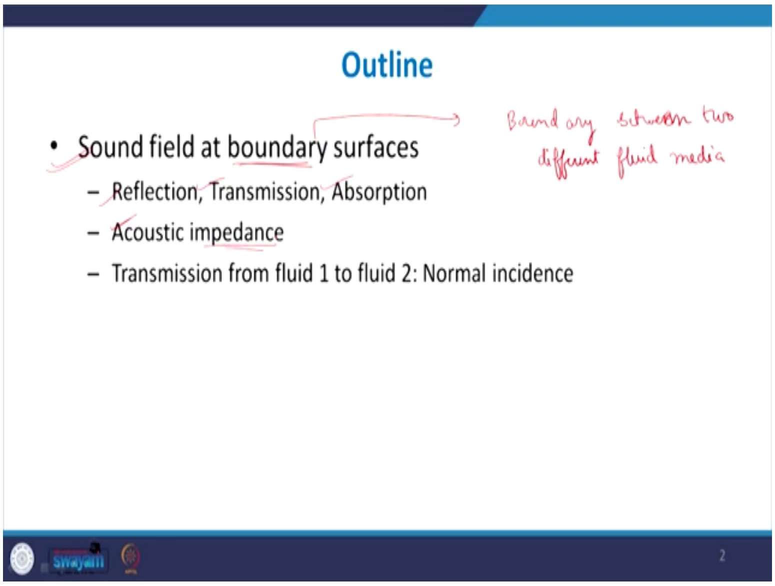
Acoustic Materials and Metamaterials

Prof. Sneha Singh
Department of Mechanical & Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 05
Sound Propagation at Medium Boundaries – 1

Welcome to our 5th lecture in this series on Acoustic Materials and Metamaterials. So, up till now we have been studying about sound wave propagation in a homogenous fluid medium, but what happens, when the same sound wave which is travelling in one medium. Suddenly, encounters a boundary of a second fluid medium. So, this is a new phenomena that we will discuss in this particular lectures. So, this is our lecture on Sound Propagation at Medium Boundaries.

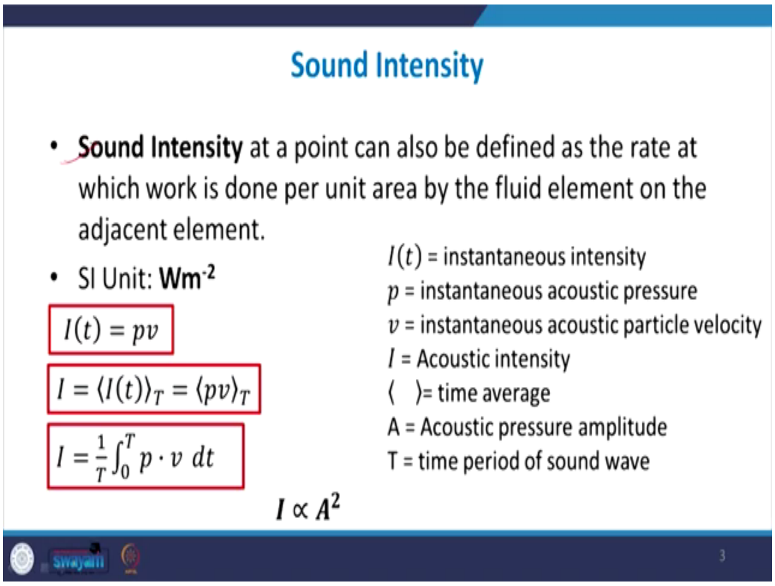
(Refer Slide Time: 00:59)



So, the outline for this lecture is that, we will discuss Milli sound field at boundary surfaces. So, hereby boundary what I mean is it is the boundary between two different fluid media. So, the so, what we effectively, I am going to teach you is that, sound wave is propagating in a homogenous medium and suddenly it encounter some boundary or interface, which is for a second fluid medium. And, then we will discuss certain terminologies like the reflection, transmission, absorption and impedance.

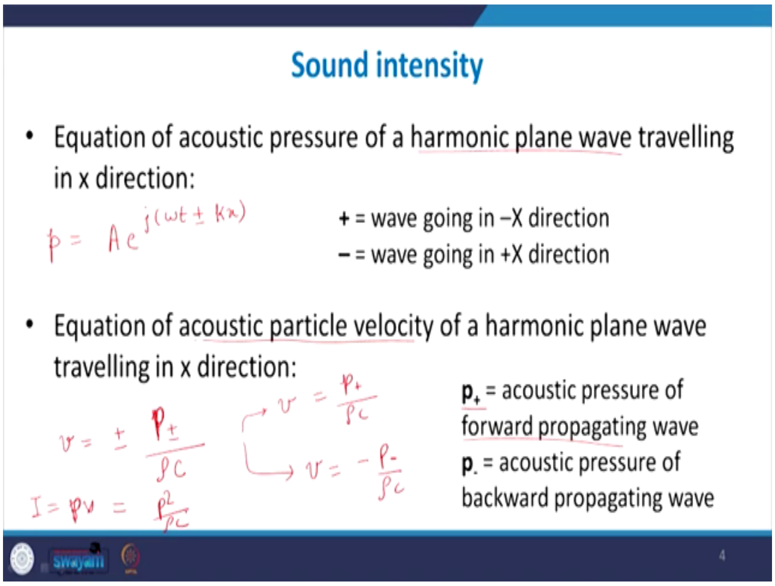
And, finally, will do a case study of transmission from fluid 1 to fluid 2 in the case of normal incidence. So, let us begin, but before we begin in the last class we studied about sound intensity and sound power. So, a few things were left in the topics, which I am going to discuss briefly here, before go going forward with our discussion on the sound propagation through the medium boundaries.

(Refer Slide Time: 02:13)



So, as you see sound intensity, which we read is defined as it is the product of pressure, acoustic pressure and particle velocity. And, the average intensity then becomes, the time average of these 2 quantities.

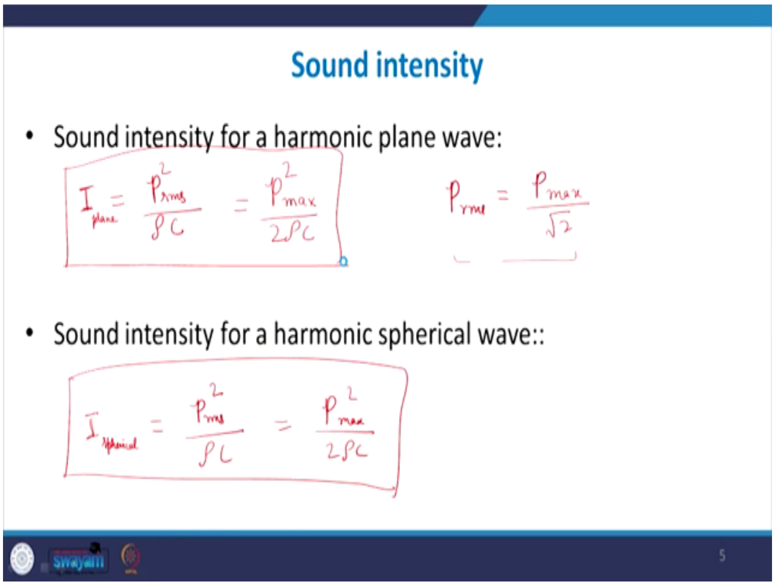
(Refer Slide Time: 02:29)



So, we know that the acoustic pressure of a harmonic plane wave, p is simply given as A e to the power j omega t plus minus k times of x plus is for the forward propagating wave. So, plus is for the backward propagating wave and minus sign is for the forward propagating wave. And, the equation for acoustic particle velocity for this harmonic plane wave we have also derive this as it is plus minus P plus minus by rho c.

So, what we get is that. It is if we considered the wave to be forward propagating here p plus is the forward propagating wave. So, if p plus is the forward propagating wave then v simply becomes v v is equal to p plus by rho c. And, if we have a backward propagating wave then v becomes minus of P minus by rho c. So, if you multiply the 2 quantities what will you get? Effectively the intensity will be multiplication of this p and v, which should be p square by rho c and then you can have the sign accordingly.

(Refer Slide Time: 03:57)

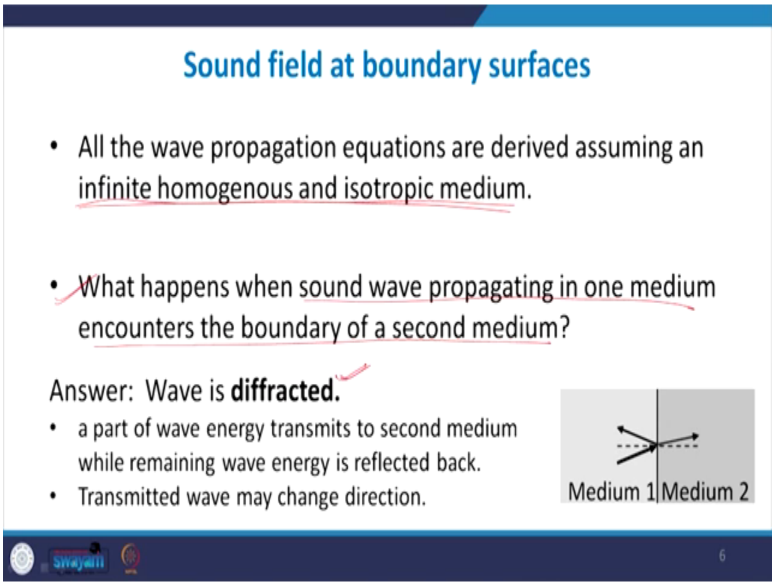


So, the sound intensity of a harmonic plane then becomes, the average sound intensity becomes Prms whole square by rho C and we know that for a sinusoidal wave. So, whether it is a cosine wave or a sine wave for any, simple harmonic sinusoidally wearing wave. The rms value is root 2 times; the rms value is root 2 is peak value divided by root 2. So, this is the property of a sinusoidal wave, if you do the integral over a time period t this is what you get?

So, you can also write this equation as the pressure amplitude whole square by 2 rho C. And, the strange thing here is that, the intensity this is the intensity for a harmonic wave, but the intensity for a spherical wave also comes out to be Prms whole square by rho C, which can be written as P max whole square by rho C, because again here we have sinusoidally wearing wave fronts. So, this equation holds true.

So, these are the intensity equations for the 2 waves. And, in the previous slide you already saw how did we derive this P square by rho C for a harmonic wave. Similar thing can be done for a spherical wave you know the equation of the spherical wave front for the acoustic, pressure, the velocity equation can also be derived using Euler's relation. So, the end result is that the acoustic intensity for both this wave is same, I mean it has a same expression.

(Refer Slide Time: 05:55)

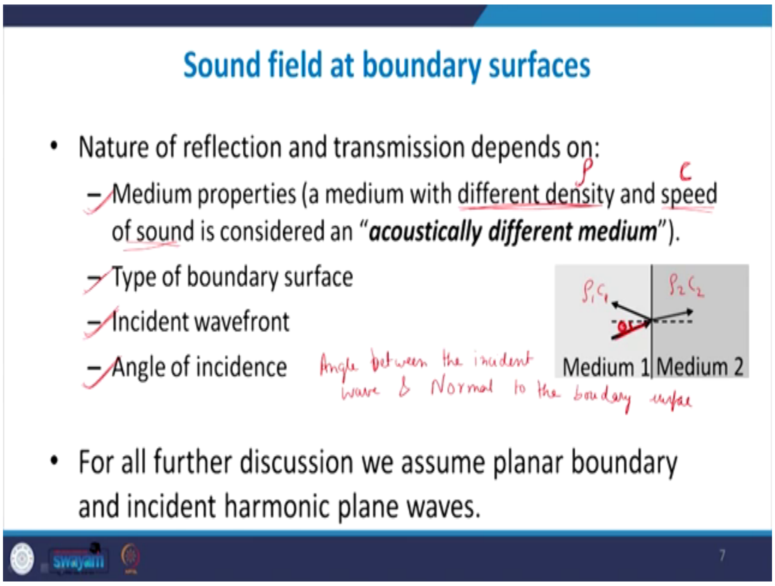


So, let us now begin our discussion with sound field at boundary surfaces. So, as I had said again till now all the equations we studied, whether it was a linear acoustic wave equation and then the special cases of harmonic plane wave front and the spherical wave front. So, in all these equations we were assuming, that the medium is homogeneous. So, what were the assumptions we made. The mean velocity is 0 and there is no extra marks that is being added to the medium and the medium remains homogeneous throughout.

So, we are assuming an infinite homogeneous and isotropic medium. So, this is the assumption we made, when we discussed the sound wave propagation till now what happens? So, by so, homogeneous medium I meant was that, the density of the medium remains constant throughout, it does not change with time or space, the bulk modulus does not change with time or space. And, similarly the speed of sound, it remains constant because it depends on the density and the bulk modulus, but what happens, when sound wave propagating in one medium encounter the boundary surface of a second medium.

So, this is medium one and suddenly there is a boundary and the second medium starts. So, the answer to this is the wave gets diffracted. So, the wave is diffracted, which means that as the wave is travelling and it gets incident or it hits the boundary. Some part of the wave will; some part of the wave energy it will go back into the first medium and some part of it will enter into the second medium with a change direction in most of the cases. So, the wave gets divided some part of it goes back some part of it enters the new medium.

(Refer Slide Time: 07:49)



And, the nature of this phenomenon as in when the sound wave is interacting with the boundary then what should be the nature of the transmission, what should be the nature of reflection, how the energy gets split, this depends upon various properties. First of all it depends upon the medium properties.

So, by medium properties we mean C, if 2 different mediums are there. So, what do you how do you define that these 2 mediums are different acoustically? How can you say that this medium is different is a different fluid medium compared to the other medium. So, for sound waves a different medium means that a medium which has got a different density and a different speed of sound. So, whenever rho and C values this rho value and C value.

Whenever these 2 values are different which means, that the medium is different. So, the word a same medium we need to have both rho and C as same. So; obviously, the way reflection and transmission will happen depends upon the rho C of medium 1 and the rho C of medium 2. So, depending on these values the nature of reflected and transmitted wave will change. The second property on which depends is the kind of surface we have the kind of boundary.

A boundary can either be a planar so, we can have one medium, another medium and a plane boundary or it can be some irregularly shaped boundary, with sharp geometrical bends or it can also be some perforated boundary. So, there can be different types of boundary. So, depending on the boundary surface also this nature of reflection and transmission is going to change. And, then depending on the kind of wave front that incident, whether it is a harmonic wave that is being incident or a spherical wave or a cylindrical or any general random noise.

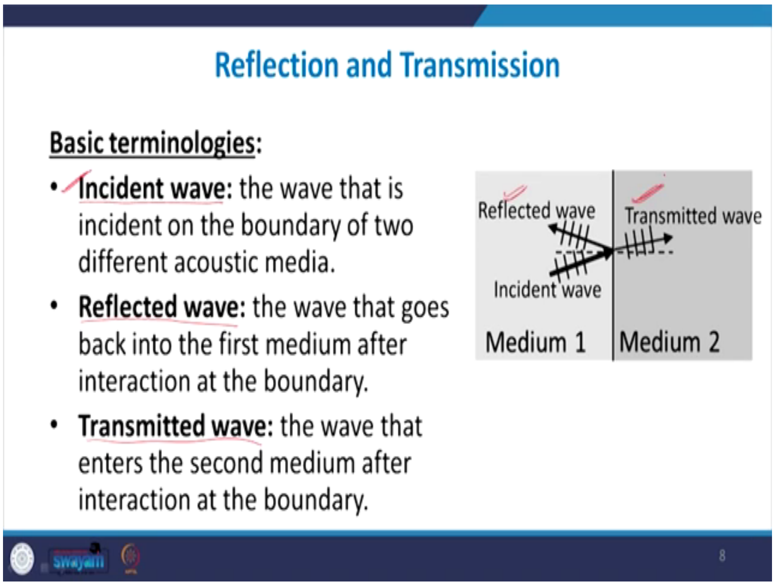
So, what kind of wave front is being incident depending on that also? We will get different reflected and transmitted waves and lastly it will also depend upon the angle of incidence. So, by angle of incidence we mean that it is the angle between the incident wave and normal to the boundary surface. So, in this case this is the direction of wave propagation.

So, this becomes the direction of the incident wave and this is the normal to the boundary. So, this becomes the angle of incidence theta. So, depending on these the nature of reflection and transmission is going to change, but now we will study some specific cases and for all the further discussions will always assume that we have a planar boundary, because it depends on the boundary surface itself.

So, we will assume it has a planar boundary and the other assumption is we assume it has a incident, the incident wave is harmonic plane wave. Now, we are studying only planar boundary this is in this particular course only planar boundaries are within our scope, because these are the most commonly found boundaries, there then irregularly shaped boundaries and the physics of those boundaries becomes even more complex.

So, we only study this one. And, for the sake of convenience we are taking the incident wave is harmonic wave, you can also take a spherical wave and what and the concepts will remain the same irrespective of the wave front. So, maybe the values may change, but the wave we are driving the reflected or transmitted wave, it will remain the same whatever wave front you take. Just for the sake of easy ease in calculation I am taking the harmonic plane wave to explain to you what happens?

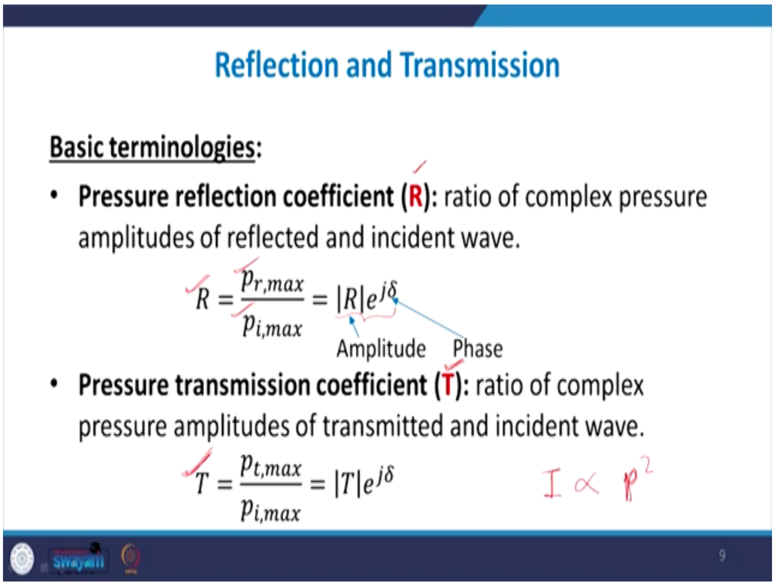
(Refer Slide Time: 11:51)



So, let us define some terminologies before we begin our discussion. So, the terminology I have already spoken about have been using this terminologies, which is incident wave reflected wave and transmitted wave.

So, incident wave is actually the wave which is hitting the boundary, it is a original wave front. And reflected wave is that part of the wave, which after it hits the boundary surface, it goes back into the same medium from which the original wave was coming. So, this here becomes a reflected wave. And, the transmitted wave is the one, which when the incident wave interacts with the boundary some part of it gets enters into the second medium. So, this becomes the transmitted wave, so this is what is the definition of transmitted wave.

(Refer Slide Time: 12:39)



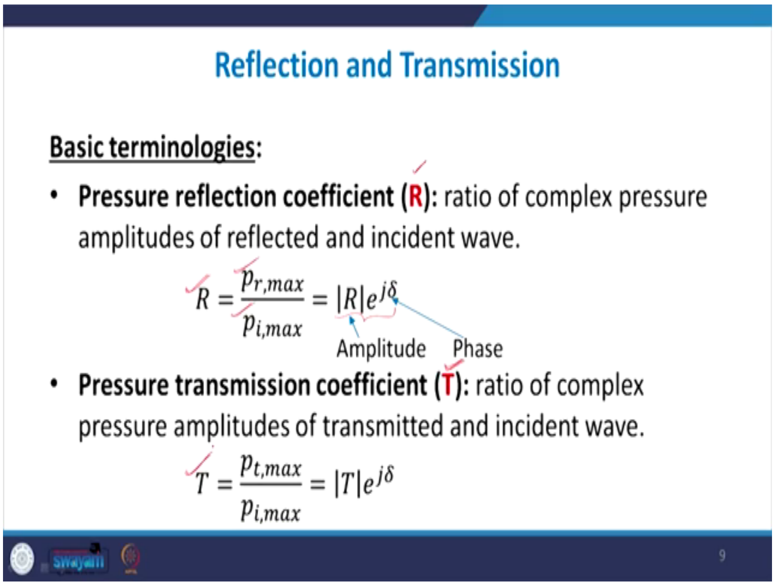
Some other terminologies are we have pressure reflection coefficient denoted by the capital letter R. So, this is defined as the ratio of the complex pressure amplitudes of reflected and incident wave. R assembly the ratio of the pressure amplitudes between the 2 and if you are taking complex pressure amplitudes, then ratio of 2 complex quantity will also be another complex quantity. Then, the second term is pressure transmission coefficient, this is denoted by capital T.

So, just like pressure reflection coefficient now this becomes the ratio of the complex pressure amplitudes of the transmitted wave and the incident wave. So, if we know the value of R and T then we can guess for example, suppose R comes out to be 0.5, which means that half of the energy is being reflected back. Because, the amplitude of this R wave the amplitude of the reflected wave is then, 0.5 times the amplitude of the incident wave.

So, overall 50 percent reflection is happening. In the same way suppose we have T equals to 0.4, which means 40 percent of the wave is getting transmitted, 60 percent is getting reflected and so on. So, this ratio in give you an idea of how the energy splitting and how the waves are splitting and how they are getting transmitted or reflected. But, mind you I said that these ratios they give you the idea of how the pressure amplitudes are splitting, they do not give you an idea of how the intensity is splitting.

Because the intensity or the energy will depend upon the square of the amplitudes. So, if t is equal to let us say 0.4, then you can say roughly 0.4 whole square which is 0.36. So, 0.36 or 36 percent sorry it is 0.16. So, let us say approximately 16 percent of the energy is getting transmitted. So, the amount of energy split cannot be found from R and T, they only give you the ratio in which the pressure waves are dividing.

(Refer Slide Time: 14:57)



So, we have defined 2 more terms, which is intensity reflection coefficient or R I. So, this is the ratio of the time averaged intensity of reflected and incident wave R I is I r, r m s by I i r m s. So, the time average intensity of reflected wave by incident wave here.

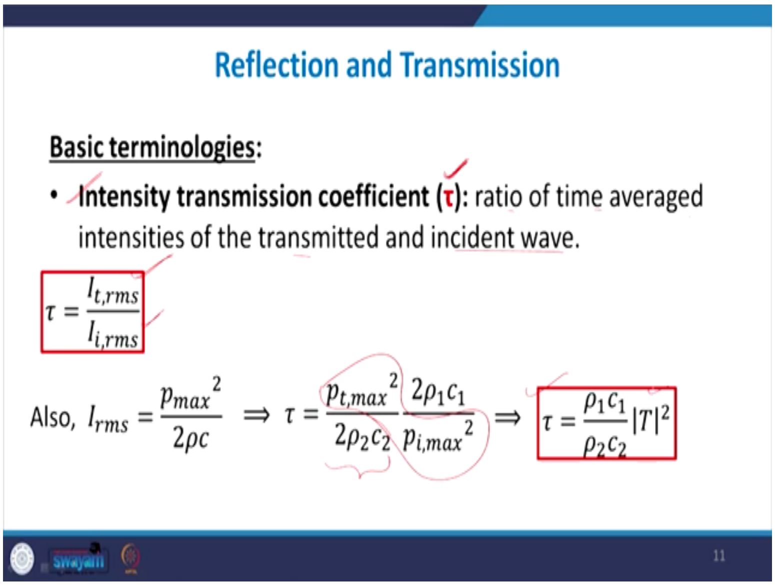
So, I rms now we know that the I rms value we have already derived in the previous slides that for a harmonic wave front it comes out to be p max square by 2 rho c, we already derived in the previous slides.

So, if it is p max square by 2 rho c. And, this R pressure reflection coefficient is a ratio of this amplitude. Then R i can be written as it will be I r rms which will be p r max whole square by 2 1 2 rho 1 C 1 divided by p i max whole square by 2 rho 1 C 1. So, here both the reflected and the incident wave they are in the same medium. So, both have the same rho and C value.

So, we are using the same rho and C values for both this mediums. So, you using this equation, if you substitute it here this is what you get. This cancels out and R i comes out to be p r max divided by p i max whole square. So, it becomes the square of the pressure amplitudes of the reflected and the intensity, the reflected and the incident wave, by definition then it becomes square of de pressure reflection coefficient. So, if R i is suppose is given as 0.3 then; that means, the 30 percent of the energy is being reflected back.

Because, it is directly proportional this R gives how the intensity is getting split an intensity is what energy per unit time per unit area. So, you can get an idea of how the sound wave energy gets distributed, what part of energy transmits to the next medium, what part of energy comes back to the same medium using the intensity reflection coefficient so, based on that value.

(Refer Slide Time: 17:11)



Similarly, we have the intensity transmission coefficient tau. Now, make note of this parameter, because this is one of the very important parameter.

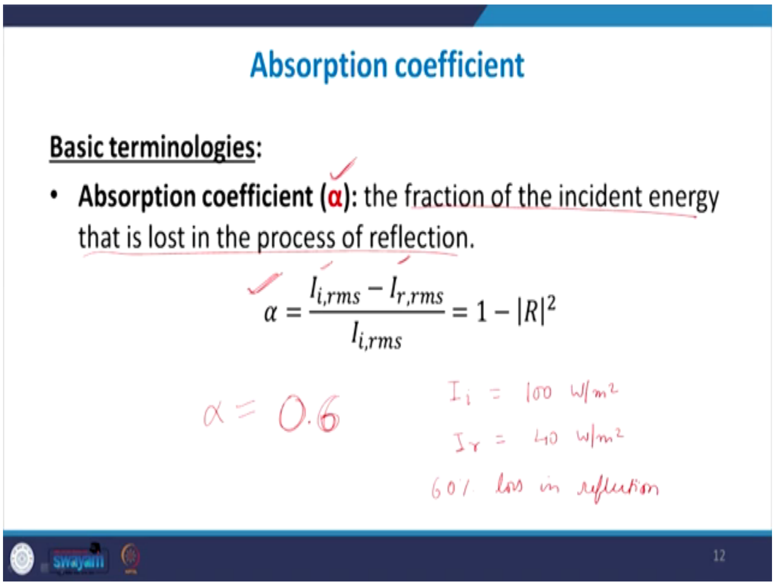
So, later when we discuss about acoustic materials. So, we will be discussing materials that are usually used for like locking sound. So, how effectively a material can block a sound wave? So, all such noise control materials, their performance is defined by using this intensity transmission coefficient.

So, using how much intensity of the wave gets transmitted we can define the performance of the materials. So, it is a important parameter. The definition is simply ratio of time average intensity of transmitted wave and the incident wave. So, I t rms by I i rms.

And, again I rms the same we have derived previously is p max whole square by 2 rho C. Now, here the difference is that the transmitted wave is in the second fluid medium whereas, the incident wave is in the first wave medium. So, we for the transmitted wave this is the term we use for the transmitted wave. So, it is pt, max whole square by 2 rho 2 C 2 this gives you the intensity of the transmitted wave, divided by pi max square by 2 rho 1 C 1.

So, overall what you get is tau is equal to this is pt max by pi max whole square. So, this is by definition the pressure transmission coefficient this, this value is the pressure, the square of the pressure transmission coefficient. So, this becomes this and we have rho on C 1 by rho 2 C 2.

(Refer Slide Time: 18:51)

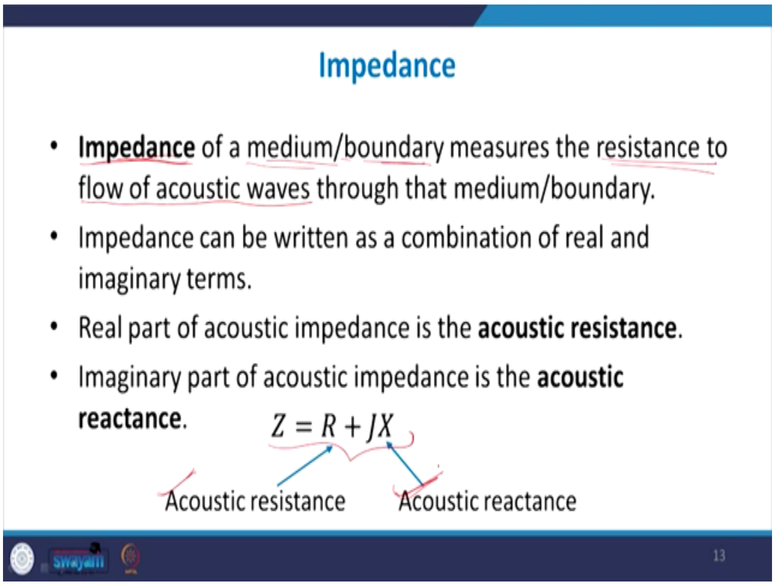


Another important terminology is called as the absorption coefficient to alpha; this is yet another important performance parameter that is used for acoustic material. So, we can define that for example, for tau or the transmission coefficient is used to know, how good a material can block the sound, how good barrier it is. And, absorption coefficient is used to know how good absorber it is, how good how is easily it can absorb the sound and it can reduce reflections. So, absorption coefficient is defined as the fraction of incident energy that is lost in the process of reflection.

So, whatever is the fraction of incident energy lost in the process of reflection? So, energy again it is directly proportional to intensity. So, alpha is intensity of the intensity of the incident wave minus intensity of reflected wave by the intensity of the incident wave. So, this gives you what fraction of energy is getting lost in the process of reflection. So, let us say alpha was 0.6, which means that suppose the sound wave that is being incident has some watts let us say 100 watts, then the wave that is coming back that will have. So, what it means is that 60 percent of that will be lost.

So, only 40 percent will be obtained as the reflected wave. So, if 100 Watts are being incident on a boundary and alpha of that boundary surface is 0.6, which will mean that the reflection, the reflections will then be 40 Watts, it will be 40 percent of 100 watts, it will become 40 watts. So, if alpha is 0.6, the I incident is let us say 100 Watts per meter square, then you can find that the intensity of reflected comes out to be 40 Watts per meter square or an average, 60 percent loss in reflection. So, that is what this indicates.

(Refer Slide Time: 21:03)

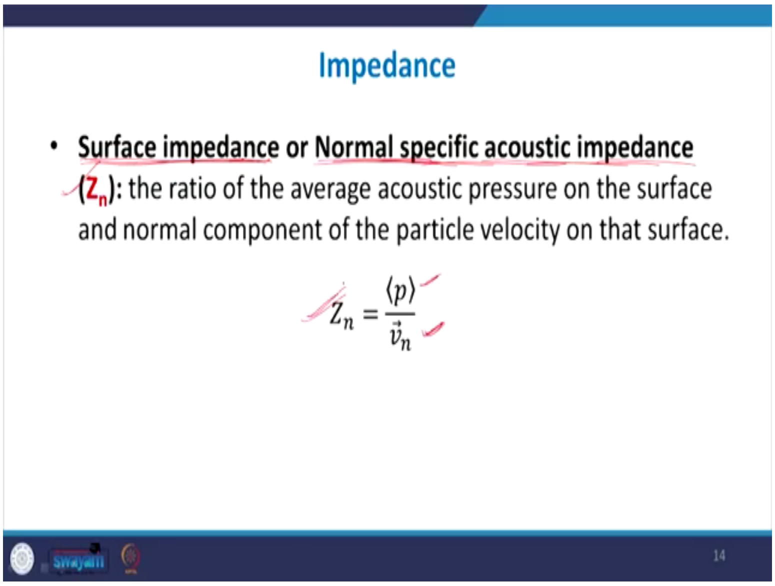


The last quantity that we are going to define is called as impedance. Now, impedance is a term which if you have you would have heard in the field of electrical technology as well. So, like we have resistance, we have reactance, we also have impedance. So, impedance of any system is the resistance to flow. So, in an electric current for example, if you put on a device in a circuit, then the impedance simply gives you what is the resistance to the flow of electric current that is devices offering.

In the same way, if we have any boundary or medium, then the impedance of that boundary will give us, how resistant the boundary is to the flow of sound waves? So, impedance qualitatively gives us for a medium or a boundary. So, what is the resistance to the flow of acoustic waves through that particular medium or boundary?

So, higher the impedance means more will be reflection very few sound waves will pass through, because it will be highly resistant to the flow of sound waves. And, this impedance can be a complex term and we can write this as a real part plus the complex part this is called as acoustic resistance and this is called as acoustic reactance.

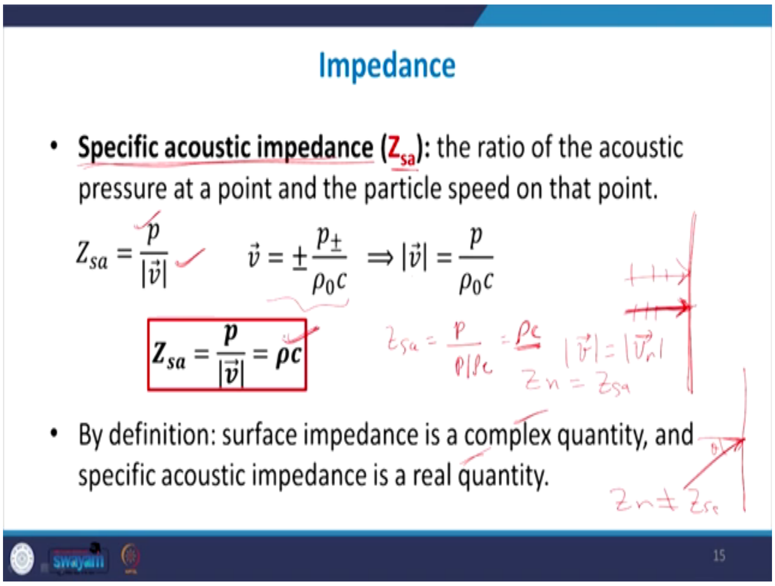
(Refer Slide Time: 22:23)



So, let us see that two different types of impedance; one we call it as surface impedance. So, these are the 2 commonly used impedance types in acoustics, we have surface impedance or normal specific acoustic impedance represented by Z n. So, this is the ratio of what is the average acoustic pressure on the surface divided by the normal component of the particle velocity on the surface.

So, as you see if impedance is very very high, which means that the particles they are oscillating, they cannot move forward. To get a particular particle velocity to move for we require a very high pressure, very high acoustic pressure. Only very high acoustic pressure of very loud sounds will be able to propagate through the higher the value of this Z n.

(Refer Slide Time: 23:15)

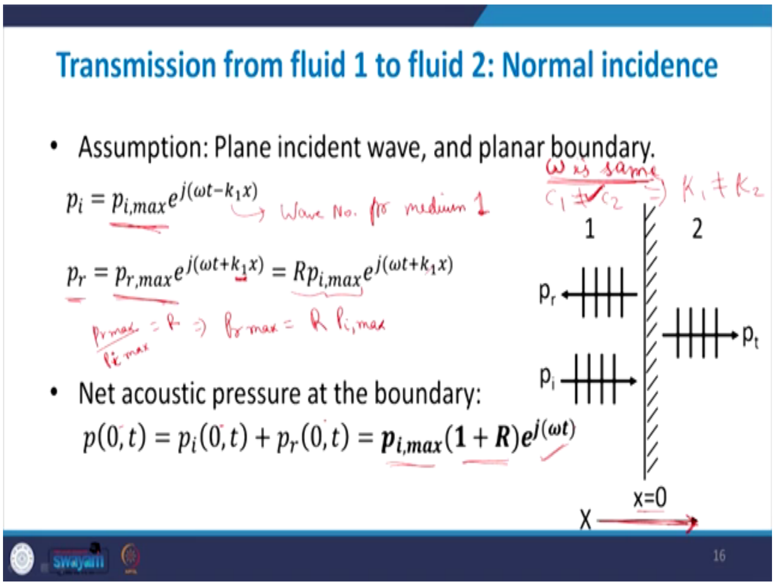


Hence, the second type of acoustic impedance is specific acoustic impedance, which is defined as it is the ratio. So, this is defined for a particular surface. So, in the surface you take what is the average fresh, average acoustic pressure acting on a particular surface or a boundary. And, then you divide by the normal component of the particle velocity acting on that surface. Specific acoustic impedance is defined for a point, it is the ratio of the acoustic pressure to the particles speed at that point.

So, suppose we have a boundary where the incident sound wave is where the sound wave is uniformly incident. So, in the in the boundary these incident wave front it is uniformly impinging. So, the difference between Z n and Z sa will only be, that here we have the normal component of the velocity and here we have the entire velocity magnitude. So, suppose there was the surface like this and this is how the wave front was incident on it uniformly, then in that case the normal component will also be in this direction and the velocity will also be in this direction.

So, both v mod and vn mod will be same. So, Z n will be same as Z sa, but if we had a boundary where instant wave front was obliquely incident, then Z n will not be same as Z sa, it will then depend upon this angle theta. So, it will be I mean the velocity will be the vn will be the velocity into cos theta. So, by definition this surface impedance is a complex quantity and the specific acoustic impedance, which is real value by real value is a real quantity.

(Refer Slide Time: 25:13)



Now, let us start with our discussion what happens when the wave front is incident normally on a boundary surface. So, here I have given you a schematic this is, this is a harmonic plane wave that is going from medium 1 to 2 to this is the boundary at x equals to 0. So, we can write the equation for the incident wave as, let us say it has same amplitude pi max. So, the general equation will be pi max e to the power j omega t minus k 1 x, because this is a forward propagating wave, this is the forward x direction that we have taken.

So, it is a forward propagating wave. So, we are using this minus sign here and k 1 becomes the wave number for medium 1 ok. So, an important note here is that whenever the waves they are passing through in the through any boundaries or they are undergoing different fluid media, then the frequency always remains the same. So, omega is same for the waves, because frequency is a quantity that depends on how the wave was generated on the first place.

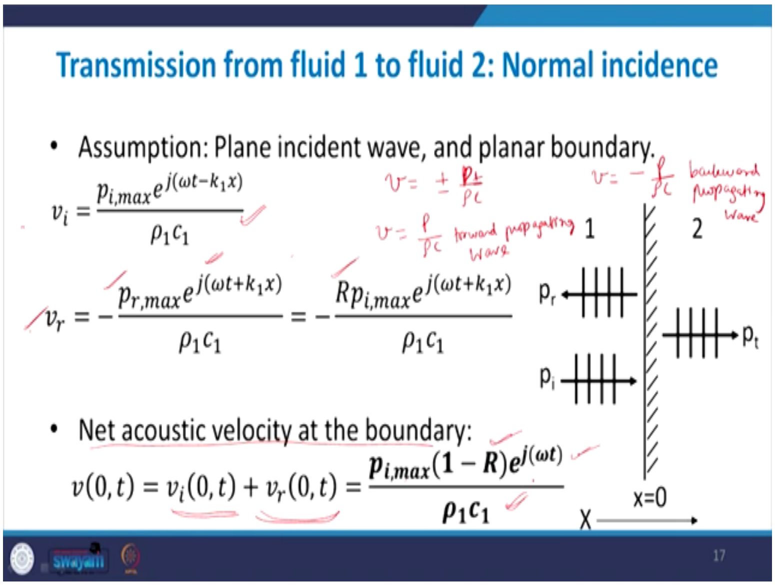
So, some vibrating surface of sphere it generated a wave, then because the source of the wave is same. Now, no matter how many different media it passes through. The frequency will remain the same. So, omega remains same, but because there are 2 different media so, C 1 will not be equal to C 2. So, it is the speed of sound that is going to be different, but omega will be same. And, that is why the vector k 1 will not be equal to k 2 because omega by C is k.

So, we are taking the k 1 value for the first medium, the reflected wave can be written as the pressure amplitude of the reflected wave e to the power j omega t plus k 1 x. So, now, because it is a backward propagating wave in this medium we are taking the plus sign and we are using the value for the first medium. And, we know that pr max by p pi max is equal to R. So, p r max can be written as R times p i max

So, we have replace this pr max by R times of pi max by definition. So, this is what we are getting. Now, let us find what is the net acoustic pressure that is present just on the left hand side of the boundary? So, in the limit of 0 negative, then the total acoustic pressure becomes the pressure due to the incident wave and the pressure due to the reflected wave at the boundary.

So, boundaries at x equals to 0. So, we put the value of x equals to 0 everywhere. So, this term cancels out, if we put x equals to 0. The overall equation that you get is you get pi max into 1 plus R e to the power j omega t from the 2 equations, we have derived previously.

(Refer Slide Time: 28:23)



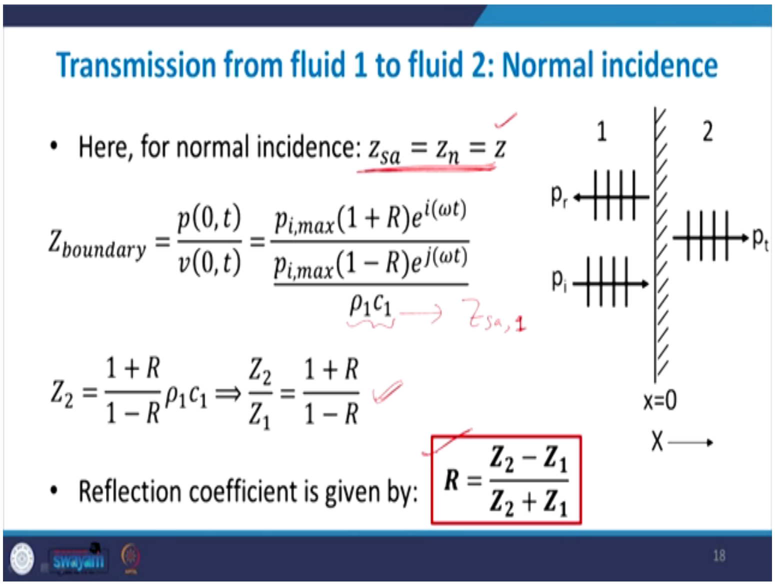
Similarly, you can find the formula for the velocities of the incident and reflected wave we know that velocity is given by plus minus P by rho C. So, because the incident wave is a forward propagating wave, so we will become p by rho C for a forward propagating wave. And, if the wave is backward propagating then it will be minus p by rho C for a backward propagating wave. So, this is how it was derived. If, it is a positive or a proper forward propagating wave we take plus sign minus sign for a backward 1.

So, v vi is a forward propagating wave. So, we simply take the pressure divided by rho 1 C 1, this becomes the v i. And, v r becomes negative of the pressure of the reflected wave by rho 1, C 1 so, minus p by rho 1, C 1. So, minus p by rho 1 C 1 again we replace pr max as R times of pi max, this is how R is defined. So, we replace this and as you see here we are using the minus sign and here we are using the plus sign for forward and backward propagation respectively.

Now, let us see what is the total acoustic velocity just on the left hand side, the total velocity hitting the surface. So, it will be the sum of the incident velocity and the reflected wave velocity at x equals to 0. So, when you put x equals to 0 in these 2 equations, then what you get is this anyways cancels out this become 0 0 0, you only left with e to the power j omega t. So, you get is pi max into 1 minus R e to the power j omega t by rho 1 C 1. So, adding the 2 terms this is what you get?

So, now you have obtained the value of the total pressure hitting the boundary and the total velocity, particle velocity at the boundary and because it is normal incidence. So, it this velocity itself is a normal component.

(Refer Slide Time: 30:49)

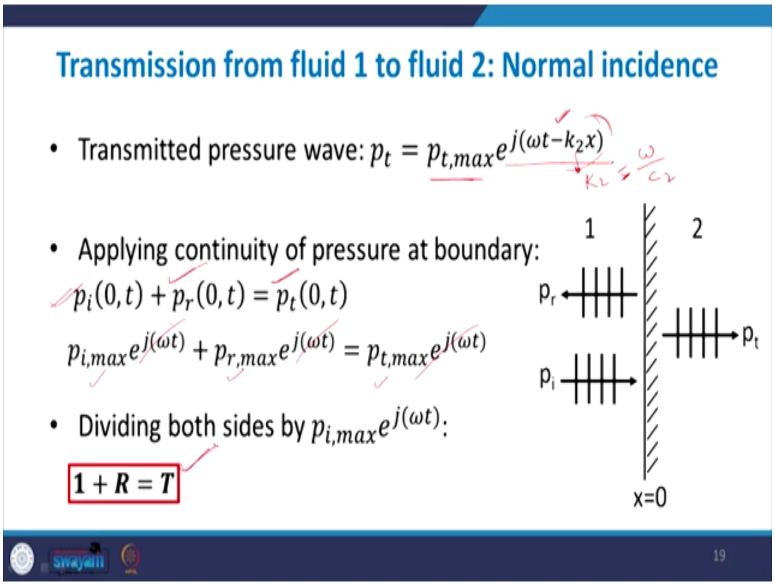


So, the Z of the boundary Z sa will be equal to Z n, which we simply call as the impedance, because it is a normal incidence. So, Z of boundary will be p by v at the boundary. And, the boundary is x is equal to 0 is the boundary. So, Z of boundary becomes p by v we put the expressions for p, which we had obtained here, expression for p and this is the expression for v. So, you summed the; you have summed the pi plus p r divided by p v i plus v r and this is what you are getting.

So, this cancels out this cancels out. So, overall what you get is Z 2 and this rho 1 C 1 is what, we know that where is it. Now, we know that the specific acoustic impedance is p by the mode of v. And, v can be written as this expression. So, the mod of v is always p by rho naught C. So, Z sa becomes p by p by rho naught of C. So, it becomes rho C or rho naught C whatever. So, rho C of that medium this is what p pyv becomes.

So, the specific acoustic the specific acoustic impedance of a medium is rho C. So, here we put this rho 1 C 1 becomes the specific acoustic impedance of medium 1. So, we have replace this value by Z 1. So, Z 2 is equal to 1 plus R divided by 1 minus R times of Z 1. So, Z 2 by Z 1 becomes this quantity, when you use the property of proportions. So, componendo and dividendo property of the proportions you can solve this equation and the R value you get is Z 2 minus Z 1 by Z 2 plus Z 1.

(Refer Slide Time: 32:43)

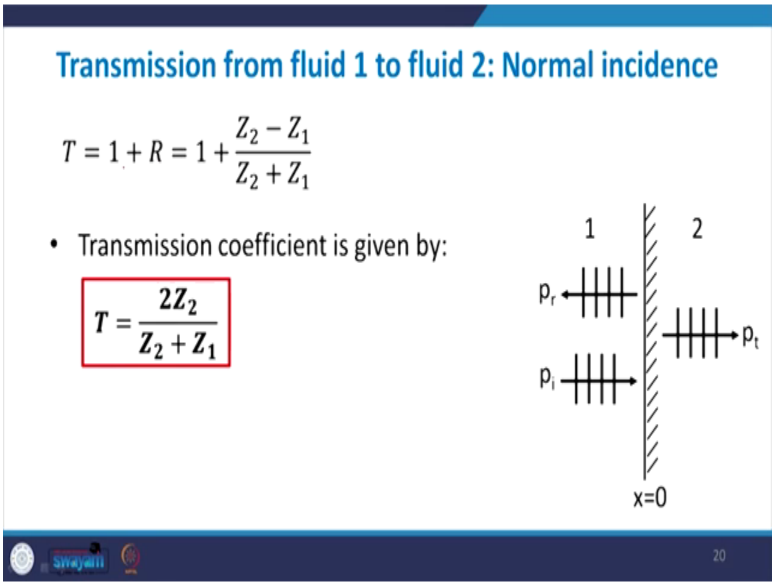


So, this is the R value and the transmitted wave can be written as p t max e to the power j omega t minus k 2 x. So, this is also forward propagating wave. So, we are taking as minus. And, for the second medium k 2, which is simply omega by the C 2 of the second medium.

So, we are taking the other wave propagation vector. Now, we apply the continuity of pressure at the boundary, which means that whatever pressure that is hitting the boundary from the left side will be same as the pressure on the right side just on the right side. So, pi 0 t plus p r 0 t should be equal to p t 0 t equating the pressure just on the left and the right side.

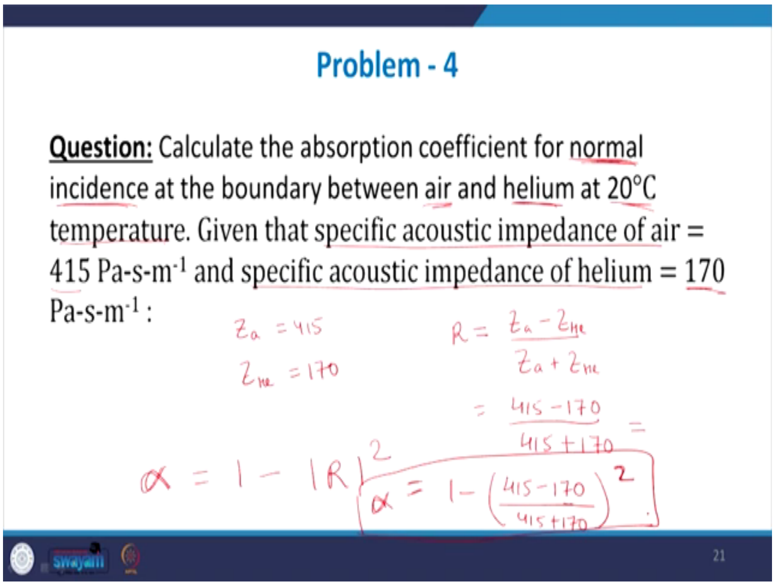
So, in that case because we put x equals to 0. So, the plus this particular term anyways become 0 for all the waves this term. So, this is the final expression we get this cancels out. So, we get pi max plus pr max should be equal to p t max, if you divide this by pi max. So, you get 1 plus R is equal to T. So, this equation holds true. And, we have already found the value for the reflection coefficient, which is the difference between the impedance divided by the sum of the impedance.

(Refer Slide Time: 34:11)



So, using this T equals to 1 plus R, which is equal to 1 plus this thing quantity, this is the transmission coefficient value, which you get.

(Refer Slide Time: 34:21)

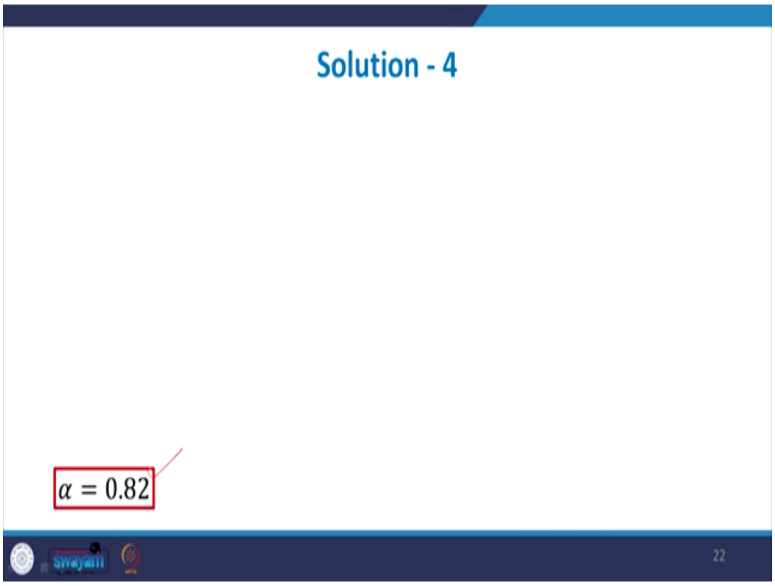


So, before I leave this lecture a small question we will solve. So, calculate the absorption coefficient for normal incidence. So, here normal incidence is given to you, at the boundary between air and helium at 20 degrees temperature. And, it is given that the specific acoustic impedance of air is 415 and the specific acoustic impedance of helium is 170.

So, we have given Z of air which is 415 and Z of helium s 170, then this is the case of normal incidence. So, for normal incidence we already derived that the reflection coefficient becomes Z a minus Z helium. So, medium 1 minus medium 2 divided by media 1 plus medium 2 impedance. So, which will be you can calculate this value, then alpha or the absorption coefficient will be 1 minus mod R square.

So, 1 minus 415 minus 170 divided by 415 plus 170 whole square. So, this will be the absorption coefficient and this is what we have to find. So, the absorption coefficient when you calculate this the solution is 0.82.

(Refer Slide Time: 35:59)



So, 0.82 becomes the absorption coefficient. We will continue our discussion on this topic in our next lecture.

Thank you for listening.