

**Acoustic Materials and Metamaterials**  
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**Lecture - 21**  
**Microperforated Panel Absorbers-1**

Welcome to lecture 21. This is our 5th week in the series on Acoustic Materials and Metamaterials and I am Dr. Sneha Singh, an assistant Professor at the Mechanical and Industrial Engineering Department at IIT, Roorkee. So, before that we began our discussion on acoustic materials and we studied about some conventional acoustic materials that are widely used today. So, we discussed about the porous absorbers.


Firstly, we discussed about the barriers and enclosure materials and then, we discussed about sound absorbers and within the sound absorbers category, we studied about Porous absorbers. Then, we studied about panel absorbers, then perforated panel absorbers and Helmholtz resonators. So, we saw that the perforated panel absorbers, they are a special case of Helmholtz resonators. Today, we will deal with micro perforated panel absorbers.

So, micro perforated panel absorbers as you can see from the name, they are derived from the perforated panel absorbers, but here its micro perforated. So, and they contain micro holes. So, what is the effect of just reducing the hole size and how it becomes a different absorber altogether? So, let us begin our topic.

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### Micro-perforated panels (MPP)

- It was found that a higher absorption magnitude can be achieved if holes are made smaller, sub-millimetre, in size.  
 $\text{micrometre} = 10^{-6} \text{ m} \leq 10^{-3} \text{ mm}$  hole size, diameter  $\ll 1 \text{ mm}$   
Micro holes
- With advent of new technology in manufacturing and machining, micro holes which are less than 1 mm in diameter could be made.
- The panels containing such micro-holes are called "**Micro-perforated panels**".



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So, to begin I will like to say what is a micro perforated panel. So, it is same as in the case of a perforated panel absorber, but the limitation in perforated panel absorber was that the absorption magnitude. So, both for Helmholtz resonator and perforated panel absorber because they both are based on the same principles of Helmholtz resonance.

So, both Helmholtz resonator and perforated panel absorber, they have low they have a limitation to the absorption peak or what is the maximum absorption that they can achieve which I had told to you in the lecture 18. So, this maximum absorption limit is always there and therefore, high absorption may not be achieved all the time. So, that is a major limitation and then, wide range absorption is not possible. So, these were the two limitations a wide range absorption and high absorption.

So, micro perforated panels they are designed to cater to one of this limitation that is to increase the absorption magnitude. So, how does it increase the absorption magnitude? So, experimentally and analytically it has been found that; so, if you see here it has been found that the absorption magnitude can be achieved, it can be increased if holes are made. So, higher absorption magnitude can be achieved if the holes are made smaller or sub millimeter in size.

So, micro means  $10$  to the power minus  $6$  meters is equal to a micron or a micrometer which means  $10$  to the power minus  $3$  millimeters. So, usually any holes that are sub millimeter or fractions of millimeter. That is if hole size here a hole size or hole diameter is taken as less than  $1$  millimeter, significantly less than  $1$  millimeter maybe  $0.1$ ,  $0.01$ ,  $0.001$  and so on; then in that case, they are considered as micro holes or small holes.

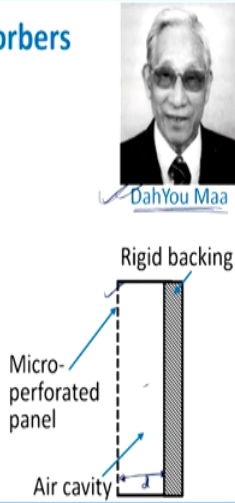
And with the advent of new technology and new machining process manufacturing search holes has become possible. So, initially is very small minute holes could not be made, but nowadays they can be made. For example, nowadays we have 3D printers, both of the metal 3D printers as well as the polymer 3D printers and they are capable of actually printing the holes which are less than  $1$  millimeter that  $0.1$  to  $0.01$  millimeter.

So, nowadays, it is becoming possible and such micro perforated panels have now getting manufactured within the last  $10$  to  $15$  years, but this concept was introduced way before the manufacturing of the micro perforated panels began. So, anyway, so that is the definition of the micro perforated panel which is the panel containing such micro holes or holes with less than quite less than  $1$  millimeter diameter.

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### Micro-perforated panel absorbers

- **Micro-perforated panel absorbers** constitute a thin sheet of hard material with numerous micro holes cut out on it and the sheet backed by an air cavity followed by a rigid backing.
- A special type of air-spring oscillator.
- First proposed by Prof. DahYou Maa



The diagram illustrates the structure of a micro-perforated panel absorber. It consists of a thin sheet of material with small holes (micro-perforated panel) positioned in front of an air cavity. This air cavity is enclosed by a rigid backing. Labels with arrows point to the 'Micro-perforated panel', 'Air cavity', and 'Rigid backing'.

**DahYou Maa**

So, the person attributed to this contribution or the person who first proposed and came up with a theoretical model of a micro perforated panel was Professor DahYou Maa. So, his works were there in his papers which I will give you a reference for in the year 1980; from the year 1980 itself he had proposed a full analytical model for micro-perforated panel.

So, micro perforated panel absorbers, what do they consist of? So, just like a perforated panel absorber. They have a thin sheet of hard material which is perforated with these micro holes. So, this is the thin sheet of hard material perforated with the micro holes and it is enclosed and it contains an enclosed air cavity here and it is enclosed with a rigid backing behind a little distance behind the micro perforated panel.

So, there can be some distance between this micro perforated panel and the rigid backing. So, just the same construction as panel absorbers with we had studied the perforated panel absorbers. So, just the same construction, but now the holes have been made smaller in size.

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**Working principle**

- When sound is incident on a micro-perforated panel (**MPP**), it causes the air to vibrate back and forth through the perforations/ holes. The air movement through the panel is opposed by the bulk modulus of the enclosed air within the cavity.
- So, in an equivalent **mass-spring model**:
  - Small tubes of air with mass that oscillate to and fro through the micro-perforated panels = **Mass**
  - Air in the cavity with its bulk modulus = **Spring**

The diagram illustrates the physical structure and its equivalent mechanical model. The top part shows a vertical cross-section of a micro-perforated panel (MPP) with sound waves incident from the left. The panel is positioned in front of a rigid backing, creating an air cavity. The bottom part shows a mass-spring model where a mass 'M' is connected to a wall by a spring with constant 'k', representing the equivalent system.

So, this is again a special spring type of air spring oscillator just and it works like Helmholtz resonator. So, as I have explained. So, I would not go to the detail of the working principle here because it is the same as the perforated panel absorbers. So, effectively what happens is that when the sound energy, it is incident on these panel, then it and then this panel itself will have its own resonance.

So, whenever the sound is incident then due to acoustic coupling at the resonance or at the resonance frequency, there will be a strong acoustic coupling and the sound energy will drive the air molecules. So, here the neck is this. So, the air molecules that are moving oscillating

back and forth through the neck or through the holes or perforations, they become the mass of this oscillator and we have an enclosed air cavity which has its own bulk modulus to resist any compression and expansion.

So, whenever the air molecules they are oscillating a back and forth, the enclosed cavity when the air molecules they oscillates towards the cavity, they will try to compress it. The compressed by a air molecules within the cavity. And this resistance to compression will then act as a restoring force which will face the air molecules on the other end and as they go and oscillate to away from the cavity.

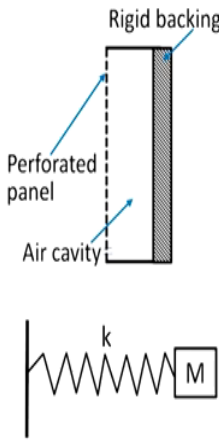
Then in that case the air molecules within that cavity will undergo a small expansion or a acoustic expansion and then, again a restoring force will be acting due to the resistance to expansion and the air molecules of the neck, they will be forced to come back again. So, sort of this particular enclosed cavity is acting as a restoring element and is ensuring that an oscillating motion continues throughout the time provided there is no damping or other opposing force.

So, this is the mass spring model, where the small tubes of air with mass that oscillate to and fro through the micro perforated panels become the mass and the bulk modulus of the air inside the cavity or its resistance to compression and expansion becomes this spring.

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### Working principle

- Thus, each perforation hole with enclosed air cavity acts as an individual Helmholtz resonator.
- So, MPP absorbers can be thought of as made of large number of Helmholtz resonators each having a thin **neck = thickness of the sheet**, and a shared **air volume = total air volume enclosed between the panel and its backing**.



The diagram illustrates the physical structure and its mechanical equivalent. The top part shows a vertical cross-section of a Helmholtz resonator: a thin perforated panel on the left, a rigid backing on the right, and an air cavity between them. Labels with arrows point to 'Rigid backing', 'Perforated panel', and 'Air cavity'. Below this, a mechanical equivalent circuit is shown, consisting of a spring with stiffness  $k$  and a mass  $M$  connected in series.

And the working principle is as I said is the same. So, because there are oscillators they will have their own particular fundamental or resonance frequency and here each perforation will then act as a individual Helmholtz resonator. So, each perforation acts as an individual Helmholtz resonator, where the holes or the holes become the neck and the enclosed cavity just behind the neck or the just behind a particular perforation that enclosed cavity, then becomes the enclosed cavity of a Helmholtz resonator. So, it is just the same.

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**Effect of micro-holes**

- When the holes become too small (less than 1 mm in diameter), then **thickness of the viscous boundary layer around the hole orifices  $\approx$  hydraulic diameter of the holes.**
- So, **viscosity cannot be neglected.**
- Therefore, **high viscous losses** take place as air passes through the perforations.
- This leads to **high absorption magnitude.**

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So, if you see so. So, now, you are seeing that basically they are working the same way as a perforated panel absorber, then what makes them special and what makes them increase the absorption magnitude? So, here it is the effect of the micro holes. So, it is because of this micro holes that magnitude increases. So, in the panel absorbers the dissipation mechanism was resonance oscillation. So, dissipation only happened because at the fundamental frequency, the target sound energy will couple with the panel and it will drive the panel, drive the air through the panel at large oscillations.

So, the work will be done by the incident sound energy on these air molecules and hence at the fundamental frequency, a lot of energy will be lost in doing work in driving these air molecules through the panels at the resonance oscillations. But in this case in the micro perforated panels the resonance happens. So, at the resonance, there will be the energy will be lost to drive the

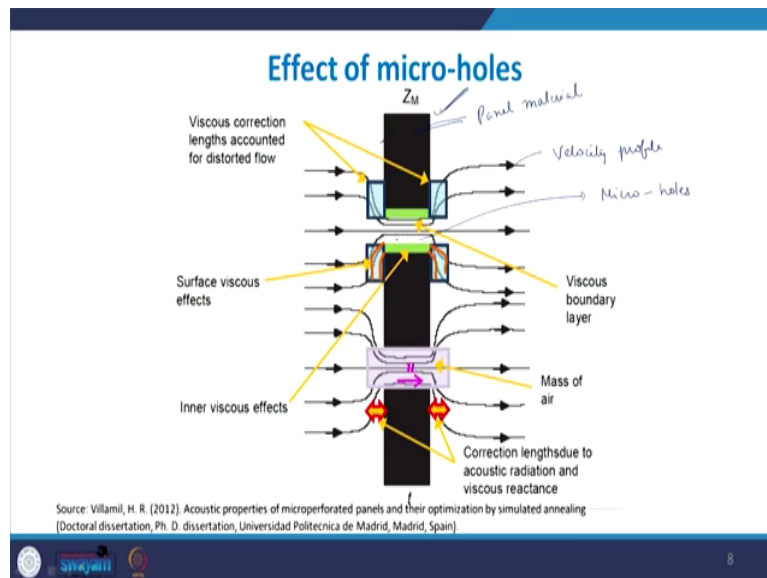


air molecules, but other than that there is also another major dissipation mechanism which is the viscous loss or viscosity.

So, here what we see is that when the holes they become too small less than a millimeter in diameter, then the thickness of the viscous layer the viscous boundary layer around the whole or if is almost same as the hydraulic diameter of the holes. So, now, the viscosity or the order of the viscous resistance is the same as the order of other forces and other pressure and other variables acting on it.

So, in that case it cannot be neglected. A very high viscous losses that take place. So, that is the difference as air passes through the perforations and therefore, a high absorption magnitude is observed.

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To explain this point further, if we have a look at this particular figure here. So, this is a magnified view of a micro perforated panels. So, here we have this is the material or the panel material and these are those micro holes. So, all these are in the micro holes. So, as you see that whenever these as the sound where the air particles they are moving around. So, as they move around and they approach the boundary of the solid, then we this due to viscosity we have a boundary layer, where the flow becomes non-uniform.

So, there because the viscous resistance tries to oppose the velocity of the flowing fluid. So, here this shows the velocity profile. So, what you see is whenever these air particles they are approaching this solid particle. So, all around the boundary wherever all around the boundary, there is a relative motion between air and the material, then this velocity profile it suddenly decreases because of the viscosity.

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### Dissipation mechanisms in MPP

- There is heavy viscous drag between boundary of air and hole boundary surfaces. Effect of viscous drag is observed throughout the hole, as hole diameter  $\approx$  viscous boundary layer.
- Air flow is opposed by the viscous force that is:  $F_{viscous} \propto \frac{\partial v}{\partial r}$  and air velocity changes radially.

So, to explain it in more detail here. So, I have just enlarged the view to in a one particular hole here. So, you have you see is this is the zone where the viscosity is the maximum. So, this is the zone where viscosity acts. So, overall what happens suppose we have some hole. So, if this is a material, here and then, the layer of the layer of air is passing through it. So, you know towards the center line, there is no change in the velocity and as you go more towards the boundary suddenly you will see this effect of viscosity.

So, the it is limited to a certain zone, near the boundary and this layer up till which or the distance up till which the viscosity acts is called as the viscous boundary layer. But in the case of micro holes they are so small in size that the viscous boundary layer itself is almost the same order as the hole size.

So, this means that almost throughout the hole, the viscosity effect can be observed. If the holes were bigger than only be observed for a small fraction of the air molecules near the boundary and the remaining air molecules they will pass through without any viscosity effect.


But because of smaller size now the entire zone comes under viscous effect and that is why heavy losses are observed and what it tries to do is viscosity this is a resistive force. So, it tries to reduce the it tries to this is it tries to reduce the velocity radially as well as actually as there is moving. So, it is acting as an opposing force to the flow of the air molecules. So, as you saw here, so the only effect; so, the main effect that becomes is that under such small magnitude of hole size, the viscous boundary layer is the same order as the hole size.

And therefore, the entire throughout the entire hole the viscous effect can be observed and there is a heavy reduction in the velocity of the or there is a heavy opposition to the flow of the air throughout the whole. So, because there is heavy opposition to the flow of the air throughout the whole, so this incident sound energy has to do now more work to drive these molecules at oscillation at resonance oscillation.

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### Dissipation mechanisms in MPP

- Incident sound energy is absorbed & dissipated by following mechanisms:
  - **Resonance oscillations** due to acoustic coupling between MPP and incident wave.
  - **Viscous losses**: air flow through the holes face heavy viscous resistance, so some energy is dissipated in overcoming the viscous forces. Viscous losses are proportional to the air flow velocity.
  - **Frictional losses**: air flow is opposed by friction between air molecules and the surface of the holes.
  - **Thermal + inertial losses**: work is done against the air mass in the enclosed cavity to bring about its periodic expansion and contraction.
- **Resonance + Viscous losses >> Friction, thermal, inertial losses**



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So, the various mechanisms operative here for dissipating the sound energy are first of all it is the sound energy is lost at the resonance because the acoustic coupling takes place between the MPP and the incident wave and then, the energy is now being used to do work in driving the; driving the MPP at resonance or driving the air molecules or oscillating them at resonance.

Other than that, this is the effect of the micro hole. So, this is also observed in PP, but this one is special case for MPP. So, here heavy viscous losses take place. So, as the air flow through the holes, it phases heavy viscous resistance and some energy is dissipated in overcoming this viscous force and therefore, viscous losses, they are proportional to the air flow velocity. So, heavy viscous losses take place. So, now, you see that viscosity is directly proportional to

velocity. So, as you will increase the velocity of the acoustic particle or as the acoustic particle velocity increases the viscous losses will also increase with time.

And then, some other loss mechanisms are also operative like frictional losses due to the when they are the when the particles, they are moving to and fro they collide with each other and there is friction between them and some losses take place in some inertial and thermal losses due to the work done against expansion and contraction of the enclosed cavity.

But the majority of the loss is due to this resonance phenomena and the viscous losses. So, this is together accounting for more than 85 percent of the losses compared to the other smaller effects. So, now heavy losses take place and that is why whenever sound energy is incident, heavy amount of it is lost; very little amount of it is reflected back. So, the absorption overall increases.

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### Impedance of an MPP

- **Acoustic impedance model** by [Maa, 1975, 1987, 1998] for MPP where  $s > r$ , and incident SPL < 100 dB:

$$Z_{MPP} = \frac{8\mu t}{\sigma \rho c r^2} \left( \sqrt{1 + \frac{x^2}{32}} + \frac{\sqrt{2}}{16} x \frac{r}{t} \right) + j\omega \left[ \frac{t}{\sigma c} \left( 1 + \frac{1}{\sqrt{9 + x^2/2}} + 1.7 \frac{r}{t} \right) \right]$$

$x = r \sqrt{\frac{\omega \rho}{\mu}} ; 1 < x < 10$

- $\mu$  = coefficient of viscosity
- $t$  = panel thickness
- $\rho, c$  = density and speed of sound in the fluid medium
- $\sigma$  = porosity of panel,  $r$  = radius of hole
- $\omega$  = incident sound wave frequency



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Now, Professor DahYou Maa, who is being a credited for coming up with a full theoretical model of an MPP, he gave in his series of papers which you can read. He gave in this series of papers, what is the he derive what is the acoustic impedance when an MPP, a layer of MPP material is placed. So, this MPP is comprising of the entire thing. It is the micro perforated panel followed by a rigid backing and then, some air cavity.

So, this entire becomes one material. So, what is the impedance that this material offers to the flow of sound waves. So, an equation was derived; it is a complicated equation. So, I am just giving you not the derivation, but just the equation itself. So, as you can see it depends on lot of parameters, it depends on coefficient of viscosity on the thickness of panel. The rho and the c values of the fluid medium, the porosity of the panel, radius of the hole and the incident sound wave frequency. And it is a complex impedance.




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### Impedance of an MPP

- Acoustic impedance model by [Maa, 1998] & [Ingard 1953] for MPP where  $s > r$ , and incident SPL < 100 dB, and with end correction:

$$Z_{MPP} = \frac{8\mu t}{\sigma \rho c r^2} \left( \sqrt{1 + \frac{x^2}{32}} + \frac{\sqrt{2}}{4} x \frac{r}{t} \right) + j\omega \left[ \frac{t}{\sigma c} \left( 1 + \frac{1}{\sqrt{9 + x^2/2}} + 1.7 \frac{r}{t} \right) \right]$$

$$x = r \sqrt{\frac{\omega \rho}{\mu}} ; 1 < x < 10$$

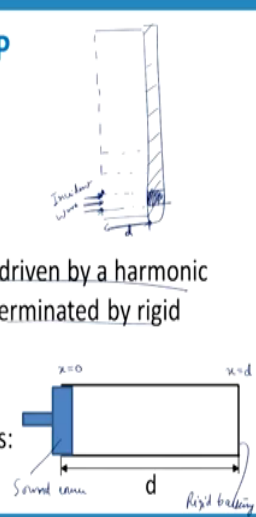



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Some end correction when the same model proposed by Maa was then, some end correction was included to it because obviously, the holes they act as open-open pipes. So, there will be end corrections involved. So, when end corrections is involved, then this factor 16 becomes 4 and the remaining things remain same. So, this is the corrected model for acoustic impedance of a MPP. So, let us see the significance of individual parameters on the acoustic impedance of MPP.

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### Impedance of an MPP

- Total impedance by MPP + air cavity:
 
$$\underline{Z = Z_{MPP} + Z_{cavity}}$$
- Per perforation, the cavity acts as a long tube driven by a harmonic plane wave source (e.g. a piston) at  $x=0$ , and terminated by rigid boundary at  $x = d$ .
- General expression for pressure inside cavity is:
 
$$p = Ae^{j[\omega t + k(d-x)]} + Be^{j[\omega t - k(d-x)]}$$



So, if you begin first, then this is the this is giving us the acoustic impedance of this micro perforated panel, but the micro perforated panel also has a cavity behind it. So, that together the impedance of the material will then be the impedance or due to the micro perforated panel plus the impedance due to this cavity. So, this will be the total acoustic impedance. So, what

happens is that this is the total impedance. Now, per perforation in the cavity that is behind it, it acts as a long tube.

So, if we have this micro perforated sort of panel here and then, we have the cavity behind it. So, effectively for every perforation, suppose this is the effective cavity for a perforation, this is the cavity for this perforation and so on. So, for every perforation, this is the cavity is now acting like this cavity acts like a long tube with a rigid end at one end. So, it is a long tube with a rigid end at a distance of  $d$ ; where,  $d$  is the air cavity depth.

So, at  $d$ , we are having a rigid backing which is the rigid wall or the backing of the panel and then, there is some incident sound is some sound energy is incident on the other end. So, if this is taken as. So, if it is take a long tube model and we take this point as  $x$  equals to 0 and this point is  $x$  equals to  $d$ . So, the cavity can be represented by a long tube; where at  $x$  equals to 0, we are getting some incident wave. So, the wave is incident at the panel. So, we have a sound source at  $x$  equals to 0 and a rigid backing at  $x$  equals to  $d$ . So, this is what is the cavity, it acts as a long tube driven by a harmonic plane wave source at  $x$  equals to 0 and terminated by a rigid backing at a  $x$  equals to  $d$ .

So, taking a general expression for this cavity because this is a constraint medium and the source acts at  $x$  equals to 0 and ends at  $d$ . So, here we have instead of taking the variable  $x$ , we are taking the variable as  $d$  minus  $x$ . Using this new variable, we have defined. So, here what we get is pressure can simply be represented with this new variable for this constraint medium as  $A e$  to the power.

So, it will be a sum of both forward and backward propagating waves. So, it will be  $A e$  to the power  $j \omega t$  plus  $k d$  minus  $x$  plus  $B e$  to the power  $j \omega t$  minus  $k$  of  $d$  minus  $x$ . So, it will be an expression. So, this is just a general form of an expression of the sound wave inside this enclosed cavity, which acts as a long tube.



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### Impedance of an MPP

- General expression for pressure inside cavity is:
 
$$p = Ae^{j[\omega t + k(d-x)]} + Be^{j[\omega t - k(d-x)]}$$

$$\frac{\partial p}{\partial x} = -Ajke^{j[\omega t + k(d-x)]} + Bjke^{j[\omega t - k(d-x)]}$$

- Using rigid boundary condition at  $x=d$ ,  $v = 0$ , or  $\frac{\partial p}{\partial x} = 0$ 

$$\Rightarrow -A + B = 0 \Rightarrow A = B$$

$$p = Ae^{j\omega t} [e^{jk(d-x)} + e^{-jk(d-x)}]$$

$$p = Ae^{j\omega t} [\cos k(d-x) + j \sin k(d-x) + \cos k(d-x) - j \sin k(d-x)]$$

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So, this we have obtained. Now, if we differentiate this expression with respect to  $x$ , then the  $\frac{\partial p}{\partial x}$  can be written as  $-Ajke^{j[\omega t + k(d-x)]} + Bjke^{j[\omega t - k(d-x)]}$ . So, it is  $-Ajke^{j\omega t + kd - kx} + Bjke^{j\omega t - kd + kx}$ . So, minus  $Ajk$  into  $e^{j\omega t + kd - kx}$  and this expression becomes here with  $x$  you have the constant plus  $j k$ . So,  $Bjk$  into  $e^{j\omega t - kd + kx}$ .

So, exponential based on the formula for the differential differentiating and exponential function you can simply obtain this. So, it is an easy differentiation. So, at the rigid boundary, you know that we have solved such kind of problems previously when we were dealing with standing waves and resonance.

So, whenever there is a rigid backing which means suddenly the impedance is infinity and the acoustic wave stopped. So, the particles cannot impinge through this rigid boundary. So, the condition is that the acoustic particle velocity has to become 0 at rigid boundary and the

acoustic particle velocity is directly proportional to this quantity that is the  $\frac{\partial p}{\partial x}$ . So, when  $v$  becomes 0 which means  $\frac{\partial p}{\partial x}$  becomes 0. So, that is the condition we have taken the rigid boundary condition.

So, if we solve this. So, let us put  $x$  equals to  $d$  here. So,  $\frac{\partial p}{\partial x}$  will be 0 at  $x$  equals to  $d$ . So, if you put the expression of  $x$  equals to  $d$  here, then this will be  $d$  minus  $d$  and this will be  $d$  minus  $d$ . So, this overall will be  $e$  to the power 0 or 1. So, this 1 constant will cancel out. This is another constant that will cancel out  $e$  to the power  $j\omega t$  and  $j k j k$  will cancel out; all the non zero constants are removed. So, what we are left with is this minus  $A$ , then some constant plus  $B$ , then some constant is equal to 0.

So, from this boundary condition what we get is  $A$  is equal to  $B$ . So, when  $A$  is equal to  $B$ , now putting this back again into the expression for pressure, what you get is you can remove the constant term. So, it becomes and this becomes  $A$  and this also becomes  $A$ .  $A$  becomes  $B$ . So,  $A e$  to the power  $j\omega t$  the constant term is taken out and inside, we are left with  $e$  to the power  $j k d$  minus  $x$  plus  $e$  to the power minus  $j k$  of  $d$  minus  $x$ . So, this is the term we are left with.

So, using the Euler's relation this can be broken down as  $\cos$  of  $k d$  minus  $x$  plus  $j$  times of  $\sin$   $k d$  minus  $x$  and this obviously, by its definition becomes  $\cos$  of  $k d$  minus  $x$  minus  $j$  of  $\sin$   $k d$  minus  $x$ . So, this  $j$  term cancels out, when you add the two. So, you only left with twice the  $\cos$  term.

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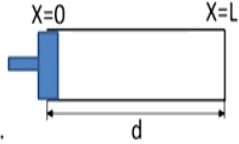
### Impedance of an MPP

- Acoustic pressure inside cavity is:  

$$p = 2A \cos k(d-x) e^{j\omega t} \quad \text{Eq. (1)}$$
- By property of acoustic medium (Ref. lecture 2):  

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\frac{\partial p}{\partial x} \Rightarrow \vec{v} = -\frac{1}{\rho_0} \int_0^t \frac{\partial p}{\partial x} \quad \checkmark$$

$$\vec{v} = -\frac{1}{\rho_0} \int_0^t 2Ak \sin k(d-x) e^{j\omega t} = -\frac{2Ak}{j\rho_0\omega} \sin k(d-x) e^{j\omega t} \quad \text{Eq. (2)}$$



So, what you get at the end is 2 times of  $\cos kd - x$  into  $A$  into  $e^{j\omega t}$ . So, this is a sort of a standing wave that is varying sinusoidally only with time, but not with respect to  $x$ . So, this is the kind of pressure that you are getting inside the cavity. So, we are deriving what will be the impedance of the cavity because so we began with this expression where the total impedance is the  $Z$  of MPP plus  $Z$  of cavity and this  $Z$  of MPP was given by the previous scientists. Now, if we have to add  $Z$  of cavity. So, let us see what is the expression for  $Z$  of cavity.

So, now, we have obtained the expression for  $p$  inside the cavity and if you go back and refer to lecture 2, where we studied about sound wave propagation; then, a few relationships were derived for any acoustic medium. One of that relationship was this and this relationship was

one of the standard relationships for any acoustic medium. So, using this relationship  $v$  or the particle velocity can be obtained as  $\frac{1}{\rho} \int \frac{\partial p}{\partial x}$ , from this equation.

So, when you solve this becomes  $\frac{1}{\rho}$  and in the integral you have  $\frac{\partial p}{\partial x}$ . So, when you differentiate this again with respect to  $x$ . So, what you get is  $2 A k \sin(kd - x)$ . So, the differentiation of  $\cos$  is minus sign and because we already have a minus sign with respect to  $x$ . So, minus minus becomes plus. So, this is the expression into  $e$  to the power  $j\omega t$ . Now, integrating this with respect to time. So, we are solving this expression here.

So, what you get is that this is this thing is a constant with respect to time, it just varies with  $x$ . Only this is the part which varies with respect to time. So, we only integrate this part and this part is taken out as constant. So, the overall expression now we get for velocities  $\frac{2 A k \sin(kd - x)}{j\omega \rho}$ . So, the integration of  $e$  to the power  $j\omega t$  is  $e$  to the power  $j\omega t$  by  $j\omega$ . So,  $j\omega$  term now appears here.

So, now, we have obtained any expression for the acoustic pressure inside the cavity and the particle velocity inside the cavity. So, now, we can get acoustic impedance.

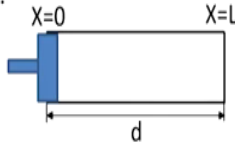
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### Impedance of an MPP

- From eq. (1) and eq. (2), acoustic impedance due to the cavity measured at the boundary of the perforation is:

$$z = \left( \frac{p}{\underline{v}} \right)_{x=0} = \left[ -\frac{j\rho_0\omega}{k} \cot k(d-x) \right]_{x=0}$$

$Z_{cavity} = -j\rho_0c_0 \cot(kd)$






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And the acoustic impedance is given by  $p$  by  $v$  pressure by velocity ratio. So, because this cavity is. So, we have this MPP here and then, there is a cavity just at the back. So, here MPP is the source and the source is incident on the MPP and then, it faces some the sound wave, it faces some impedance due to the MPP itself and then just behind the MPP we have cavity. So, it faces some impedance due to cavity. So, the boundary here is at  $x$  equals to 0. So, for this particular cavity element the impedance becomes  $p$  by  $v$  at  $x$  equals to 0.

So, when you divide the two expressions. So, you have this expression and this expression for  $p$  and  $v$ . When you divide the two,  $2A$  cancels out;  $\cos$  by  $\sin$  becomes  $\cot$  and the power  $j\omega t$  also cancels out. So, what you are left with its  $\cot$  of  $k d$  minus  $x$  and this term becomes this term goes to the top. So, it becomes  $\cot k d$  minus  $x$  minus  $j\omega$  naught  $j\rho_0$  naught  $\omega$  by  $k$ .

So, this is what you get  $j\rho_0 c_0 \cot kd - x$ , when you just divide these two terms here. So, this is the end expression you put the value at  $x$  equals to 0. So, this is the impedance of the cavity that we have found. So, let us do a quick parametric study on how the verrier; how the impedance varies with the MPP parameters. So, now, that we have expression for  $Z_{MPP}$  and  $Z_{cavity}$ . So, this is given to us.

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### Effect of MPP parameters

- Total impedance of the material = impedance by MPP + impedance by air cavity:

$$Z = Z_{MPP} + Z_{cavity} ; Z_{cavity} = -j\rho_0 c_0 \cot(kd)$$

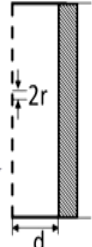
$Z_{cavity} \uparrow, \text{ as } d \downarrow$

- Sound absorption coefficient by MPP absorber:

$$\alpha = 1 - \left| \frac{z-1}{z+1} \right|^2$$

$R = \frac{z-1}{z+1}$ 
 $\rho = \frac{z_c - 1}{z_c + 1}$ 
 $A = \frac{z_2 - z_1}{z_2 + z_1} = \frac{z_2 - \rho c}{z_2 + \rho c}$

$\frac{z_2}{\rho c} = Z = \text{relative acoustic impedance w.r.t air } \rho c =$



$t = \text{panel thickness}$

So, the first control parameter. So, control parameters are those parameters in the design of an MPP that will change its acoustic impedance. So, the first parameter that affects the acoustic impedance of this particular material is going to be the cavity depth because  $Z$  of cavity depends upon the cavity depth. So, depth of cavity so obviously, the  $Z$  will be dependent on the target frequency; but other than that, what are the design parameters on which it depends?.

So, it will first depend on the cavity frequency sorry the cavity depth. So, as you see here this is the model here  $\cot kd$  as the this argument as  $d$  increases, the value of  $\cot$  will go down. So, as the argument increases within minus  $\pi$  by 2 to  $\pi$  by 2 as the theta angle inside  $\cot$  increases  $\cot$  value goes down. So, following that relationship as  $d$  is decreasing  $\cot$  will increase. So, the magnitude of  $Z$  cavity is going to increase. So, this is the first relation how the. So, this is what we have found and then, we know that absorption is given by this, but we will use this later.

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### Effect of MPP parameters

- Acoustic impedance model by [Maa, 1975, 1987, 1998] for MPP where  $s > r$ , and incident SPL < 100 dB, with end correction:

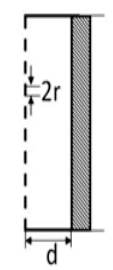
$$Z_{MPP} = \frac{8\mu t}{\sigma \rho c r^2} \left( \sqrt{1 + \frac{x^2}{32} + \frac{\sqrt{2}}{4} x \frac{r}{t}} \right) + j\omega \left[ \frac{t}{\sigma c} \left( 1 + \frac{1}{\sqrt{9 + x^2/2}} + 1.7 \frac{r}{t} \right) \right]$$

$x = r \sqrt{\frac{\omega \rho}{\mu}} ; 1 < x < 10$

$Z_{MPP} \uparrow, as \sigma \downarrow$

$Z_{MPP} \uparrow, as r \uparrow \downarrow$

$Z_{MPP} \uparrow, as t \uparrow$



$t = \text{panel thickness}$

Now, let us examine the  $Z$  of MPP and what are the parameters that affected. So, what we get is that; let us begin first with porosity. So, if you see here  $x$  is the term which is independent of porosity. So, and  $r$   $t$  all of them are independent of porosity. So, this entire term is independent of porosity. This is independent of porosity. This is independent of porosity.

So, porosity only appears in the denominator. So,  $Z$  equals to some constant by porosity plus  $j\omega$  some constant by porosity. So, it is inversely proportional to porosity. So, as the porosity will decrease, the acoustic impedance is going to increase. So, this will be another relationship. So,  $Z$  is approximately inversely proportional to the porosity which we have found because all other quantities are constant here. Now, with  $r$ ; so, before we see what is the relationship between  $Z$  and the radius. Let us see relationship between  $Z$  and  $t$ .

So, again all this quantity. So, here you have  $t$  here in the denominator and here you have numerator and here you have in the denominator. So, when you multiply them together, this factor will be some constant into  $t$  and this will be some constant independent of  $t$ . So, we get something multiplied by  $t$  plus something plus  $j\omega$ , then again something that is multiplied by  $t$  plus something that is multiplied by  $t$  plus some independent quantity.

So, overall, we are getting  $Z$  as some  $t$  plus some constant. So, it is sort of a linear relationship plus  $j$  both for  $t$  plus some constant. So, this is the kind of thing we are getting. So, we are getting some  $t$  factor plus some constant and so on. So, as  $t$  increases, both the real and the complex the imaginary quantity will increase. So,  $Z$  MPP will increase as  $t$  increases because  $t$  appears effectively in the numerator.

And now, if you have a look at  $r$ ; then,  $r$  follows a bit of a complicated relationship. So, here what you see is that and this is in the denominator  $r$  square, but  $x$  square itself is some  $r$  times of something. So, root of  $r$   $r$  comes out. So, you have  $r$  by  $r$  square. So, this becomes  $1$  by  $r$ ; almost of the order of  $1$  by  $r$  and this becomes  $r$  square  $r$  square. So, this becomes a constant.

Again, this is constant this because this is some something of  $r$  and this becomes something of  $r$ . So, what we are getting is that in this for  $Z$  term, what you are getting is here you are getting  $r$  both in the numerator and the denominator. So, some constant  $A$  times of  $r$ , some constant  $B$  times of  $r$  and so on. So, getting a more complicated relationship.

So, what it has been found through some experimental and numerical studies is that  $Z$  MPP increases as the micro hole radius increases only up till a certain extent and then, it follows a



reverse relationship and it starts to decrease as the hole sorry a hole radius is increased. So, it this is the relationship that it follows a dual relationship. So, we have these as the various design parameters that affect acoustic impedance.

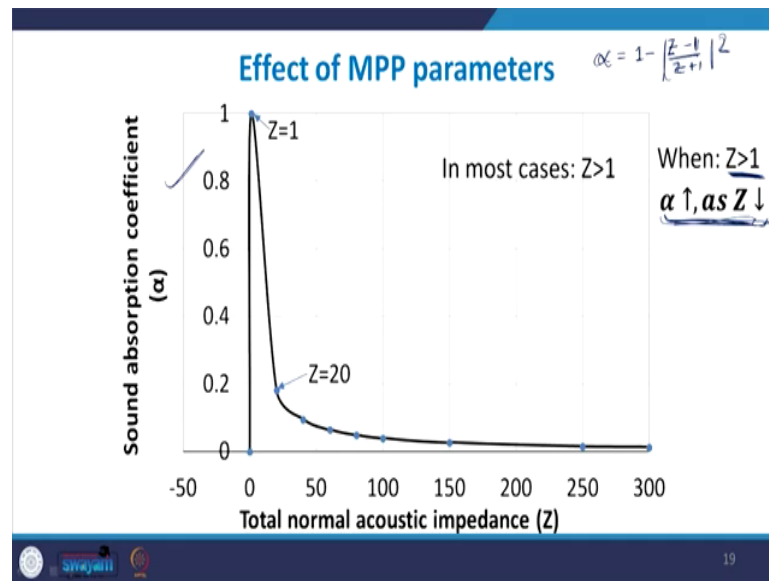
So, let us see how the acoustic impedance and alpha are related to each other. So, we know that  $r$ , when we studied about sound propagation through medium boundaries we know that  $r$  is equal to  $\frac{Z_2 - Z_1}{Z_2 + Z_1}$  and here  $Z_1$  is this particular medium of air here.

So, this is some  $\rho c$ . So, this becomes  $\frac{Z_2 - \rho c}{Z_2 + \rho c}$ . So, the incident medium has it has  $\rho c$  and the new medium which is the perforated MPP material, this has some new impedance which is a complicated term we have found. So, now, if we divide this entire thing by the  $\rho c$  which we what we get is  $\frac{Z_2}{\rho c} - 1$  upon  $\frac{Z_2}{\rho c} + 1$  will give you  $r$ . If you take this  $\frac{Z_2}{\rho c}$  is equal to simply the  $Z$  which is the relative acoustic impedance with respect to air, with respect to air.

So,  $Z$  is taken as a relative acoustic impedance with respect to air, then you get that term as  $r$  is equal to simply can be replaced by this relative acoustic impedance of the of the MPP material minus 1, where 1 is the relative acoustic impedance of air into this. So, we have expressed it in relative terms by dividing numerator and denominator by  $\rho c$ . So, this is the expression we are getting alpha is equal to  $1 - \text{mod of } r^2$  and  $r$  is given by this expression. So, we have already discussed this equation of alpha and  $r$  before when we were three to four times before whenever we are studied about sound absorption coefficient.

So, let us plot the graph of this particular function. So, let us plot alpha versus  $Z$ . So, how does alpha vary with  $Z$ . So, I did a numerical study and this is my result.

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So, as  $Z$  is varied between 0 to some infinity or large number let us say 300 and I saw how does the  $\alpha$  vary with it. So, this is the graph of  $\alpha$  versus  $Z$ , where  $\alpha$  is given by  $1 - \left| \frac{Z-1}{Z+1} \right|^2$ . So, this is the graph of this particular expression. So, what you get is that whenever  $Z$  is greater than 1  $\alpha$  increases with a decreasing value of  $Z$ . So, as you increase  $Z$ ,  $\alpha$  decreases. So, as you increase  $Z$ ,  $\alpha$  decreases or the other way around as you decrease  $Z$ ,  $\alpha$  increases.

So, there is an inverse relationship and in most of the cases because the because the impedance air, the air impedance and unbounded air impedance is  $\rho c$  and then, we have some MPP material placed on it. So, most of the cases it is the MPP which is restricting it. So, compared to the air, the MPP material will have an impedance which is greater than the air.

So, in most of the cases  $Z$  will be the relative impedance of the MPP material is going to be greater than the relative impedance of an unbounded air medium. So,  $Z$  will be greater than 1. So, that is why we are only consider this particular case and the relationship that we obtain is alpha increases as  $Z$  value decreases.

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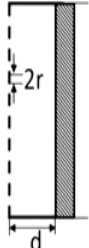
### Effect of MPP parameters

- Therefore the control parameters for MPP performance are:
  - Pore radius ( $r$ )
  - Porosity ( $\sigma$ )
  - Panel thickness ( $t$ )
  - Cavity depth ( $d$ )




$Z \uparrow, as \sigma \downarrow$   
 $Z \uparrow, as r \uparrow \downarrow$   
 $Z \uparrow, as t \uparrow$   
 $Z \uparrow, as d \downarrow$

When:  $Z > 1$

 $\alpha \uparrow, as \sigma \uparrow$   
 $\alpha \uparrow, as r \downarrow \uparrow$   
 $\alpha \uparrow, as t \downarrow$   
 $\alpha \uparrow, as d \uparrow$



t = panel thickness




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So, now we already have this the control parameters that we have found for MPP are pore radius, porosity, panel thickness and the cavity depth. So, it depends on all these 4 properties. So, the  $Z$  depends on these properties and we found this is the kind of relationship and because we are taking this particular case because for most of the cases 99 percent of the cases MPP will have an impedance more than that of air.

So, in this case alpha follows a negative a reverse relationship then Z. So, this is the overall relationship, how does the various design parameters of an MPP influence its absorption. So, we will have a look at this parameters again in our next class. So, see you for the next class.

Thank you.