanishes, and this component vanishes, so you are left with only the omega t and the y component term. So, you get p i max e to the power j omega t minus k 1 y sin theta i, similarly for this and this. So, this is minus y because it is propagating along positive y negative x, but positive y, so it is minus in every case.

Now, so this is the overall expression: e to the power j omega t is a constant which is canceled out from every end, so this is the utmost equation that we are getting this is the final equation. Now, in this particular equation this has to be satisfied for the entire boundary, so for any value of y this is the y co-ordinate right.

So, for any value of y at x equals to 0 this particular equality must hold true. So, this is a complex coordinate and for this equality to hold true these must be same these values they must be same. Then only this equality can hold true because this is a complex quantity here.

So, for this to hold to these exponential parts have to be the same which means that this particular argument has to be the same so, that this equality can hold true for every value of y, so this particular thing will have to be the same first.

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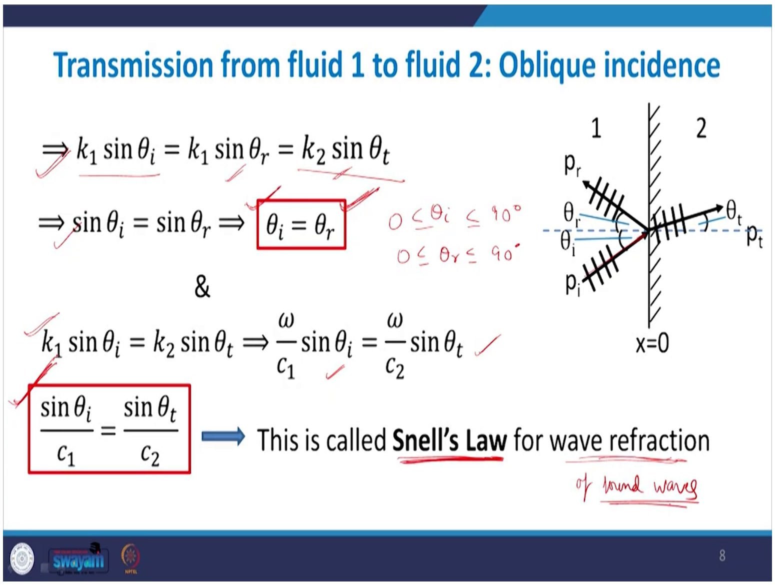

So, these exponents have to be the same, so what you get is k 1 sin theta i k 1 y sin theta i should be equal to k 1 y sin theta r which is equal to k 2 y sin theta t, so y is the common term which we have eliminated. So, what we get ultimately is k 1 sin theta i should be equal to k 1 sin theta r which is equal to k 2 sin theta t. So, this means that sin theta i is equal to sin theta r, and because they neither theta i nor theta r will be more than 180 degrees. Because it would not be more than 90 degrees because this is the angle this is the angle which is theta i.

So, if theta i can be from 0 to 90 degree and then value more than 90 degree means it is going beyond the medium 1 into medium 2 which is not possible. So, they both are they both have the domain between 0 to 90 degree. So, in that case these angles must be the same because both have to be between 0 to 90 degree. If it is more than 90 degree which means that the wave is going into the second medium which is not what we have what is being assumed,.

so this is the wave in the first medium, so the maximum theta can be I, theta can be 90 degrees. So, in 0 to 90 degree if 2 and 2 sin angles are same which means they those theta values have to be the same. Similarly, we have k 1 sin theta from this particular relationship this one and this one, k 1 sin theta i is equal to k 2 sin theta t.

So, which means omega by c 1 sin theta i is equal to omega by c 2 sin theta t. So, the ultimate relationship that we get is sin theta i by c 1 is equal to sin theta t by c 2. This is a very important law which is called as the Snell’s law for wave reflection.

A similar law exists for electromagnetic waves and in the field of optics, so when the light is incident and the light ray gets deflected they have to follow the Snell’s law which states that the ratio of the science of the angle of incidence and transmission should be the same as ratio of their respective speed of sounds. So, the same Snell’s law applies for wave refraction of sound waves, so whatever we are studying here these are all sound waves.

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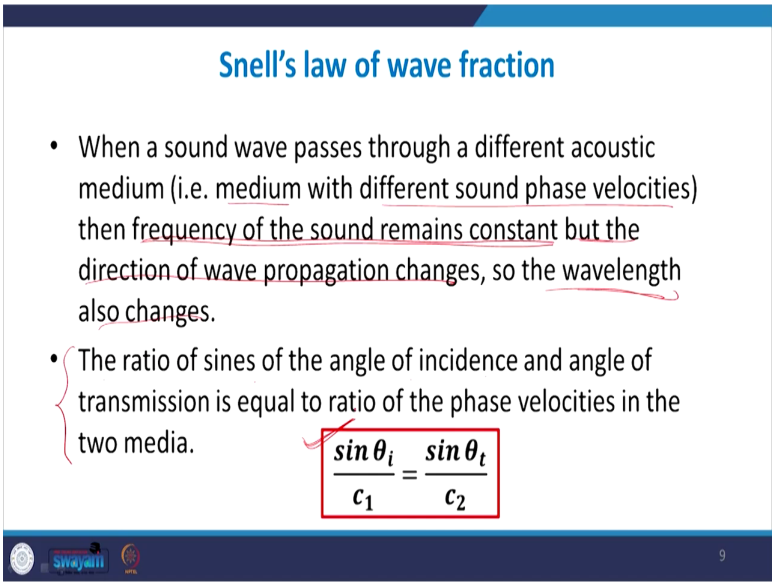

So, to state it more clearly what does the Snell's law say? So, overall what it says is that when a sound wave passes through a different acoustic medium that that is a medium which has got different sound phase velocities. The frequency will; obviously, be the same because they are all created from the same source.

So, when the sound wave is passing through a different medium frequency remains constant. But, the direction of the wave propagation can change and the wavelength will also change, this is in the case of oblique incidence.

So, the ratio the Snell's law according to Snell's know the ratio of the science of the angle of incidence and angle of transmission is equal to the ratio of the phase velocities in the 2 media this is how d. So, this is the relationship between the 2 angles, so we got these two relationships at theta i is equal to theta r and this is the third second relationship ok,

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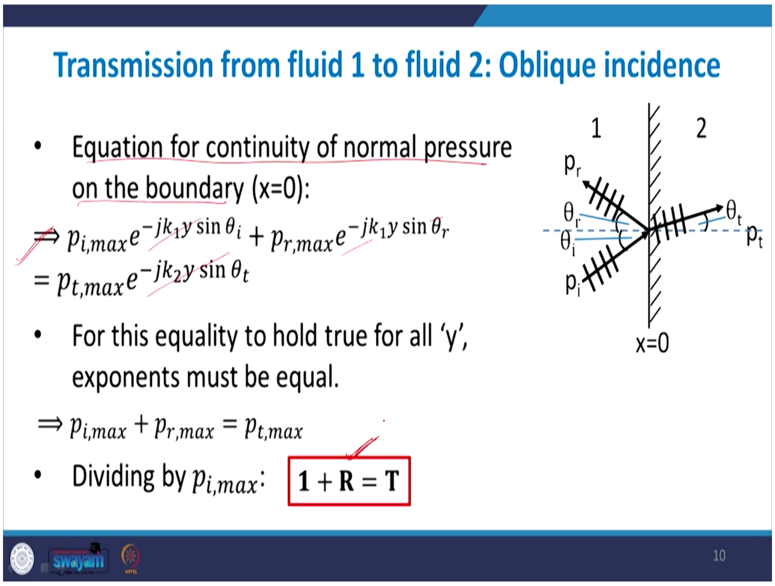

Now, we have got these relationships let us again go back to the problem. So, we have applied the equation for continuity for normal pressure on the boundary, so this is what we were getting right. Now, these exponents are same, so let us cancel out these exponents.

So, we are left with p i max plus p r max is equal to p t max, you divide the whole thing by p i max, so you get 1 plus R is equal to t. So, same relationship as in the case of normal incidence the pressure transmission coefficient is 1 plus the reflection pressure reflection coefficient, so 1 plus e is equal to t.

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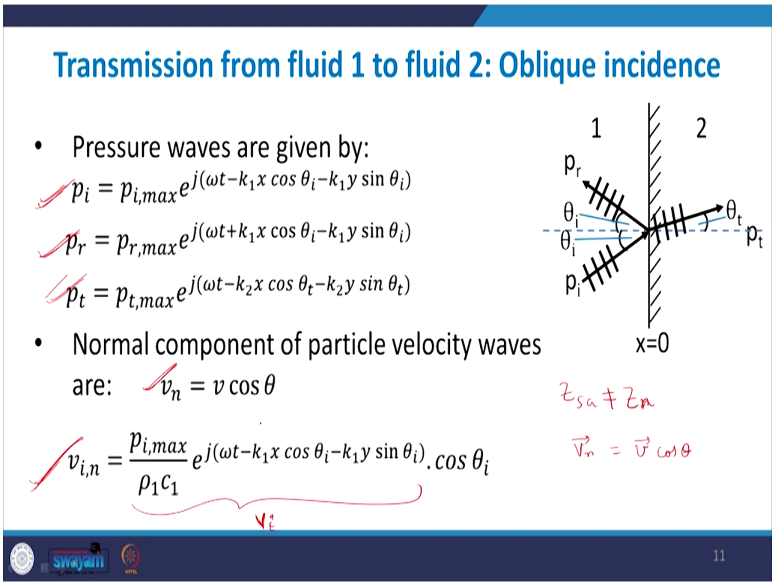

Now, we let us derive the equation for the impedance of the system. So, p i we found the expression for p i was this p r and p t, so this was the expressions we found in the previous in the first part of the lecture. So, the normal component of the velocity here because it is in this case v is incident at some angle

So, Z sa will not be equal to Z n, so the normal specific acoustic impedance and the specific acoustic impedance will be different. So, and the normal component of the velocity will be what it is the actual velocity into cost times the angle that it is making with the normal of the boundary, so it becomes v cos theta.

So, this is what we have taken, so the normal component of the incident wave is what it will be p the pressure of the incident wave divided by rho 1 c 1. So, this is the actual velocity of the incident wave which is the pressure of the in the pressure of the incident wave divided by rho and c 1 then multiplied by cos theta will give us the normal component.

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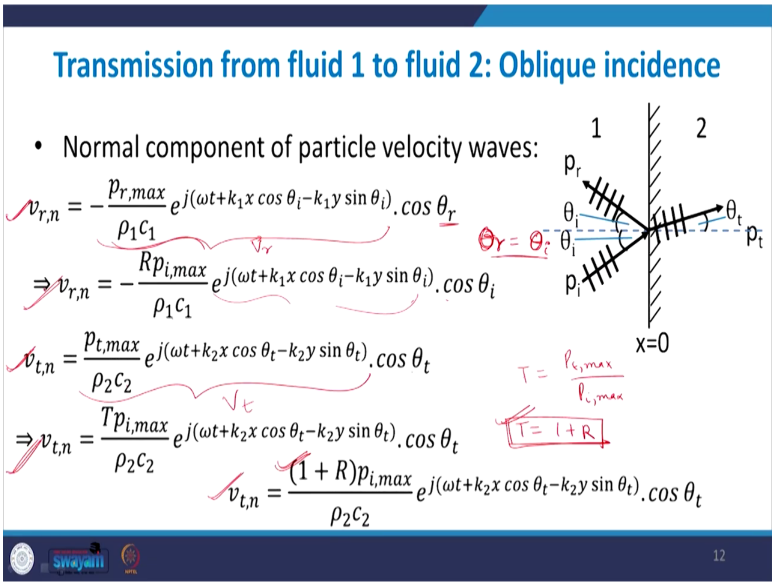

Similarly, the normal component of reflection coefficient will be minus p by rho 1 c 1 this particular quantity will give us actually if v r. And multiplied by cos theta R will give us v r n and this cos theta R will be here theta r is equal to theta i, so we replace this theta r with theta i

So, the ultimate expression we get for normal component of the reflected wave is minus R times of p i max by rho 1 c 1 into this comp this expression into cos theta i. Similarly, you can find the normal component of transmitted wave it is again because this is a forward propagating wave we have this by rho 2 c 2 into cos theta t.

So, this is the expression this is the expression for v t multiplied because theta t will give us the normal component of v t. So, we can replace this p t max as t times of p i max because, t is p t max by p i max. So, p t max will be t times of p i max and similarly we have replaced here r.

So, these are the various expressions we have got for the normal component of the velocity as well as for the pressures of the three waves. So, what you see here is that T is equal to 1 plus R we have already found from the continuity of pressure that T is equal to 1 plus R. So, putting replacing this T with 1 plus R this is the final expression we get for the normal component of the transmitted wave.

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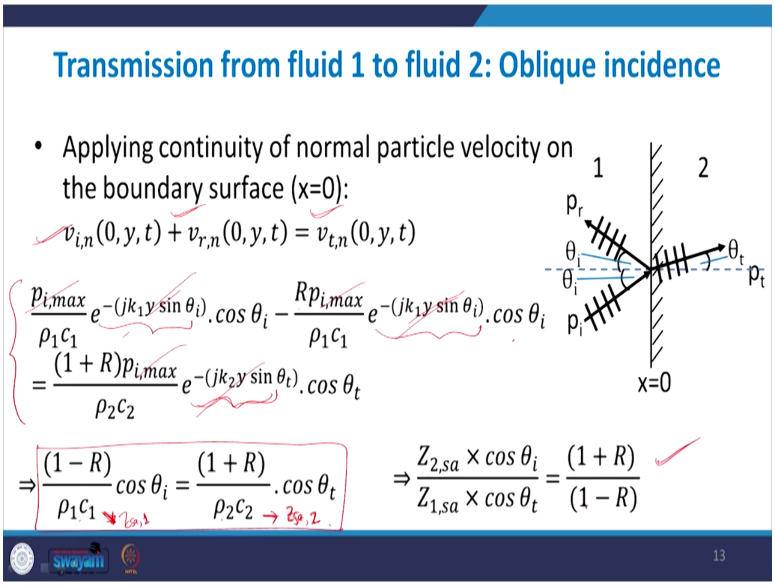

So, applying the continuity of normal pressure velocity on the boundary surface what we get is v i n plus v r n should be equal to v t n and so. We are applying the continuity of normal pressure along the boundary. So, in that case because the boundary itself is not moving the boundary has zero velocity, so we are just equating the velocity just on the left and the right hand side.

So, putting these expressions, so these were the expressions that were derive these are the long expressions we derived for the three waves and we put x equals to 0 in this case, so this particular component will vanish. So, putting x equals to 0 what we get? The ultimate expression we are getting is this is the ultimate expression that we are getting. So, again because these exponents are same these are all same.

So, these can be this is the constant which can be removed from this equality these exponents are same. So, what we are left with is and we can remove p i max also, so we left with it is 1 minus R by rho 1 c 1 cos theta i. So, we have this is 1 minus R times divided by rho 1 c 1 into cos theta i is equal to 1 plus R by rho 2 c 2 into cos theta t, so this is the expression we get from this equality.

Now, rho 1 c 1 and rho 2 c 2 are the specific acoustic impedances, so this is the specific acoustic impedance of medium 1, this is the specific acoustic impedance of medium 2. So, if you replace these values here what you get is 1 plus R by 1 minus R is going to be Z 2 sa into cos theta divided by Z 1 sa into cos theta t.

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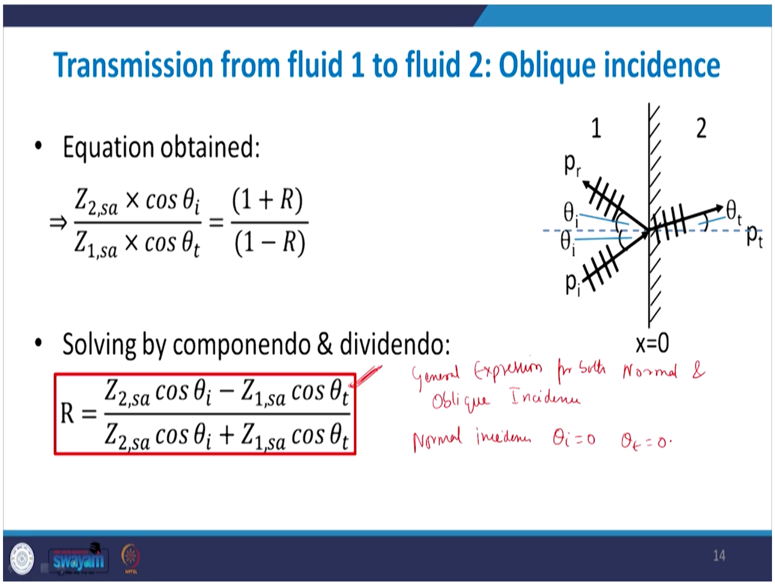

Again solving this equality using the component 2 and dividend row method what we get is R will become Z 2 sa cos theta i minus Z 1 sa cos theta t divided by Z 2 sa cos theta I plus Z 1 s a cos theta t. So, it is quite similar to the expression that we obtained for normal incidence but now here we have multiplied it with the cost of the respective angles.

So, when we multiply it with the cos of respective angles, now you can see here that this is a general expression that is applicable both for normal and oblique incidence; general expression for both normal and oblique incidence because in case of normal incidence it is incident normally. So, theta i will be 0 and theta t will also be 0, so both theta i and theta t are 0, so cos theta will become 1 cos theta t will become 1, so the expression will reduce to simply Z 2 minus Z 1 divided by Z 2 present 1.

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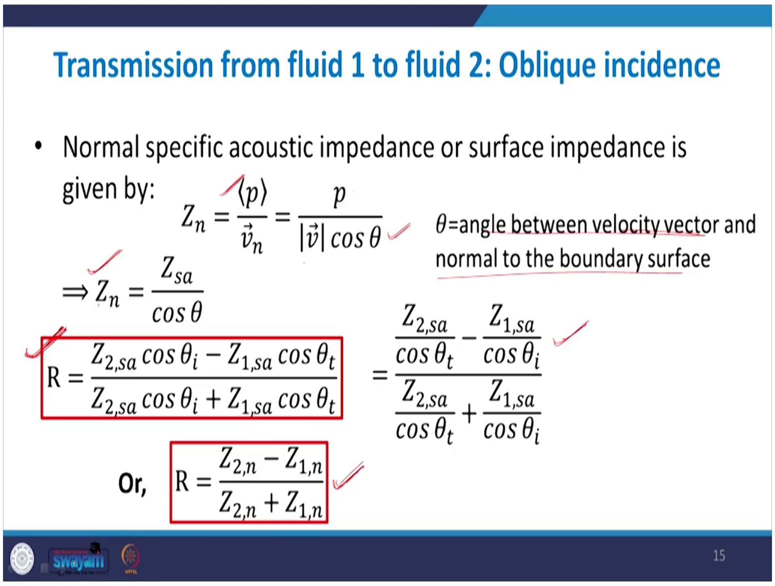

Now, but in this oblique case we have obtained this general equation. Now, there is normal specific acoustic impedance is defined as p divided by the normal component of the velocities which becomes p mod v cos theta where theta is the angle between velocity vector and the normal to the boundary surface.

So, Z n is what it is p by Z sa is p by mod v, so this becomes Z sa by cos theta. So, if you use this equation here, so this was the general expression we got in terms of specific acoustic impedance. So, when you divide both the sides with cos theta t cos theta i, so you dividing it off wave i cos theta t cos theta i.

So, this is the expression you get replace it with this value, so you get Z 2 n minus Z 1 n divided by Z 2 n plus Z 1 n. So, the normal specific acoustic impedance will follow the same expression for oblique and normal incidence that is Z 2 minus Z 1 by Z 2 plus Z 1. But, this will be the general expression in terms of this specific acoustic impedance ok.

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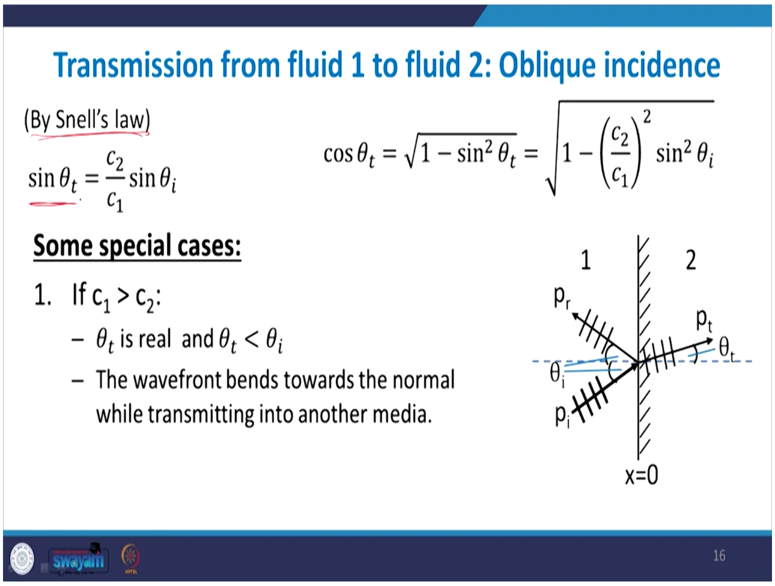

Let us study a few special cases what happens? Now by Snell's law we already get that sin theta t by c 2 is equal to sin theta i by c 1. So, the value of sin theta t in terms of theta i is c 2 by c 1 times of sin theta i. And cos theta t is under root of 1 minus sin square theta t we replace it with this particular value, so this is the expression we get for cos theta t.

So, what is the what happens when c 1 is greater than c 2 this is the first case. So, if c 1 is greater than c 2 which means that this sin this particular sin theta t is some fraction of sin theta i here so, which means that the angle the overall value of sin theta t is smaller than theta i.

So, sin theta is smaller than theta i which means that the theta t angle is going to be smaller than theta i. Theta t is real and the transmission that theta t is going to be smaller than theta i which means that suppose this is the wave front. So, the transmitted wave will bend towards the normal. So, it will bend towards the normal this angle will be smaller than this angle.

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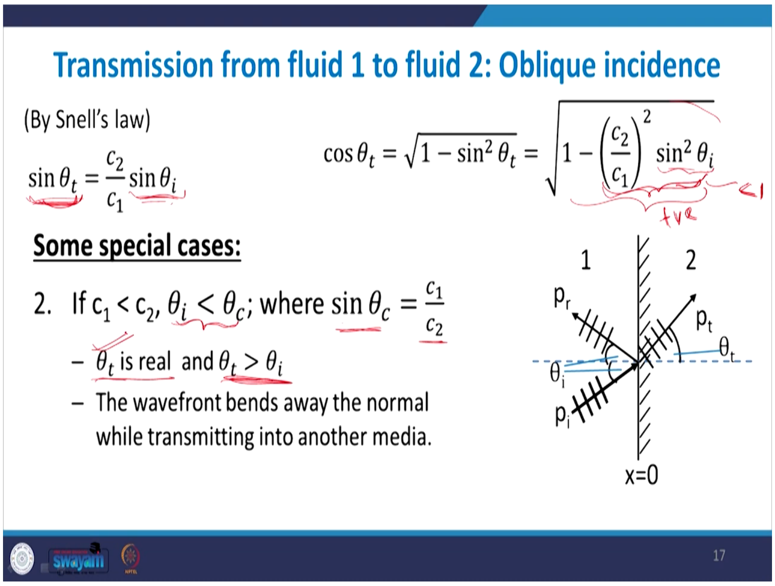

If the second case we consider now suddenly we have c 1 is smaller than c 2, and also theta i is smaller than a critical angle which is defined as c 1 by c 2 sin of the radical angle is c 1 by c 2. So, because c 1 is smaller than c 2 which means the sin theta t is a is some fraction is equal to something a quantity greater than 1 times theta i.

So, which means that sin theta t is going to be greater than theta i sin theta t is going to be greater than sin theta i. So, which means that theta t is going to be greater than theta i from this particular relationship. So, which means that the wave front will bend away from the normal, and you can also check here that cos theta t is under root of this quantity.

So, this quantity must be positive for a real solution. So, this is sin squared theta i, if we put this value sin theta c is equal to c 1 by c 2 then this value becomes 1, so this quantity becomes 1 minus 1 which is 0. But this theta i is smaller than this value, so which means that this thing that we are getting, so initially if you are multiplying it with theta c we are getting 1 value.

Now, if you multiply it with a quantity something that is smaller than the previous quantity, so overall will be this overall thing will be less than 1. So, the under root term will always be positive, so the theta t will be real and the wave front will bend away from the normal.

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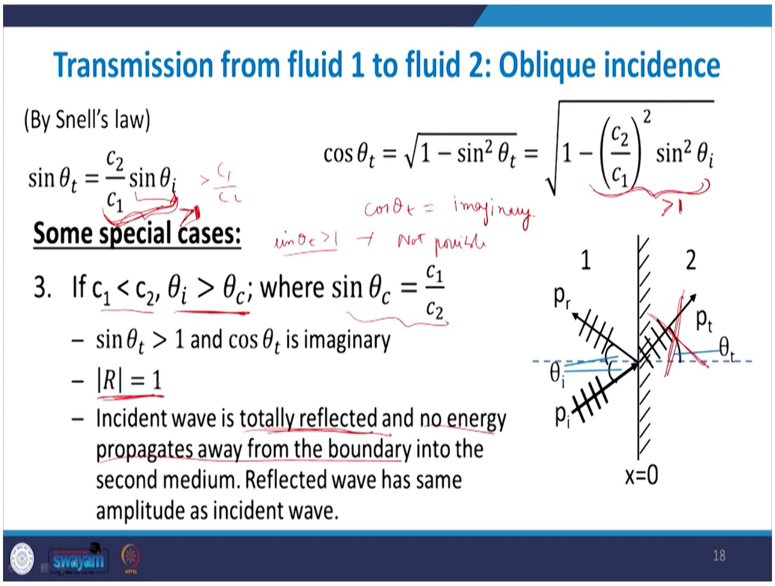

And the third case is when this holds true this and this is greater than theta c. So, in that case just from the previous discussion this quantity will now become greater than 1. So, cos theta t will become a imaginary in quantity. So, which means that this becomes imaginary and even here what you get is this is going to be a quantity which is greater than greater than c 1 by c 2, so this overall quantity is going to be greater than 1.

So, this quantity is going to be greater than 1 and this quantity is also going to be greater than 1. So, cos theta t comes out to be imaginary and sin theta t comes out to be something greater than 1 which is not possible because sin can always be between 0 to 1 minus 1 to 1 that is the value of sin; so, which means that no transmission takes place.

So, when the theta i becomes greater than this particular critical angle and c 1 is smaller than c 2 in that case no transmission takes place, so this does not take place. The entire wave front is totally reflected no energy propagates away from the boundary and the mod R in this case is equal to 1. So, with this I would like to end the discussion on the sound propagation in at oblique incidence, and in the next class we will discuss a new topic, so.

Thank you for listening to this lecture.