Acoustic Materials and Metamaterials
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Lecture – 03
Sound Wave Propagation in Fluids- II

Welcome to our 3rd lecture on Acoustics Materials and Metamaterials. So, in the last lecture; we were discussing about Sound Wave Propagation in Fluid. And we were able to derive a linear acoustic wave equation which is given as nabla square p plus nabla square p minus 1 by c square times of del square p by del t square is equal to 0. So, and then we discussed about a harmonic plane wave.

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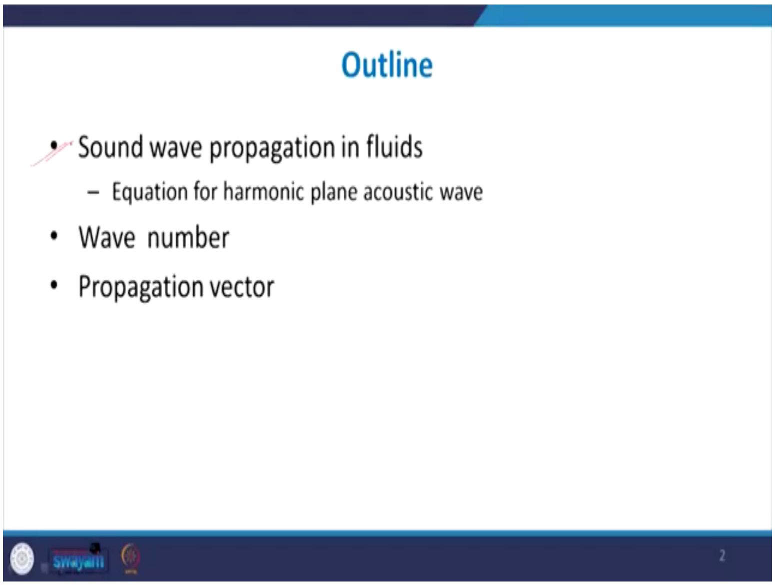

So, we will continue our discussion on harmonic plane wave. So, the outline here is that we will study the harmonic plane, the equation for harmonic plane acoustic wave and then we will see two different parameters that is a wave number and a propagation vector.

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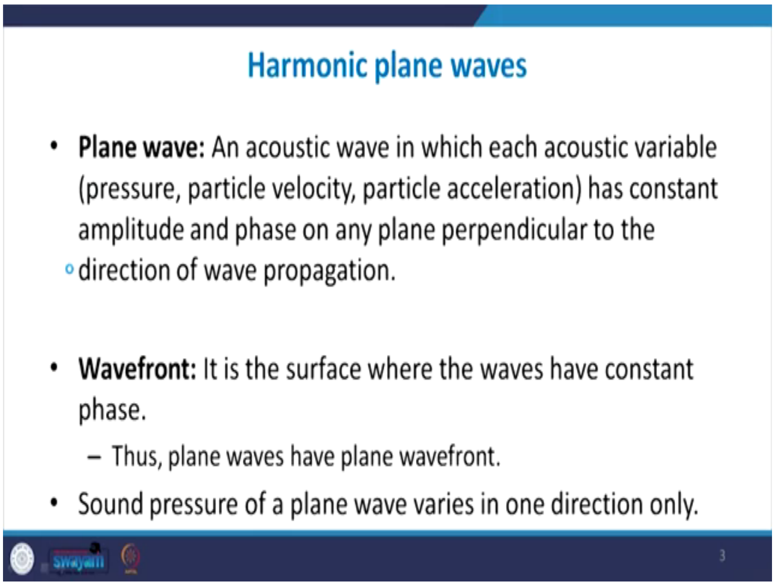

So, as discussed previously a harmonic plane wave is which has a plane wave front that is; if we take any plane perpendicular to the direction of wave propagation, then the amplitude remains constant for the wave throughout that plane and the phase also remains constant. And one important thing to note is that the pressure vary for a harmonic wave the pressure varies only in one direction.

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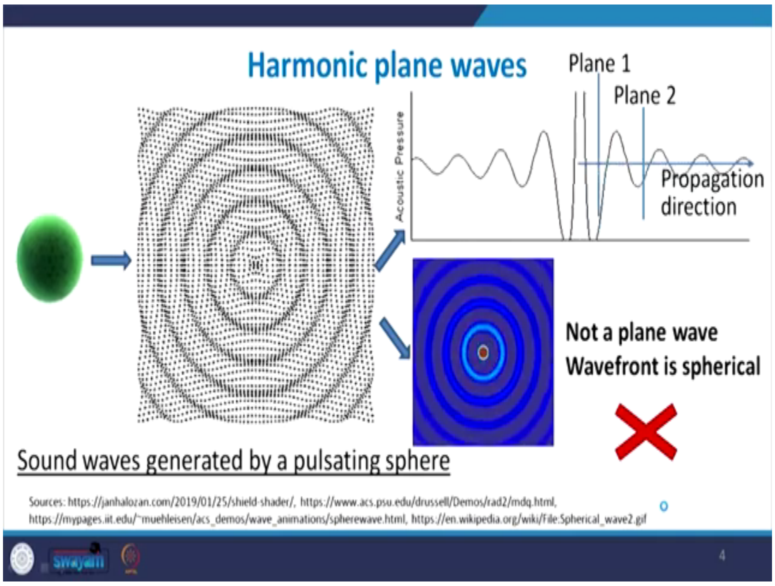

So, we were showing a few examples; like for example, this is spherical this is not a harmonic plane wave, but this form this is our example of a harmonic plane wave.

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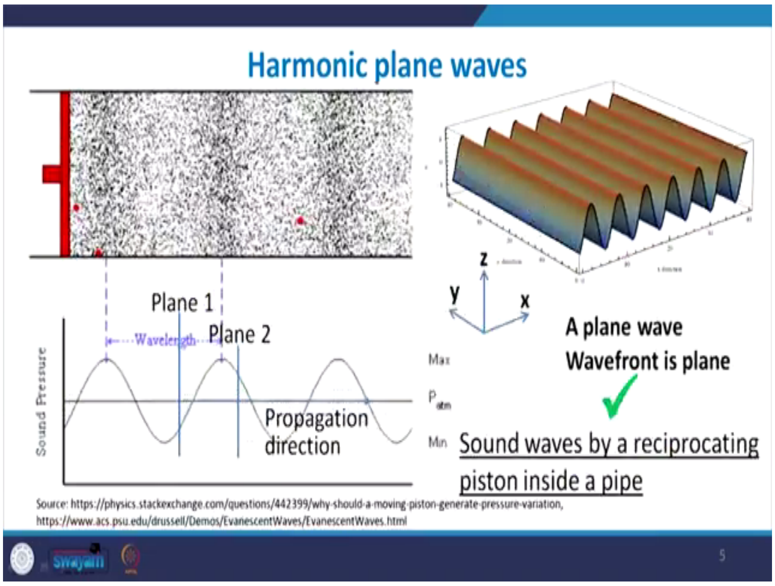

So, as you can see here; the wave front is this plane here the wave is propagating along the positive x direction. So, the wave front is actually a plane that is y z plane the y z plane becomes the wave front. So, it is normal to this x direction. So, at any all the various y z planes, if you cut it you will see that at any point the amplitude remains constant throughout as well as the phase remains constant.

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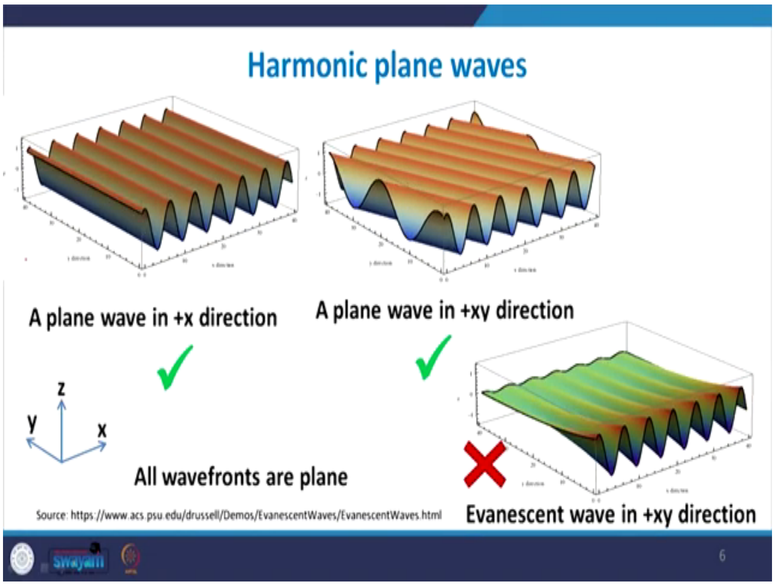

Then, we were showing you the wave form of I was showing you; this is the wave form of a wave traveling in the positive x direction. This is the wave form of a harmonic plane wave traveling in the positive x y direction and this was not a harmonic plane wave, because here the amplitude varies amplitude does not remain constant over the wave front.

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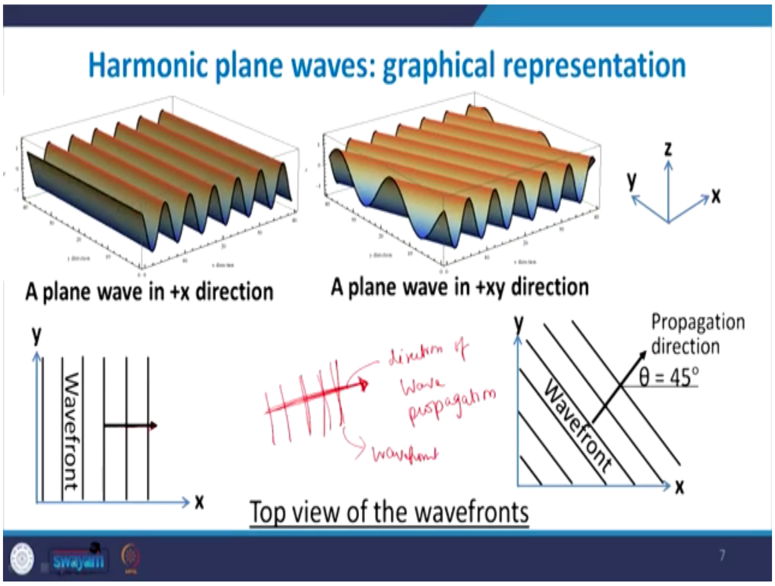

So, this wave front; if you see the top view. This is the wave and it is propagating along the positive x axis. So, if we take a top view only the x y plane, then this is the direction of wave propagation and the wave front is normal to the direction of wave propagation. So, for all the harmonic plane waves; this symbol will be used or arrow will be used to represent the direction of wave propagation and the lines that are normal to this direction will it will simply give you the wave front.

So, this is the wave front; the wave front is actually continuous; if you take any point it will be a wave front. So, this is the typical symbol we use to represent a harmonic plane wave. So, here if you see in this particular diagram; here the plane is propagating along the positive x y direction at 45 degrees angle. So, this is the direction of wave propagation and perpendicular to that we have drawn the wave front. So, wave front is always perpendicular to the direction of wave propagation.

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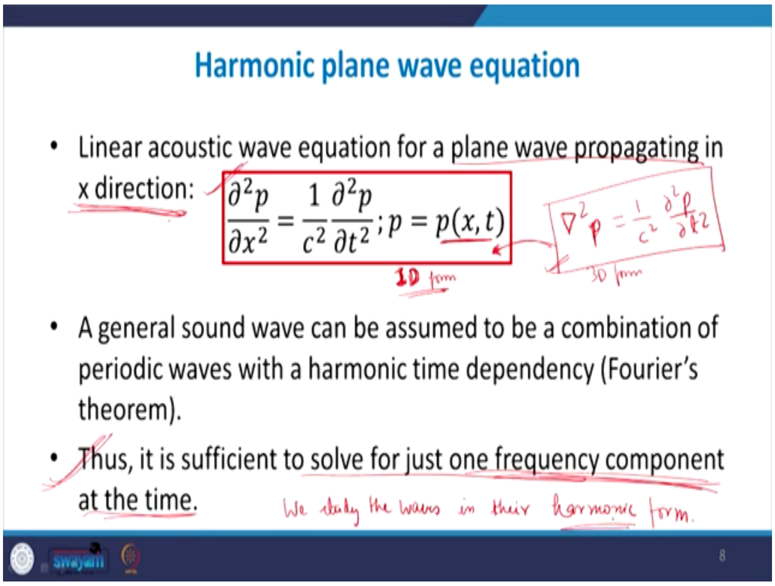

So, let us derive the equation for a harmonic plane wave. So, here we will derive the equation for a harmonic plane wave. So, we have already seen the equation that is nabla square p is equal to 1 by c square times of del square p by del t square. This was a general equation for sound wave propagation in fluids. And now, here let us say the plane wave is propagating along the positive x direction and the direction in which the plane wave propagates.

So, the property of a harmonic wave is that; it only has a single direction it is not a 3D wave it propagates along a single direction. So, here we have taken the direction of wave propagation is plus x. So, the other components like del square by del y square del square by del z square they vanish. So, we only look at the x component. So, from this equation; we are getting the

1D form. So, this was a 3D form and this harmonic plane wave is; where the plane where the acoustic pressure varies only along the one dimension propagation is only along the one dimension.

So, this is the respective one dimensional form. So, it is del square p by del x square is equal to 1 by c square times del square p by del t square. So, p here is a function of x and t. Now, in real life when we encounter sounds; we do need not have only periodic waves a general sound can be composed of many frequencies it may or may not be periodic in nature. But, by Fourier series; we know that if we have any random signal then that random signal can be written as a sum of science or a sum of cosines with a varying harmonic component.

So, we can represent any random signal as signal which is a sum of cosines with different amplitudes and different frequencies and these frequencies they vary as integral multiples. So, this is the main Fourier's theorem and this is very important in the field of acoustics because, whenever we study a general wave.

So, maybe it is a random wave, it is periodic or non periodic, but by using the Fourier's theorem; we can represent any wave as a sum of different sinusoidal waves. And if we if we know the nature of 1 sinusoidal wave then we can simply integrate, we can change the frequencies as integral multiple of the fundamental frequency and we can create a general solution for a wave.

So, whenever we study about the waves; we usually observe one frequency component. So, because a wave can have many different forms it can be random or it can be periodic in nature, but even random waves can be represented as a sinusoidal wave my Fourier's theorem. So, when so whenever we study the waves; we only study 1 particular frequency component. So, we study the wave as in the harmonic form.

So, we study the waves in their harmonic form. So, once we know their harmonic solutions. So, we assume that the wave we only assume that this wave is varying sinusoidally. So, whether it is some cosine function or it is a sine function. So, the wave is varying sinusoidally. So, once we derive the equations with respect to sinusoidal wave form, then we can use the same thing for random waves also using the Fourier's theorem. So, for the sake of simplicity; we only study the harmonic waves.

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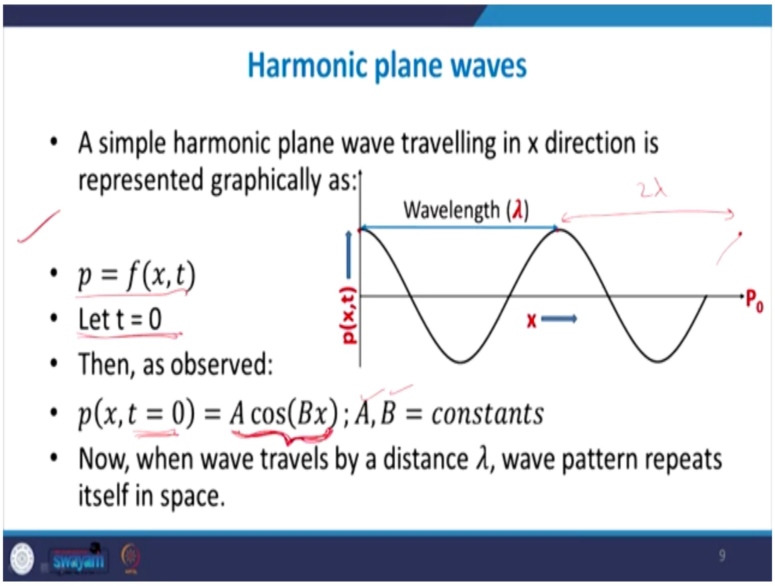

So, here we use this assumption. So, a simple harmonic plane wave; that is traveling in the x direction that is the equation we want to get. Now, we know that this wave will be a function of both space and time. So, it will be p x comma t. So, let us see here. So, this p is a function of both space and time. So, let us first freeze the time and see how does it vary over space.

So, we have frozen the time we have taken t as 0. And we only observe; how this wave varies with respect to space. So, let us say the t function we have removed, so t is 0. So, it may have some t term as well this. This cosine will have some t term also, but we have assumed t is equal to 0. So, now, all we are left with is only the x term. So, we have assumed that it is a sinusoidal wave.

So, let us assume a general solution A cos B x. This is how this is a general form of this wave and here A and B are constants which depend upon the conditions in which the wave is generated. Though, here if you see that after every distance of lambda; the wave pattern repeats itself right. So, the value of p at this point will be the same as the value of p at lambda and it will be same as the value of p at 2 lambda. So, after every addition of lambda; you get the same value of acoustic pressure. So, overall this particular function has a period of lambda.

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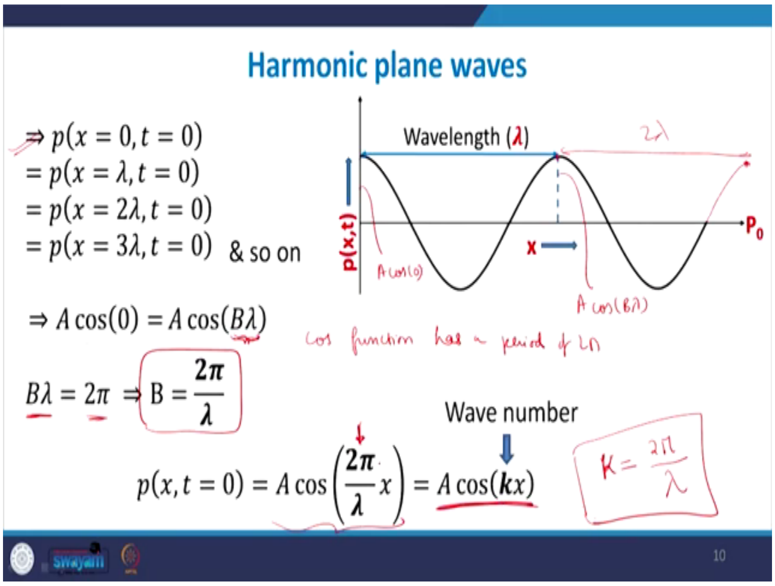

So, to explain it further, so as I have explained before; the value of p at x equals to 0 and t equals to 0 will be same as the value of p at x equals two lambda and t equals to 0. And similarly it will be the same as the value of p at x equals to 2 lambda, 3 lambda and so on. So, after every lambda the value becomes the same.

So, A cos 0 from this equation; p at x comma z x equals to 0 and t equals to 0 is the general form of the solution was this was the general equation A cos B x. So, at x equals to 0 we have A cos 0 and at x equals to lambda; we have A cos B lambda and these two values have to be the same. This value is A cos 0 and this value is A cos B times of lambda and both these values are same.

So, with this, what we get is that B lambda. Now, we know that cos function has a period of 2 pi. So, after every 2 pi; the cosine function repeats itself and the same happens with sine function and tan function. So, all these sinusoidal are trigonometrical functions. They have a period of 2 pi. So, this means that this thing must be 2 pi; that is why the waveform is repeating itself.

So, we equate this, so B lambda becomes 2 pi. So, B becomes 2 pi by lambda. Now, this is a very important value. So, we have written this in the function A cos 2 pi by lambda times x; this is a very very important value. So, we replace this by in the constant k, this is called as the wave number and the wave number is given by 2 pi by lambda, it is a very important parameter.

So, we have got the equation for how this plane wave varies with respect to x. Now, let us see what is wave number. So, as we said this proportionality constant that we are getting 2 pi by lambda is x is k.

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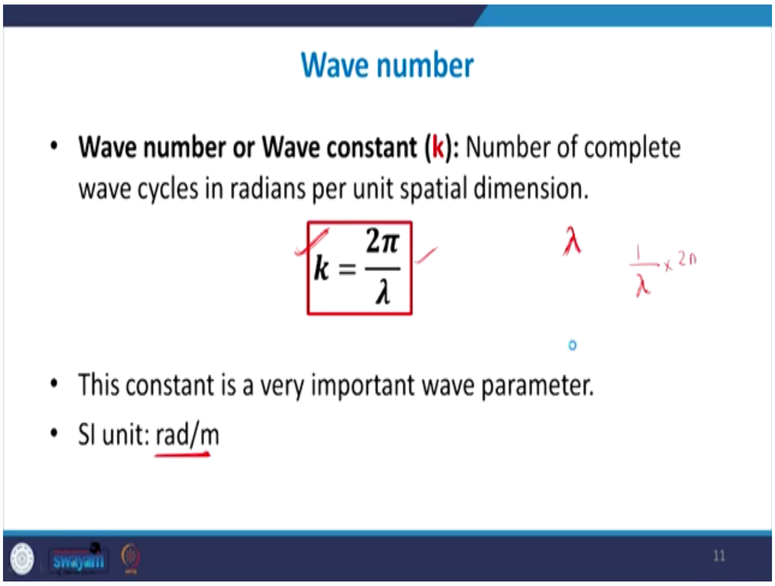

So, wave number or simply the wave constant is represented by the symbol k and mathematically it is 2 pi by lambda and it is defined as the number of complete wave cycles in radiance per unit special dimension. Which means that; if you take 1 meter. So, within a 1 meter of space how many 2 pi revolutions that particular wave is making. So, this is given by 2 pi by lambda. So, again the S I unit of this is radians per meter.

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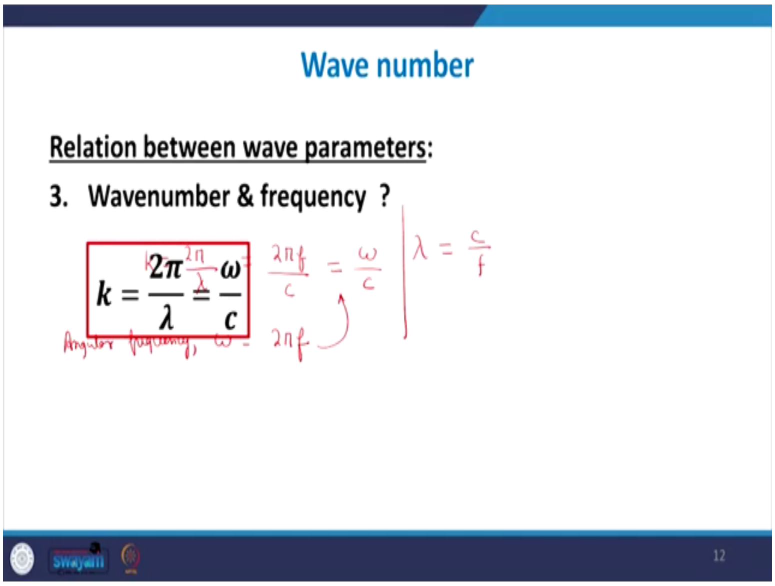

Now, we have defined what is wave number and we got why it is 2 pi by lambda because, what is the number of revolutions it is making per unit dimensions. So, if that 1 if 1 wave cycle takes lambda. So, lambda meters; you have 1 wave within lambda. So, within 1 meter you will have 1 by lambda waves, then how many revolutions will you have within 1 meter 2 pi by lambda.

So, with this, let us derive the relationship between wave number and frequency. Now, wave number is simply k which is equal to 2 pi by lambda and lambda we have derived earlier is c by f. So, it becomes 2 pi f by c and we have angular frequency which we call as omega is 2 pi f. So, again putting this value here what we get is k becomes omega by c. So, that is the equation you get. So, k equals to 2 pi by lambda which becomes omega by c.

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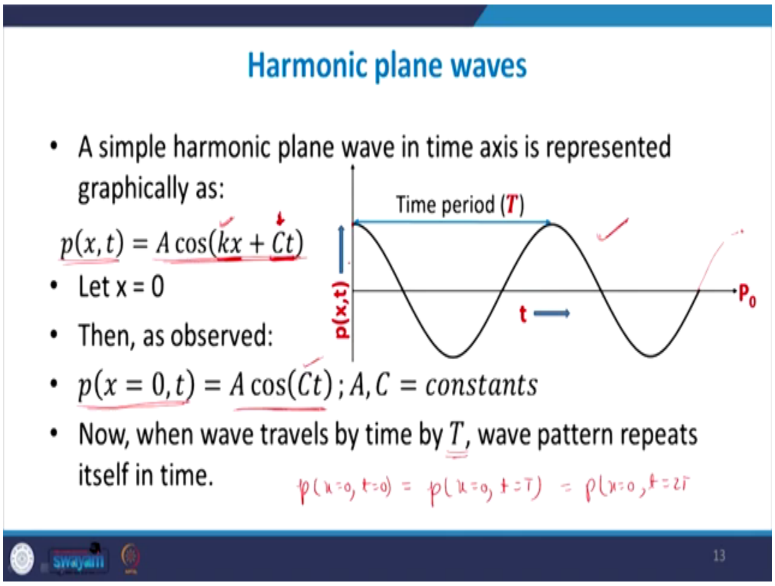

Now, that we know; what is the shape of this particular wave function with respect to space. Now, we will see how it varies with respect to time. So, now, let us assume because we are studying harmonic waves. So, it has some sinusoidal variations. So, it is A cos some variation with respect to space and some variation with respect to time as well. And for this x; we have already found the constant as k. Now, let us assume that we now we have to find what is this constant with respect to time.

So, this is the general wave form for a wave varying with respect to x and t. Now, in this case let us now freeze the freeze a one particular point. So, we let us say let us say we are taking the point as x equals to 0. And now, we will observe how this point at x equals to 0 varies with time. So, this is the wave form of that point. So, p at x equals to 0 comma t. Now, it becomes A cos C t and we will now derive what is the value of this constant C. Now, again following the same logic. Here, the wave pattern repeats itself after every t seconds where t is the time period of the wave.

So, which means that p at x equals to 0 and t equals to 0 will be same as p at x equals to 0 and t is equal to capital T and which will be same as p at x equals to 0 and t equals to twice T and so on. So, after every t the value of p becomes the same.

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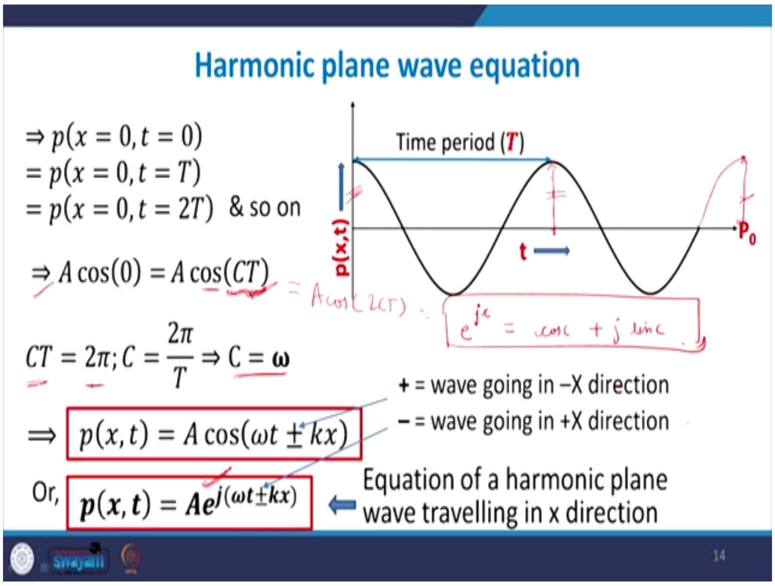

So, we have equated this. So, this value is the same as this value which will be the same as this value and so on. So, with every time addition of capital T; we are getting the same acoustic pressure. Because. so putting this here; what we get is the wave function was something like this A cos C t. So, we put t equals to 0. So, we get A cos 0 and we put t equals to capital T; we get A cos capital C T and so on. And this will be equal to A cos 2 C T and so on.

So, we are equating these two values. So, again this is a cosine function, so which means that it has a time period of 2 pi. So, after every 2 pi it repeats itself. So, this value must be 2 pi, because after every repetition of this value; we are getting the same function. So, this must be 2 pi because 2 pi is the time period of the cosine function. So, since cos repeats itself after 2 pi. So, C T equals to 2 pi C is equal to 2 pi by capital T. And we know that 2 pi f is equal to omega and f is equal to 1 by T this implies omega is equal to 2 pi by capital T.

So, we use this relationship, so 2 pi by capital T becomes omega. So, we have got the separate second constant. So, what we got was that now we have got the general form. So, we are adding both plus and minus signs for a forward propagating wave and a negative and the negative x propagating wave. So, if the wave is propagating forward along the x axis, then in that case we use the negative sign because, as you see when you increase the value of x here. Then the cos value will also increase.

So, as x increases, we descend down in the cosine function. So, minus is added to get the forward propagating wave and plus is added to get the reverse propagating wave and for easier calculations now, we take this as we explain we express it as in the form of an exponent. So, we take an exponential form. So, the same equation can be written as A e to the power j omega t plus minus k x and by Euler’s relationship A e to the power j times of anything any constant let us say c is cos of c plus j times of sin of c.

So, we have taken this complex function to represent both the complex and the imaginary part of the wave. And the reason for representing this acoustic solution in the form of an exponential function is that. Because, as we have seen that the wave equations are usually differential equations it involves double derivatives and so on. So, it is easier to; it is easier to differentiate the exponential function compared to the sine and cosine function.

So, just for ease of calculation; we assume this exponential function. So, whenever you do some practical questions then you assume this exponential function; which is given as this. Then you can derivate you can differentiate it with respect to time space etcetera. Because, it is must easier to differentiate it and then once you get the final solution then you can only take the real part of the final solution and that becomes your answer. So, just for the ease of calculation we take a exponential function.

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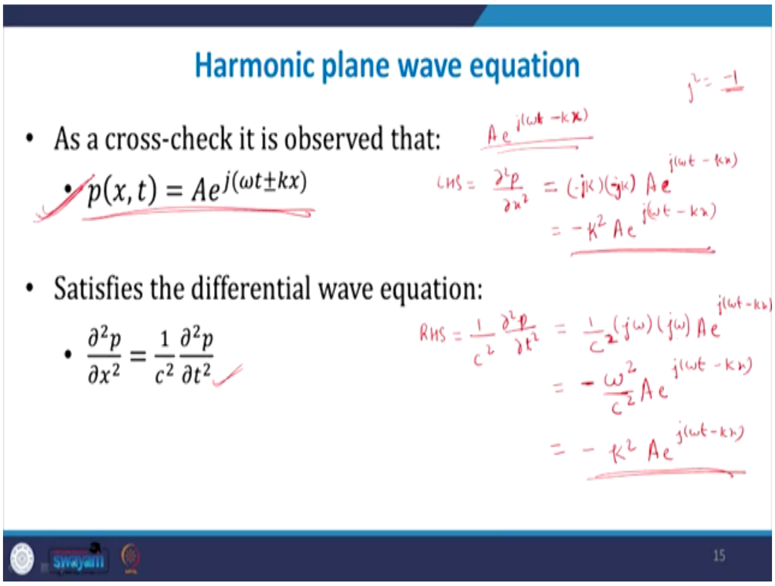

Now, let us say as a cross check; now this is what we have found. A general equation for a harmonic plane wave. Now, if you put this value here, let us see whether it satisfies this equation or not.

So, let me just do it for 1 particular case; which is let us say a e to the power j omega t minus k x. Let us do it for the more difficult part. Then del square p the left hand side is del square p by del x square. So, if you double derivate it with respect to, if you double differentiate it with respect to the space x, then what you get is; minus k into minus k A e to the power j omega t minus k x. So, this is the because, exponential function the derivative of that is the same as the exponential function itself.

So, only this constant is multiplied ok. I forgot to put the j. So, what we get is minus j k. So, this is the minus j into k is what is multiplied with x. So, what we get is j square j square is obviously minus 1. So, j square j square minus one. So, we get plus k square minus j into minus j will be plus sorry it will be minus. j square is minus 1 and minus into minus is plus.

So, we get overall a minus sign here and we get k square times of A e to the power j omega t minus k x. Let us look at the right hand side of the equation it is 1 by c square times of del square p by del t square which is equal to 1 by c square double differentiating it with respect to time what we get is, j into omega; j into omega A e to the power j omega t minus k x.

So, what we get is j j again j square is minus 1. So, we get minus sign, minus omega by c A e to the power sorry minus omega square by c square it was c square here. A e to the power j omega t minus k x. So, here omega by c is k. So, it becomes minus k square a e to the power j omega t minus k x. So, as you can see the left hand side this is the left hand side this is the right hand side they both are same. So, this thing is satisfying this equation. Similarly, if you use the plus sign you will see that it satisfies this equation. So, the equation is correct ok.

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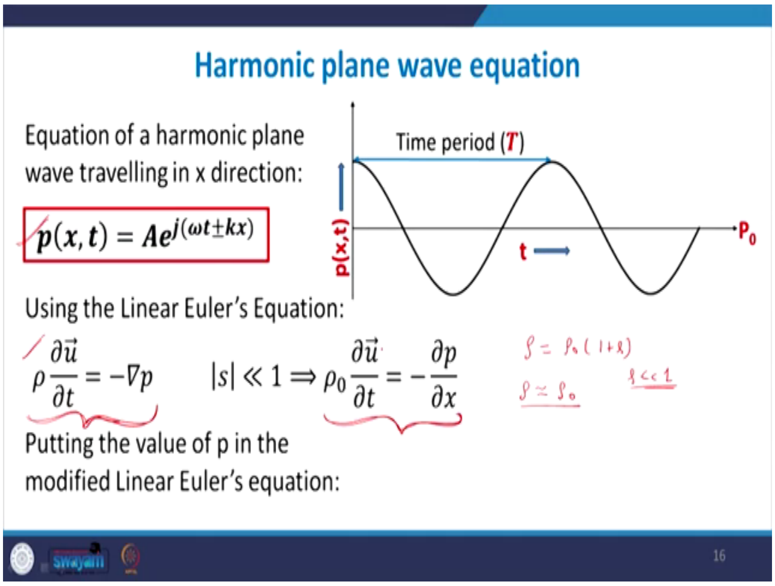

Now, we have derived the equation for the acoustic pressure of a harmonic plane wave. Let us derive the equation for the particle velocity. So, we will use the same Euler relation that we had seen in our previous class which was rho del u by del t is equals to minus lambda times p. If you want to know about this equation you will have to see the previous lecture where we had derived this equation. So, in this case; now, also we know that rho is equal to rho naught times 1 plus s. So, here since s is very very small, so for approximation we have taken rho as equals to rho naught.

So, this is the kind of equation we are getting. So, minus lambda times p because it is a harmonic plane wave propagating only along the x direction. So, we only take the x component of nabla. So, this is the equation we get. Now, we know the function of p; we know what is the function of p. So, we can put this value here to get the value of u. So, let us solve this in class.

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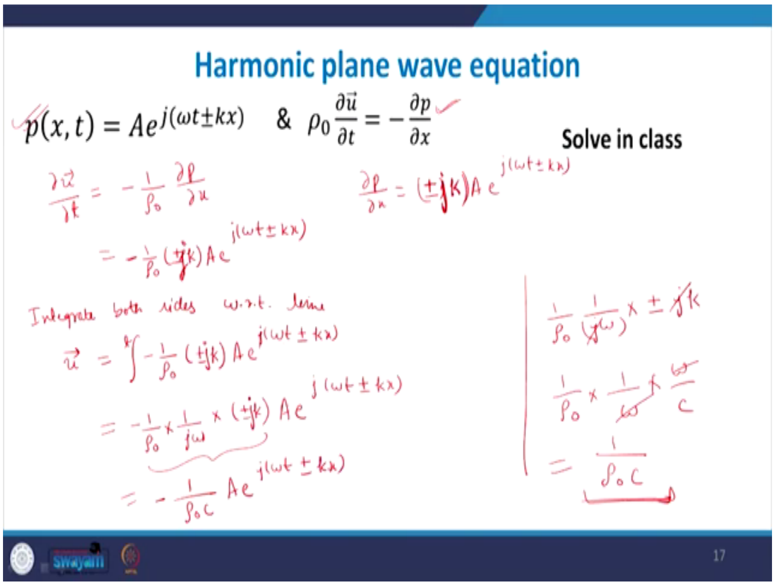

So, p function is known to us and this is the equation through which we will get the function for the velocity. So, let us say del u by del t is equal to minus 1 by rho naught del p by del x. And del p by del x is what? Del p by del x is going to be; if we differentiate this you will get plus minus k times of A e to the power j omega t plus minus k x

So, you put this value here. This is minus 1 by rho naught into plus minus k A e to the power j omega t plus minus k x. So, this is del u by del t. So, if we integrate now if we integrate both sides with respect to time. Then what we get? We will get the velocity function, which will be the integral over time of this function.

So, you integrate this, then integral will be this will become this the constant here will go towards the denominator. So, the integral will be minus 1 by rho naught into 1 by j omega times plus minus k A e to the power j omega t plus minus k x and k is omega by c.

So, what you get here; just solving this constant what you get is here, 1 by rho naught 1 by j omega and this was again I have left j here this will have a j because j is multiplied to both. So, here also there will be a j and a j here also. So, we have plus minus j k, j will be there. So, we have j omega multiplied by plus minus j of k. So, 1 by rho naught into j j cancels out 1 by omega and k is omega by c. So, overall what we are getting is; 1 by rho naught times c this is the constant we are getting. So, it becomes minus 1 by rho naught times c A e to the power j omega t plus minus k x

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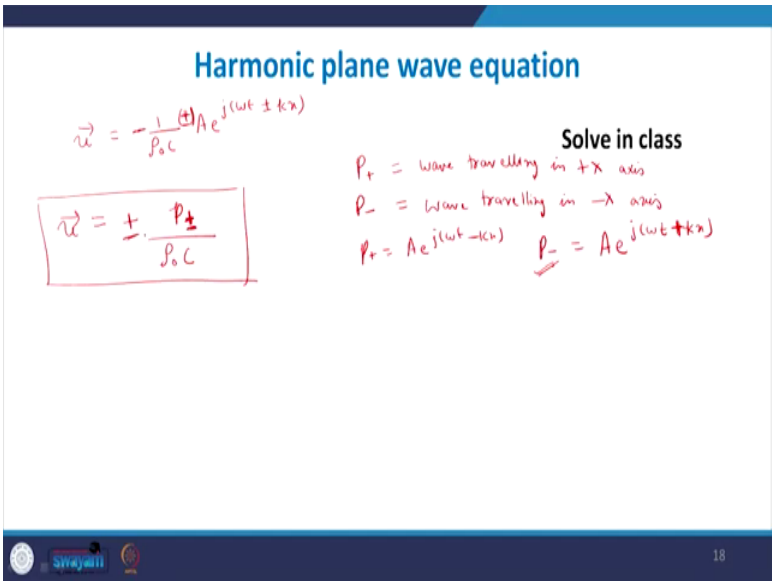

So, again solving this what we get the u of what we got was minus 1 by rho naught times c into whatever is the function for pressure. Now, let us denote two different variables let us say p positive means the pressure for a wave that is traveling in the positive x direction. So, wave travelling in positive x axis or forward propagating wave and p minus is the wave traveling in negative x axis or the backward propagating wave. So, this p positive will be what it will be a e to the power j omega t minus k x and p negative will be A e to the power j omega t plus k x. So, here if we have a plus sign then this plus sign will vanish.

So, overall what we will get is. So, this plus sign will vanish; suppose, we had a negative sign, this minus sign will remain. And we had a plus minus here right, depending upon what equation we are considering. So, we had this and a plus minus sign here. Depending upon what equation we are considering. So, if you are considering this equation; this equation the negative one. Then a negative sign will come and negative will become positive. So, we will get a positive sign when we are considering positive wave front.

So, it will be p by rho naught times c, but if we are considering this backward propagating wave which has a plus sign. So, if because it has a plus sign, so this minus sign will remain and this will be the plus sign. So, minus into plus will become minus. So, this is the overall equation. So, as we have derived, so when we have a forward propagating wave; it is plus p by rho c when we have a negative propagating wave or a backward propagating wave we get minus p by rho naught c.

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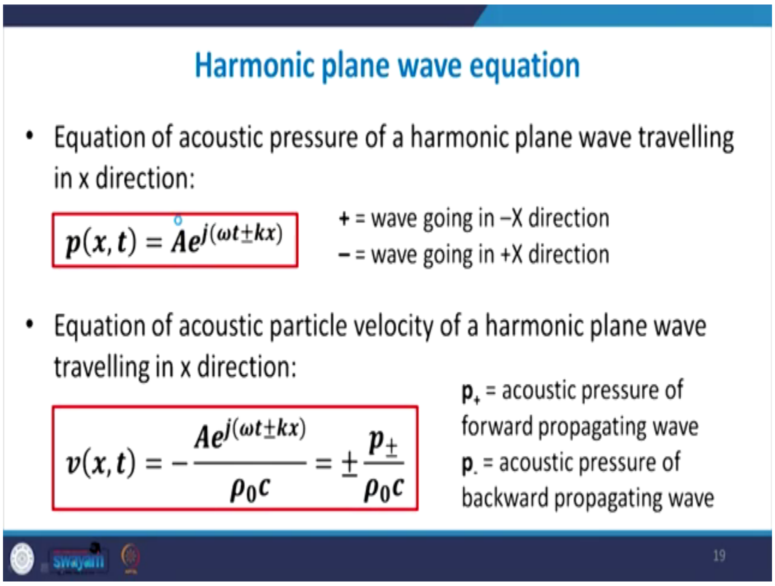

So, now we have both the equations for harmonic wave, if pressure as well as the velocity. So, the pressure is given by this equation and the velocity becomes this equation.

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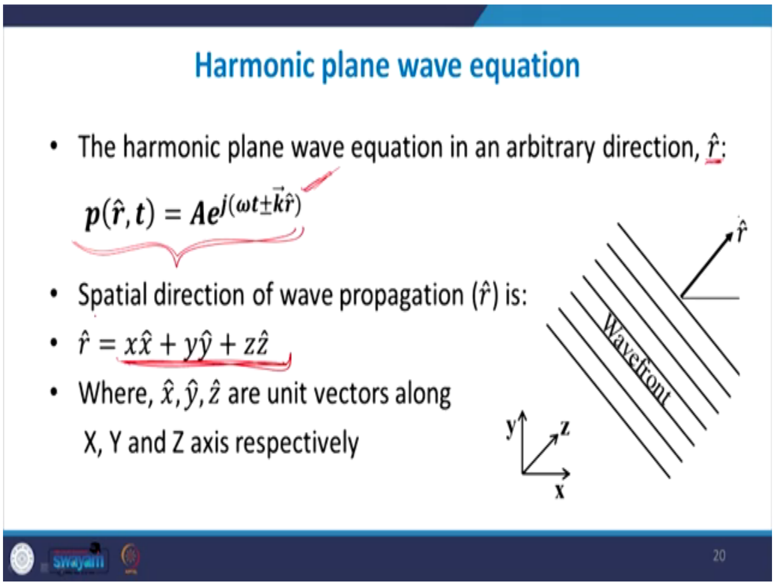

Similarly, if we are deriving now, this was for a wave propagating along plus x direction, but if we had a wave that was propagating in any arbitrary direction r, then for that particular direction also we will have the same relationship. So, it is propagating along r and this is the kind of equation we are getting. Now, we can and this r is nothing but the, it is a direction vector; this r is giving you the direction of wave propagation.

So, this is the general form. So, if we have any general wave propagating along a direction r then this equation can be used and when it is propagating along X we replace this r by x. Suppose, it is propagating along Y direction; then we replace this r by y. So, this r can be any random direction given by this. Where, x cap y cap and z cap are the unit vectors along X Y Z and X Y Z could be the coordinates of r. So, we again we have just given a random vector here.

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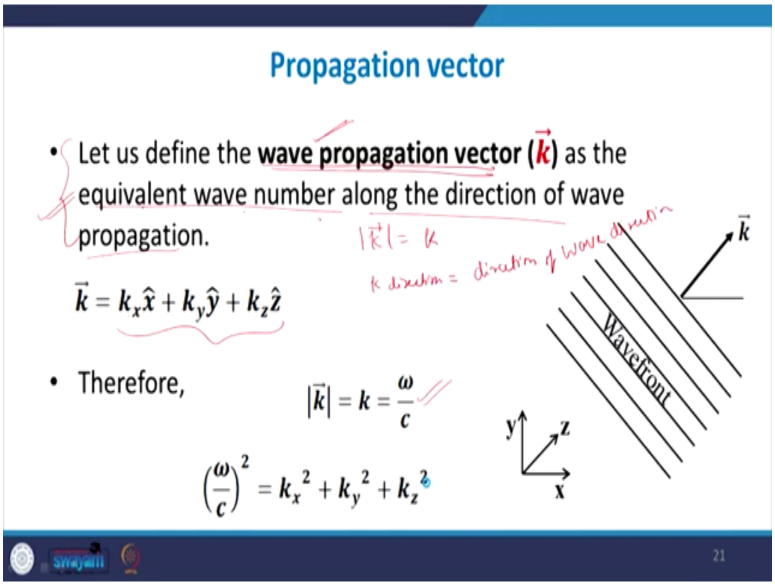

So, now we mention important quantity called as the propagation vector. Now, you already know what is wave number. The propagation vector is the same as wave number in magnitude. So, it is simply propagation vector is a vector; which is along the direction of wave propagation and that value of the vector is equal to the value of the wave number. So, this is the definition of this propagation vector represented by k vector. So, here it is defined as the equivalent wave number along the direction of wave propagation.

So, this has this is an equivalent wave number along the direction of wave propagation. So, the mod of k will always be the wave number k and the direction will be along and the direction of k will be direction of wave propagation. So, now, the only difference between a wave number and the propagation vector is that wave number is a value it is a scalar quantity and when you have that scalar quantity and you also add the directionality which gives you which direction the wave is propagating, then that becomes a propagation vector.

So, this totality becomes a propagation vector; the value as well as the direction. So, this can be written as a component along x y and z axis and this as i already said the magnitude is omega by c. So, the if you do the mod square of this you get omega by c whole square which will be equal to k x square plus k y square plus k z square. Now, the reason for giving this particular equation is that; if you have any random direction which is a component of x y and z. So, if the wave is propagating along an arbitrary direction r cap.

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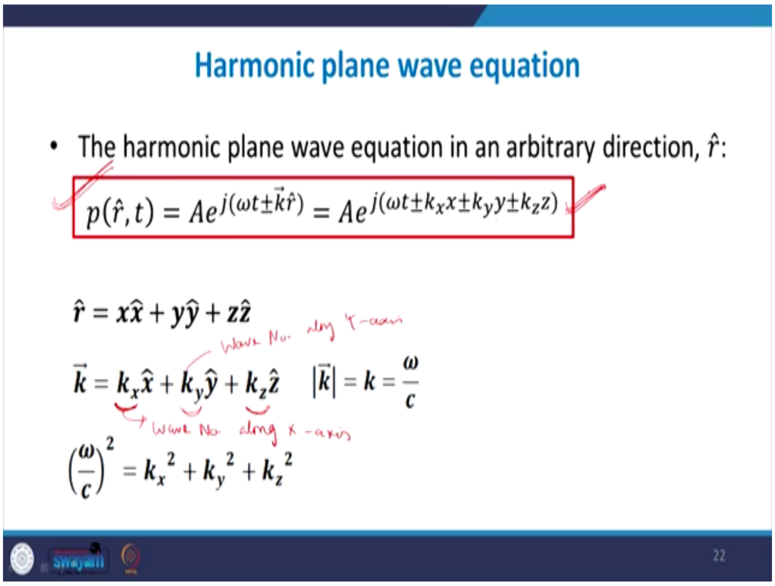

Then using these relationships; we can simply represent it as component along the x y and z axis. So, this is the equation for this, A e to the power j omega t plus minus. Now, we have decomposed this vector k as of the wave vector some component along x axis y axis and z axis. So, this is simply the wave number along x axis. So, what is the wave number along x axis and similarly this is the wave number along the y axis.

So, you get the value of the wave number along different axis and this vector together becomes the equivalent wave number. So, this is a general form of representing a wave in x y and z direction. So, in the next class; we will solve a few numericals based on whatever we have studied. So, thank you for listening for to the presentation.