Acoustic Materials and Metamaterials
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Lecture – 28
Member Type Acoustic Metamaterials - 1

Welcome to lecture 28 on the series on Acoustic Materials and Metamaterials. So, in this lecture we will begin our discussion on a new type of acoustic metamaterial which is a Membrane Type Acoustic Metamaterial.

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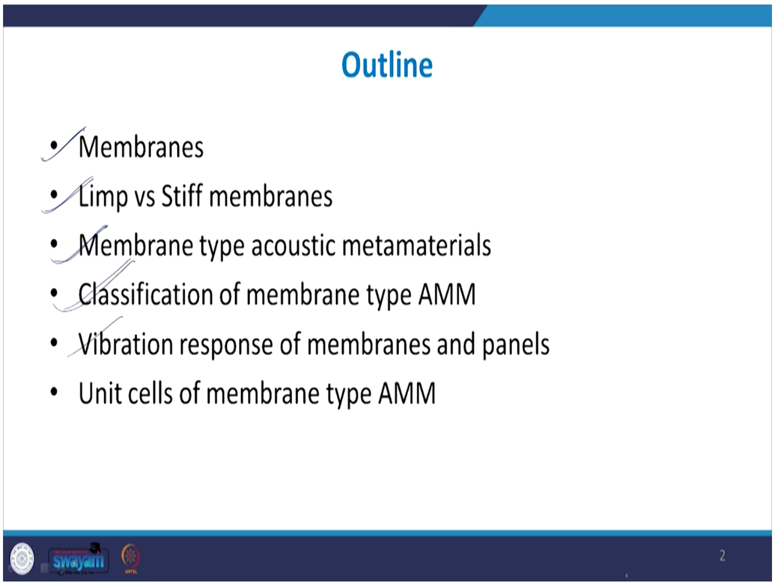

So, first of all I will describe to you, what do you mean by membranes. what is the difference between a limp and a stiff membrane then I will go into the details of what is a membrane type metamaterial or a membrane type acoustic metamaterial, what are the different types of such membrane type AMMs and then we will look into one small derivation on, what is the vibration response of a limp membrane and how does it make it a (Refer Time: 01:12) how does this benefit the membrane type AMM so that they can be used for the adaptive noise control and finally, I will end with the discussion on the different unit cells that are proposed for a membrane type AMM.

So first of all what are membranes? So, if you look around in your real world, you will see that for example, the diaphragm of the loudspeaker or the diaphragm of a microphone these are all thin panels of pliable these are thin pliable sheets of a material. So, what is meant by the word pliable is that, it is like a thin sheet of material which can be folded, which can be bent, which can be which is malleable. So, it can be bent or molded or folded into different shapes. So, that is a thin pliable sheet of material. So, membranes are thin pliable sheet of materials and some of the very best examples could be that if you see any percussion instrument like a tabla or a drum. So, if you see in this figure here.

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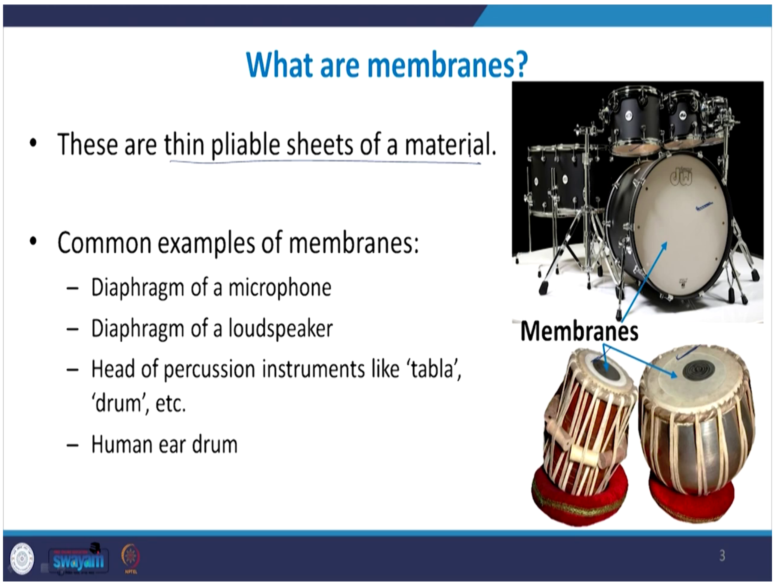

So, here the top of the tabla or this drum which actually hit it starts to vibrate and it creates a sound. So all this is an example of membrane which are nothing, but thin pliable sheets of material, even the human ear drum is a membrane.

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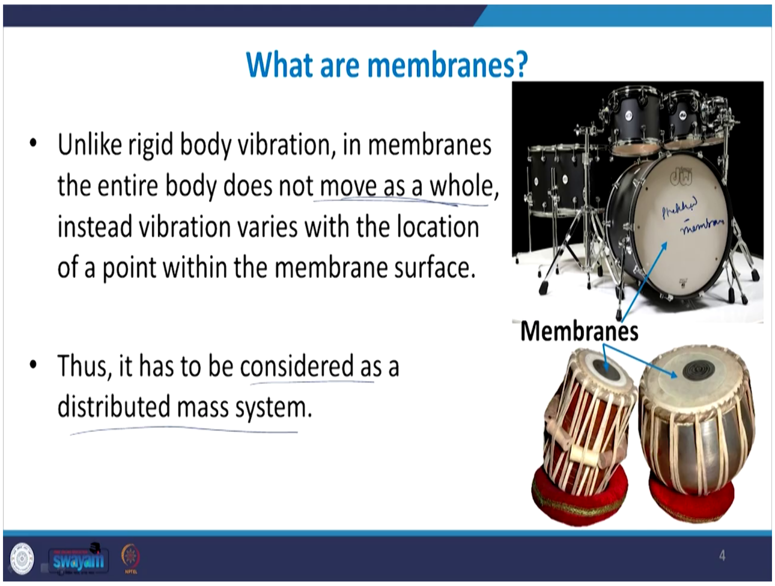

So, unlike rigid bodies, so if you have a rigid body if you give some excitation the whole body moves as a the body moves as a whole he does the vibration or the deformation does not the vibration does not depend upon the location of that particular member inside the body. But if you such thin membranes so, in that case they act more as a distributed mass system. So, when some excitation is given to them then the vibration response will be dependent upon the point in the membrane. So, it will be dependent upon the x y z coordinates of the point in the membrane. So, that is why they are not considered as moving as a whole or stiff bodies, but rather are distributed mass system.

So, the membranes they can be of 2 types they can either be very limp or they can be stiff and very stiff membranes. So, in the previous lectures we discussed about panel resonators or a panel absorber and then we also discussed about micro perforated panel absorber. So, in all these case we had the thin sheet of material, but that material was hard and stiff. So, it was a panel. So, even they can be called as membrane, but they are more commonly called as panels.

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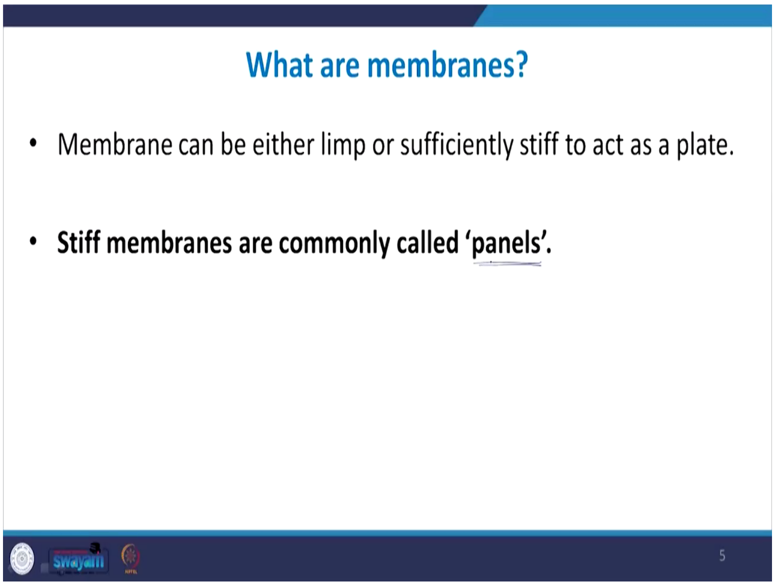

So, stiff membranes are commonly called as panels.

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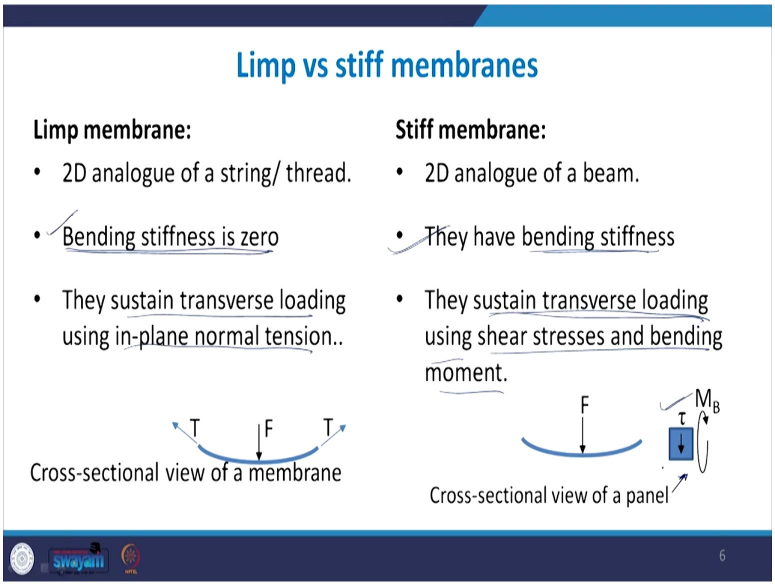

So, what is a difference between a stiff and a limp membrane? So, you can visualize it in this way that if you have a stretched string and if you revolve that stretched string in 360 degrees. So, you get a 3 D version and that is a membrane. So, membrane is like a 3 D version of a stretched string in the same way if you have a beam and if you revolve it 360 degrees then you get a plate kind of a material. So, it is so, the plate or a stiff membrane becomes a 3 D analogue of a beam.

So, over here what you see is that in the limb membrane is it behaves as if it is a stretch string, but in 2 dimensions. So, I am sorry I was talking about 3 D it is a 2 D analogue. So, here this string is in one dimension, but when you talk about this string in 2 dimensions revolve it 360 degrees you end up with a stretched membrane. So, the main criteria of defining this as limp of stiff is that the bending stiffness for if a bending stiffness of a membrane is almost closed to 0 or it is a very small value it can be treated as a limp membrane and if the bending stiffness of a particular material or a membrane is a non 0 high value which cannot be neglected then it becomes a panel or a stiff membrane.

So, in the case of limp membrane when a transverse load is given to the membrane then in plane normal tensions are generated. Whereas in the case of such beam type panels when the transverse loading is given to them shear stresses and bending moment are generated this shows a cross sectional view of this case.

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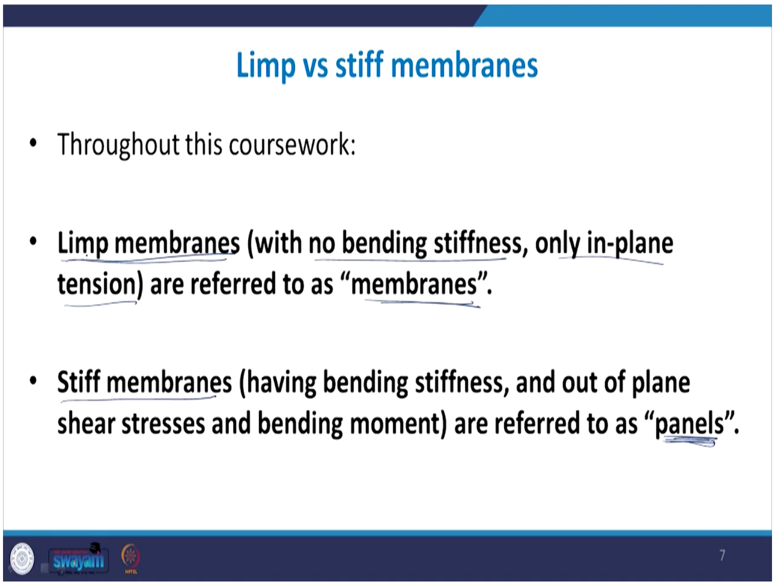

So, overall in this the main distinction between a limp and a stiff membrane is that limp membrane the bending stiffness is negligible that is not the case with stiff membrane and in the limp membrane whenever some transverse loading or excitation is given then tension is developed across the membrane whereas, in the case of a plate or a panel when you gives some excitation then shear stress will be developed and bending moment will be developed and things like buckling and bending of the particular plate can take place.

So, from here onwards throughout this course work when I am discussing about membrane type acoustic metamaterials, what I will mean by membrane is it will be the limp membranes I those membranes or structures with no bending stiffness only in plane tension acting on them when a transverse load is applied to them stiff membranes will be referred to as panels. So, let us begin with the discussion on such membranes which are limp.

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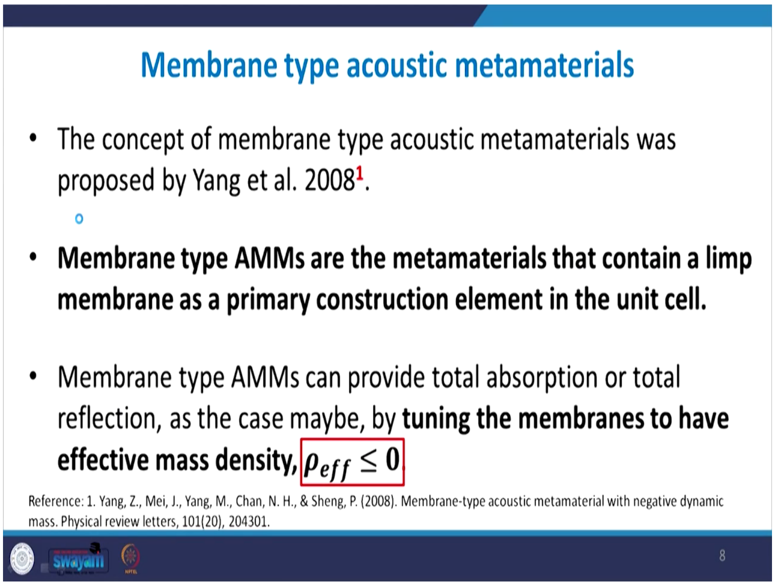

So, what is a membrane type acoustic metamaterial? So, from the name itself you can guess that this is a acoustic metamaterial which is made up of membrane. So, here it is a acoustic metamaterial where the unit cell contains a stretched membrane as it is primary construction unit. And it was first proposed by Yang et al in 2008 and I have given you the reference to the paper here which you can read for you if you are further interested, but I will be discussing all the findings of the paper in my lecture also. So, this kind of metamaterial is actually a negative density metamaterial.

So, here it can it can be it is able to provide a total absorption of sound waves or a total reflection of sound waves as the case may be by tuning the membranes to have the effective mass density less than equal to 0. So, in the regions where this density becomes negative these membranes so extraordinary properties they can behave perfect they can behave like a perfect sound blocker or in certain cases they can behave as a perfect sound absorber. So, these are negative density acoustic metamaterials.

So, let us see what are the various classifications? So, so far 2 types of acoustic, 2 types of membrane type acoustic metamaterials have been proposed.

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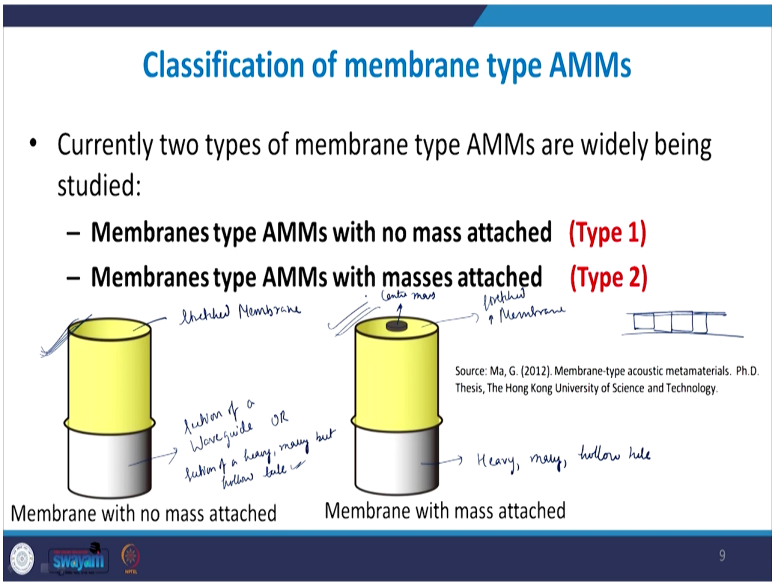

And this shows you the unit sales for the 2 types of membrane type acoustic metamaterials. So, here in the first case you have this is a wave guide this is a section of a wave guide or you can say this is a section of a heavy massy, but hollow tube. So, hollow tube which is thick in the material whose material is very thick and dense. So, such kind of hollow tube is used as a wave so, it becomes a small section of the waveguide and on the top of this tube stretched membrane is attached.

In the second kind of material it is the same thing you again have a heavy, massy hollow tube and on the top of the hollow tube an elastic membrane is attached which is stretched. So, we can call this as a stretched membrane and this is also a stretched membrane. But in the second a unit cell there is also a center mass. So, there is a mass attached at the center of this elastic membrane. So, these are the 2 different types of units cells that have been proposed and usually they are connected in series. So, one unit cell followed by another unit cell followed by another unit cell and so on and a big long tube can be constructed with small sections like this unit cell.

So, in the some of the later lectures on membrane type acoustic metamaterial I will actually show you how these unit cells are arranged to make it into a big structure. So, first of all let us just study about the unit cells and how these unit cells impart this negative density to these acoustic metamaterials. So, before I begin with the derivation of a effective mass density for the particular units cell I will first discuss that if there is a stretched elastic membrane here what will be the vibration response of this stretched elastic membrane when some sound wave hits it or some form of excitation is given to it, how will this particular membrane respond to any acoustic excitation.

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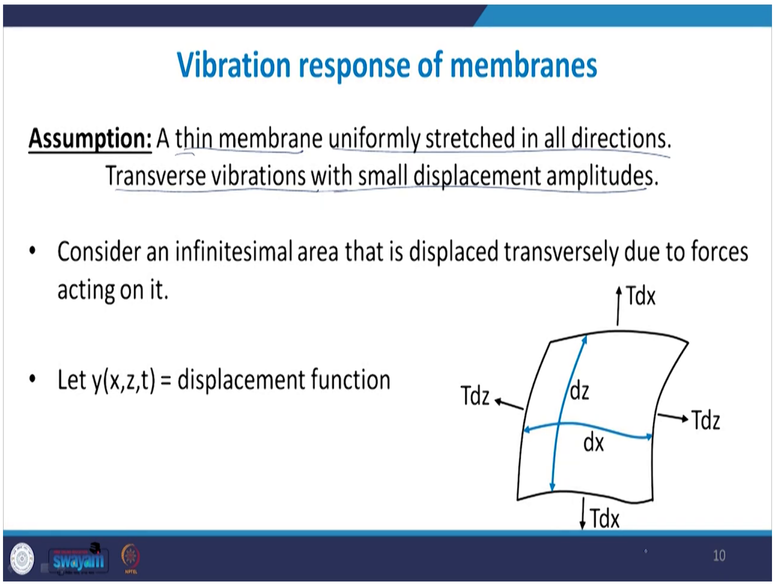

So, here let us consider we are considering a thin membrane which is uniformly stretched in all directions and some transverse vibrations with small amplitude and it undergoes transverse vibration so, with small displacement amplitudes. So, you can imagine for example, you have a drum so, in this kind of instrument here. So, what I am what we are going to study here is that let us say we have a drum here.

So, this is the stretched membrane here a stretched membrane and then suddenly somebody hits the drum. So, he or she that is hitting the drum is providing a transverse a transverse excitation. So, this is the plane of the membrane and normal to the plane of membranes some excitation is given in the form of for example, hitting the drum or hitting the table, then what will be the form of vibration response.

So, let us derive this. So, here what the first assumption is that the tension that is developed will be uniformly. There is a uniform tension or the membrane has a uniform tension it is stretched uniformly. So, if I consider a small infinitesimal area of this membrane and the displacement then it would be a function because; obviously, here the displacement will depend upon where it is located in the membrane. So, it will be a function of both x, z and t, here x, z is the plane of the membrane and y is the transverse direction.

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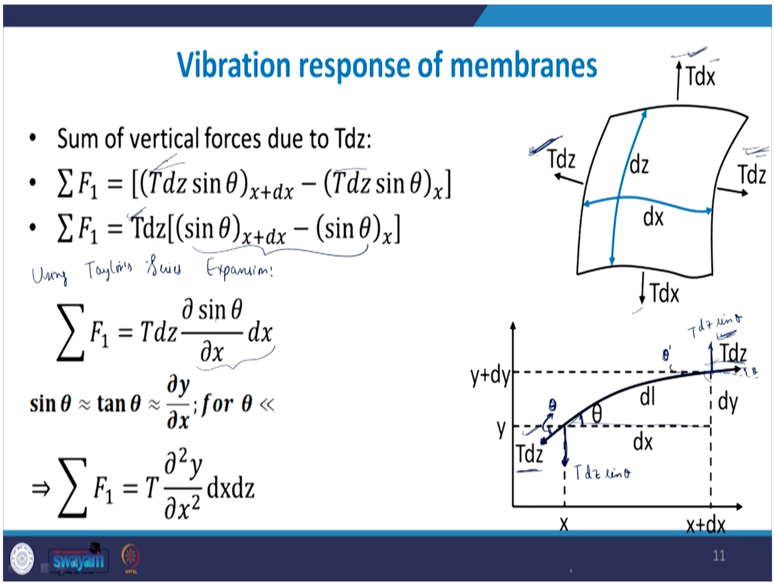

So, again showing you this elemental area, so, you know that tension is force per unit length. So, if this is an elemental area where this is this length is dx, this length is dz. So, the total area is dx into dz and the tension is acting uniformly everywhere so T is the tension acting uniformly everywhere. So, the force acting along this direction will be what it will be tension multiplied by the length across which it is acting. So, it will be T into dz because tension is the force per unit length. So, force is equal to tension multiplied by the length across which it this tension is acting.

So, in the same way in this direction the force will be tension multiplied by the length across which it is acting so, it will be T dx. So, now, that we have the forces acting along the different directions let us consider a side view of this membrane. So, this is how the membrane looks like from the side and we are considering this dx, the dx it is a way considering how does it look like along the x axis.

So, this is a stretched membrane. If it was not stretched sorry this is a membrane that has already been excited and it started to vibrate. If it was not vibrating it will there was no excitation then in the steady state it will be horizontal right because the membrane is along the xz plane. So, it will be horizontal, but now because of the excitation it undergoes some transverse displacement.

So, let us say this is the displacement shape of the membrane and in the area we are considering a point starting from x to a point starting ending at x plus dx. So, this is the dx length we are considering which has been displaced now. So, what is the sum of the vertical forces due to the this tension the tension acting along this dx is T dz, this is the tension acting these T dz. So, what T dz tension is acting here and similarly at T dz some tension is acting here sorry the force is acting here, T is the tension and T into dz is the force acting along that direction. So, this is the force acting along these directions and theta is the displacement.

So, what we see here is that let us find out what is the vertical force acting along the y direction. So, if this is T dz which is the force along the length of the membrane then it is vertical component. So, if this is the angle theta this will be angle theta sorry if this is the angle theta this will be angle theta. So, this will be the angle theta here and therefore, it is vertical component will be somewhere across this which will be T dz sin theta. Similarly whatever is the value of so, this is the value of theta at point x then there will be some value of theta at point y also.

So the way it is defined is it is the deflection from the horizontal. So, from the horizontal this is theta dash here then the vertical force here would be what. This is theta then the vertical component of this would be this becomes theta and the vertical component becomes T dz of sin theta. So, the vertical component in both the ends is T dz into sin theta, but it is this theta value which then varies as a function of x. So, we can write the vertical forces so, the net vertical force will be a difference between the 2. So, it will be T dz sin theta at the point of x plus dx minus T dz sin theta at the point x.

Now tension is uniform throughout and dz is the length which is also constant with respect to x. So, we can take out these constants outside. So what we get is T dz times of sin theta value at x plus dx minus sin theta value at x. So, this gives us the net vertical force acting and so, now, the net vertical force acting due to the tension T dz. So, this, now if you use Taylor series expansion so, if we using Taylors series expansion if we use this case then this particular thing becomes if you use Taylors series expansion on this expression then this can be given as T dz and this expression becomes del by del x of sin theta into dx because sin theta x plus dx minus sin theta x divided by dx is simply del sin theta by del x. So, from that Taylor series expansion this is the overall value what I am getting here ok.

Now we know that all the acoustic processes they involve very small fluctuations so, theta is small. So, when theta is very small then sin theta is approximately equal to tan theta which can be written as del y by del x. So if we replace this sin theta by del y by del x what we get is T dz del by del x of del y by del x so, it is the double derivative of y. So, this is the expression T del square y by del x square into dx dz. Similarly if you solve what is the net vertical force acting what is going to be the total vertical force due to this T dx and the same way you carryout.

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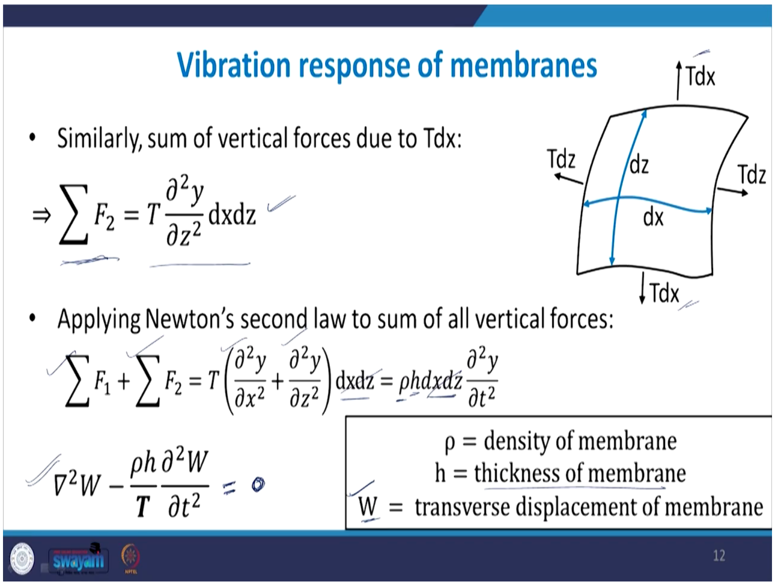

So, you will get is the net vertical force due to this particular kind of tension is T into del square y by del x del z square into dx dz. So, here we differentiated it with respect to x in the other case we differentiate it with respect to z. So, this is the expression you get. So, the total vertical force then due to both these components in along the x axis and along the y axis the total vertical force due to both these their deflection along the x and y axis is then given by summation of F 1 plus F 2 which is T del square y by del x square plus del square y by del z square into dx by dz

And by Newton's second law this net vertical force will be equal to the mass into the acceleration at that point. So, it will become the mass into the acceleration of that elemental area of the membrane. So, this particular forces mass which is density into the volume. So, if h is the thickness of the membrane and dx and dz become the area which we have considered.

So, thickness into area gives you the total volume multiplied by density which gives you the mass and acceleration is del square y by del t square.

Now, let us replace and give a general variable W for transverse displacement of membrane because sometimes a membrane can be along xy axis or yz axis and so on. So, it can be along any such plane yz plane xz plane and so on. So, let us change the variable into a new variable called w which is the transverse displacement. So you can replace this and the expression becomes nabla square and this becomes nabla square of W this dx dz cancels out is equal to rho h by T del square W by del t square. So, this becomes the overall equation this I have put as 0 this is the free vibration equation nabla square W minus rho h by T del square W by del t square.

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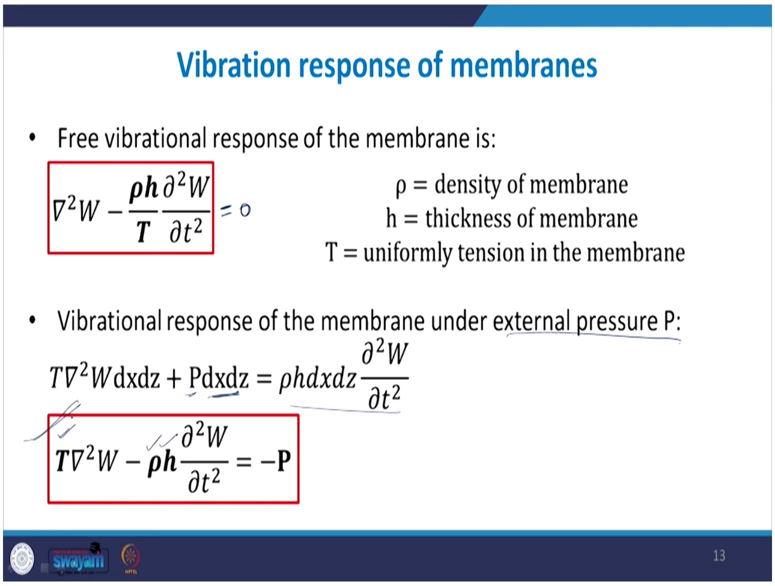

Again this becomes equals to 0 for a vibration response. Now if some external pressure is acting then the same equation can be used. So, you had T into nabla squared W. So, T into nabla square W. So, you had T into nabla square W into dx dz into dx dz plus the external the total when some external pressure is acting then the total force will be the sum of the forces due to this tension plus the force due to the pressure acting.

So you will also have this pressure times the area. So, this will be the net force which will be equal to mass into acceleration so, this expression. So, when you solve it this dx dz cancels out from every end. So, what you get is T nabla square W minus rho h del square W by del t square is equal to minus P, where P is the external pressure being applied, this is the thickness of the membrane, this is the density of the membrane and this is the uniform tension across the membrane.

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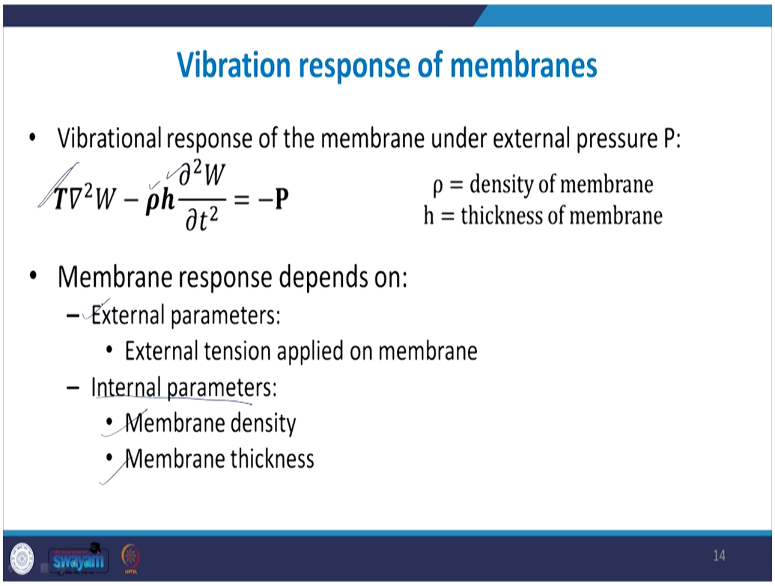

So, overall what we get is that when a membrane when we have a stretched membrane a uniformly stretched membrane and some excitation is given to it then the response of the membrane depends upon some external parameters as well as internal parameters. So, it; obviously, depends upon this inherent properties which is the membrane density and thickness rho and h value. But it also depends upon an external parameter called the external tension that is applied to the membrane.

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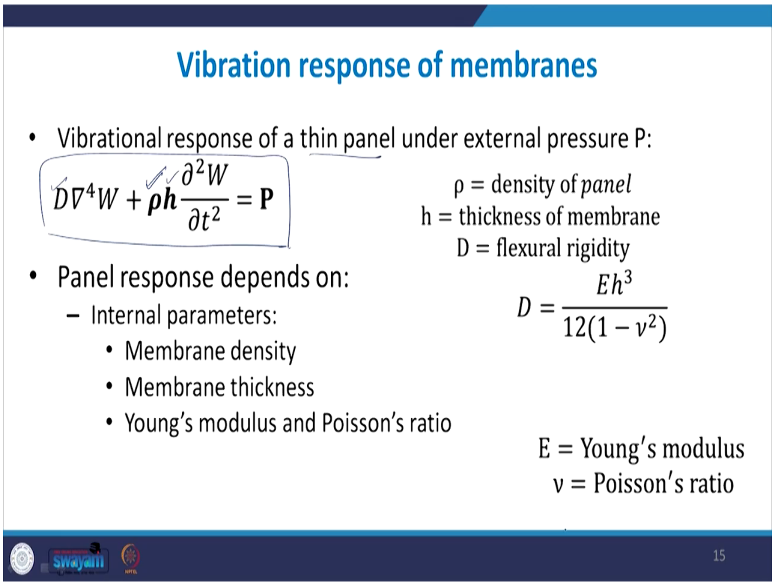

Whereas if you have a thin panel then the vibration response is given by this. So, in this case it depends upon again the membrane density the panel density the panel thickness it also depends upon it is Young's modulus and Poisson's ratio so, all in built properties.

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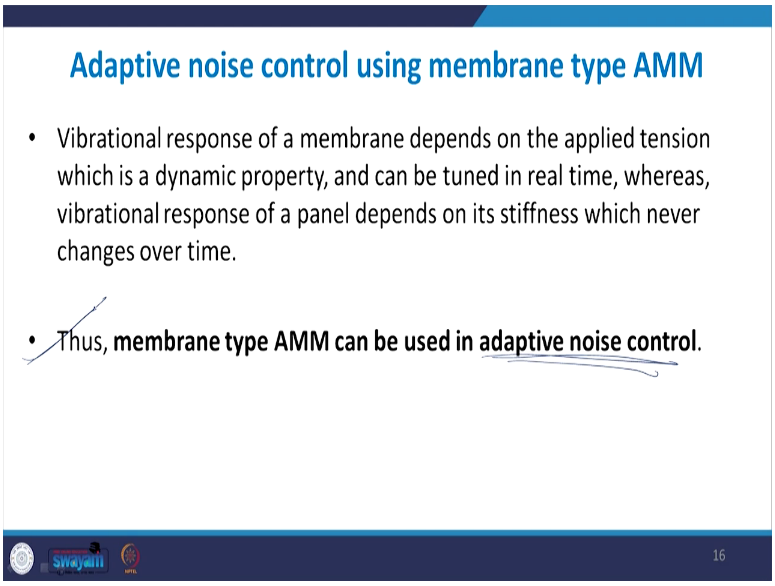

So, what we see from these 2 vibration response is that, in case of a membrane the vibration response will also depend upon it is tension. So, the response will depend on the tension. So, if suppose you have a mechanism through which you can tune the tension in the real time. So, let us say we had a stretched membrane and some sound waves were hitting it or some excitation was given. So, based on what type of excitation is given we are tuning the tension in the membrane so we had some mechanism to stretch it destretch it and so on.

So, if you can change the tension in the real time then we can use it to change it is response in the real time and when we change the response of the membrane in the real time then we can use it for adaptive noise control. So, this is. So, I discuss this particular thing just to show you that, the membrane vibration what is the advantage of using a membrane, the advantage of using a limp membrane is that, the response of this also depends upon an external property called tension and when this tension is tuned in the real time then adaptive noise control is possible ok. So, after learning about one potential benefit of using a membrane let us just briefly see what are the 2 types of unit cells.

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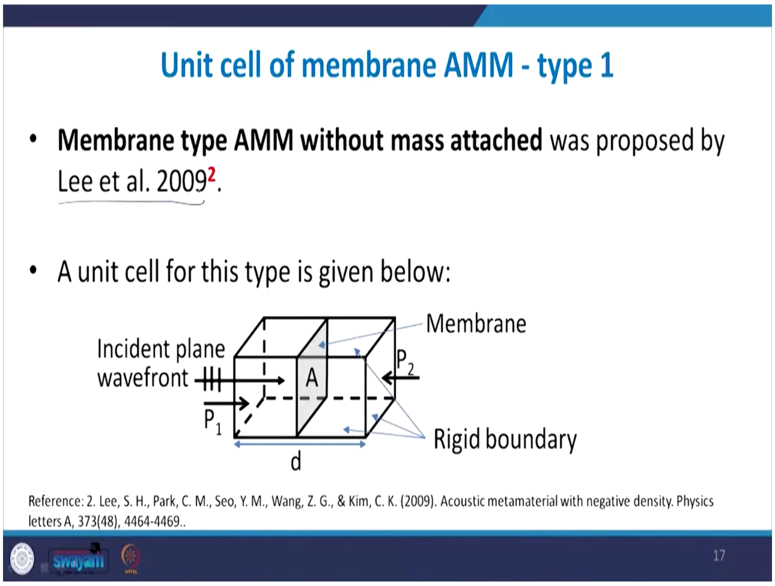

So, both of them are proposed by Lee et al 2009. So, the first type of unit cell was proposed by this scientist Lee et al in 2009. So, again I have been showing you here we have a sub wave length wave guide, these are all rigid boundaries and a stretched membrane is in between and the plane wave front is incident.

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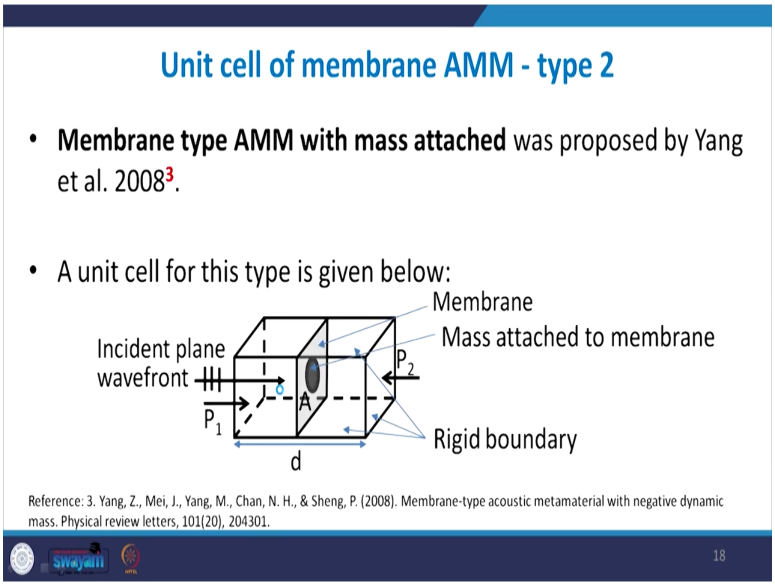

And in the second case you have a wave guide a section of a waveguide or a hollow tube and inside this hollow tube you have a stretched membrane and a center mass attached. So, in our next lecture I will go through how to represent these 2 unit cells in the form of a mass spring model and then how to derive the effective mass density.

So, thank you for listening see you for the next lecture.

Thank you.