Robotics and Control: Theory and Practice

Prof. N. Sukavanam
Department of Mathematics
Indian Institute of Technology, Roorkee

Lecture - 09
Manipulator Jacobian

In this lecture, we will see the definition of Manipulator Jacobian and the procedure for computing the manipulator Jacobian. So, the previous lecture we have seen the relation between the differential changes between two coordinate frames.

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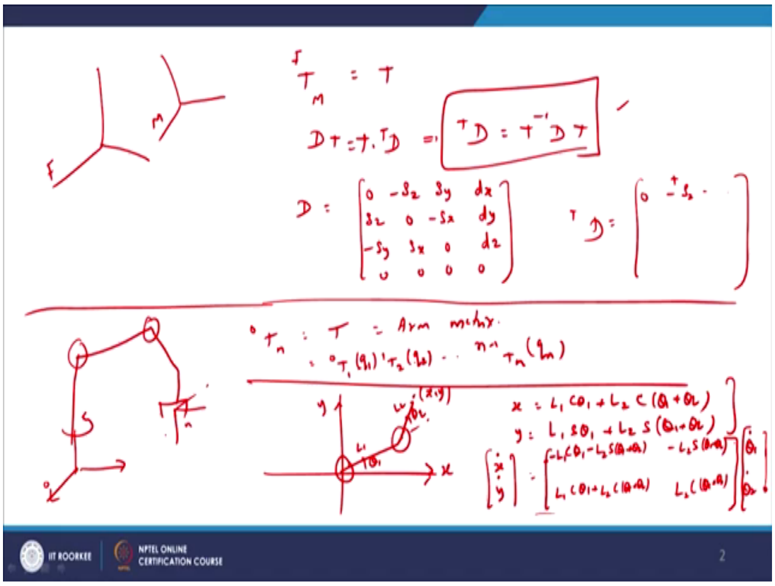

So, we have seen the following that if you have two frames a fixed frame and a moving frame and if the relation between F and M; that is M with respect to F is denoted by the homogeneous transformation T.

Then a small change in the linear motion and the angular motion of T with respect to the base F, if we denote it by D. And if the corresponding changes with respect to the frame itself is denoted by D T; then these two are related by the following D into T equal to T into D superfix T.

So, or in other words it is denoted by T inverse D T is the relation between the differential changes with respect to current frame that is in the left side and the differential changes with respect to the fixed frame; that is D given by D; where D is nothing, but 0 minus del z; del y, d x; del z and minus del x; d y minus del y, del x 0; d z. So, here d x, d y, d z denotes the small motion along the x, y, z direction and the del x, del y, del z are the small rotation about the x, y, z axis of the fixed frame and if we put D superfix T; these corresponding value are del z; t etcetera.

So, this represent the corresponding motion with respect to the current frame itself; this one. So, the two relations are given by this and then we have seen the formula relating the D superfix T and D; so that the formula was derived in the last lecture. Now, we will see using this; the definition of manipulator Jacobian is derived here in the context of robot manipulators.

So, if we consider a robot manipulator; so we have already seen that using the D H procedure; we can relate the coordinate frames at the base and the end effector. So, if you say 0 denotes the coordinate; base coordinate frame and n denotes the end effector coordinate frame, then the arm matrix is denoted by 0 T n. The end effector coordinate frame return with respect to the base coordinate frame; it is the arm matrix.

And this matrix is product of the coordinate transformation matrices fixed at each and every joint of the robot manipulator. So, this is nothing, but 0 T 1, 1 T 2 etcetera n minus 1; T n, where i minus 1 T i represent the; ith coordinate frame with respect to the i minus 1 coordinate frame; so this elaborately we have seen. And each matrix is depending on the; so they depend on the joint variables q 1, q 2, q n.

So, if the ith is say revolute joint then q a is called the joint angle and if the ith joint is a prismatic joint; then q a is the joint distance. So, these are the variables q 1, q 2, q n corresponding to a n arm manipulator. Now, the manipulator Jacobian is defined in the following way.

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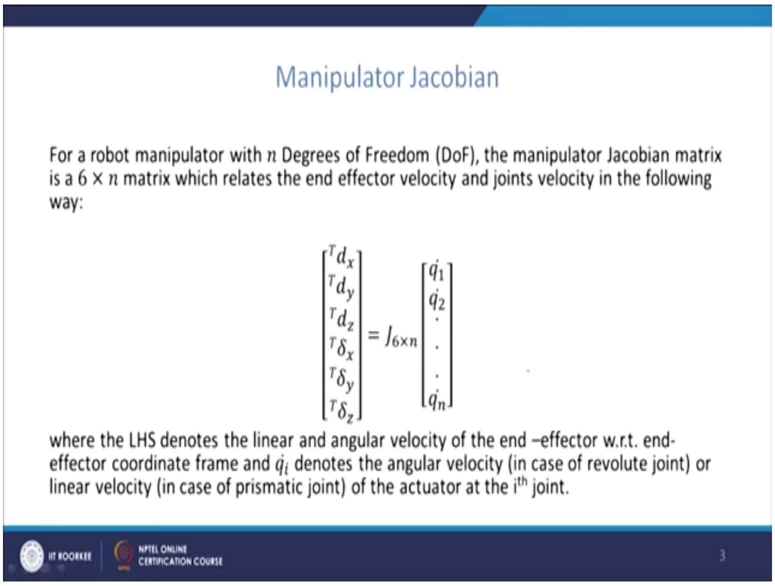

It is a relation which relates the d x T, d y T etcetera and del x T, del y T, del z T; these are the differential changes; the linear changes, the first three one along the x y z direction of the current frame that is the end effector frame and del x, del y, del z superfix T; they are the differential rotations about the x y z axis of the end effector frame; so that is the left side.

The right side we have q 1 dot, q 2 dot etcetera; these are the differential changes at the for the of the joint variables q 1, q 2, q n or we can also say that the left hand side denote the linear and angular velocities of the end effector frame with respect to itself and q 1 dot, q 2 dot etcetera denote the velocities of the joint variables and so this two are related by the J matrix that is called the Jacobian matrix.

And so because the left; left hand side contains 6 variables, it is a 6 cross n matrix; the Jacobian matrix. So, this will be very useful in many practical problems because when we want to if; if you are giving q 1 dot, q 2 dot this much of velocity at the actuators of each joint; then how much the end effector will move? With what velocity the end effector will move? Its linear velocity and angular velocities are given in the left side.

Or we can also see in more many practical problems; if you are interested in moving the end effector with their particular velocity, this left hand side velocity; then how much of the angles; angular velocity should be given at each actuator is can be calculated by this relation. By taking the; by solving this system of equation we can get the q 1 dot, q 2 dot etcetera as the solution of this system of equation.

So, this will be very useful in real life situation where we want to control a robot manipulator for a particular end effector velocity. So, how to find this Jacobian matrix is what we will see in this particular lecture.

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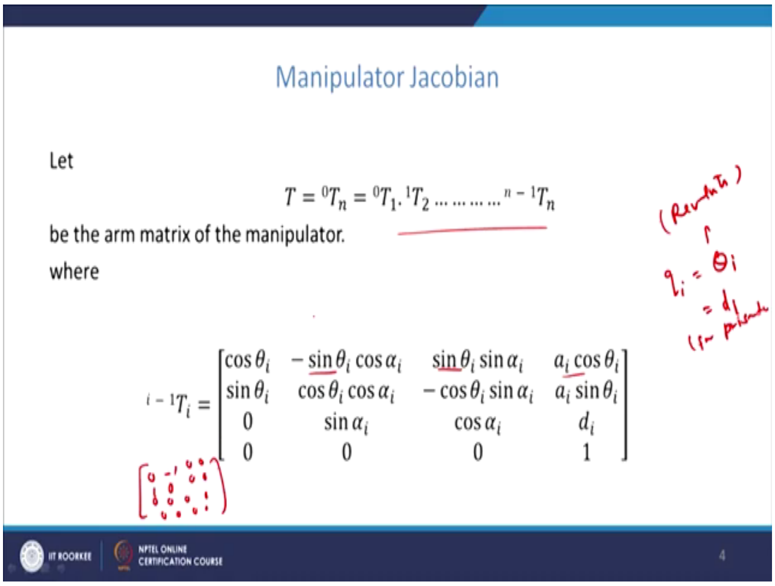

So, the end effector 0 T n is given by the product of 0 T 1, 1 T 2 etcetera as we have seen the previous slide. The ith frame with respect to i minus 1th frame is given by this expression.

Here, theta i denotes the joint angle and d i denote the joint distance etcetera. So, if the; if the joint i is a revolute joint, then theta i is the variable there and if it is a prismatic joint; then theta i may be a constant and d i is the variable for that particular joint.

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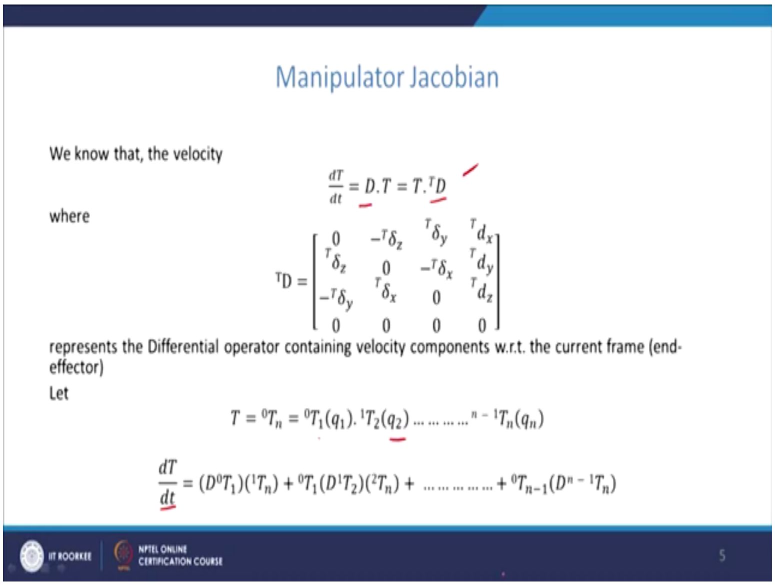

So, now we have already seen this particular formula; that is the velocity of the frame T is given by D into T; where D is the differential changes or the velocity of the frame with respect to the base frame D; base frame the 0th frame.

And the same thing can also be expressed as T into D T; that is the velocity of the frame with respect to itself is given by D T; so this relation we have already seen. And now the each matrix i minus 1 T i is a function of this q i. Now, differentiating with respect to T D T by D T; so we have to differentiate each component separately; D T by D T is total derivative and so it will be the sum of all the partial derivatives in the right hand side.

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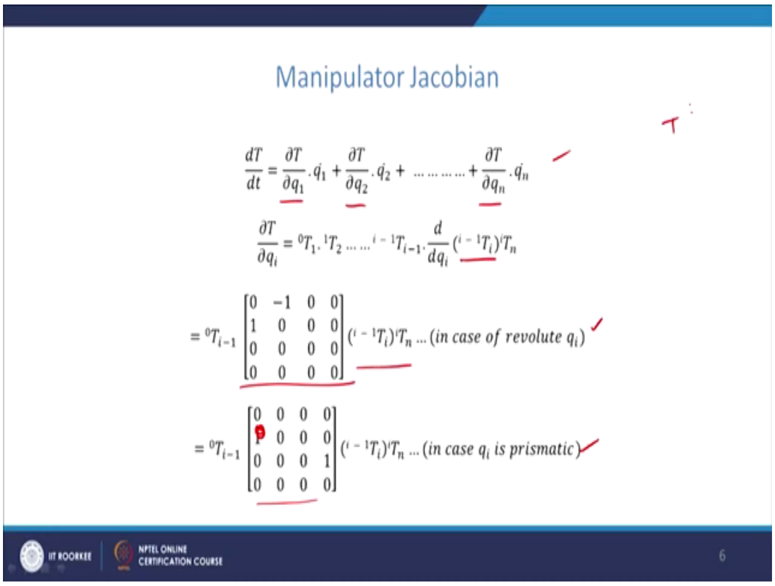

So, we will write it in the following way; del T by del q 1 into d q 1 by d t plus del T by del q 2 into d q by d t etcetera is the standard formula of the total derivative using the partial derivatives.

Now, if you want to calculate del T by del q i for the ith variable. So, that can be written like this because this q i will appear only in the i minus 1 T i matrix. As we have seen here, i minus 1 T i matrix contains the variable theta i. So, we; we write this q i equal to theta i, if it is a revolute joint and it is d i; if it is prismatic, this is for revolute joint.

So, q i is the general notation, but theta and d are the particular notation for the joint variable. So, this q i appears in the i minus 1 T i matrix. So, we want to differentiate this with respect to this. Now, if you directly differentiate that matrix if you; if you see this matrix and if you differentiate directly with respect to q i; q i means theta i for example, then it is minus sine theta i and this will become minus cos theta i into cos alpha i because we are differentiating only with respect to theta i.

So, we can see that the first row when we differentiate it is nothing, but the minus 1 multiplied by the second row. And if you differentiate the second row; it is nothing, but the same as the first row. So, what we can observe here is; if you multiply this matrix by 0 minus 1 and 1 0. So, if you multiply this matrix with 0, minus 1, 0, 0, 1, 0, 0, 0 etcetera; then we can see that this second row will become the first row with a minus sign and the first row will become the second row that is nothing, but the derivative of this matrix.

So, we can see that the derivative of i minus 1 T i is given by this matrix multiplied by the same i minus 1 T i matrix. Similarly, if it is a prismatic joint only d i are the; is the variable. The derivative of this with respect to d i is nothing, but so the derivative of this will become simply; here it is 0. So, we substitute in the d t by d; small t for each partial derivative in the following way; finally, we will get del T by del q i.

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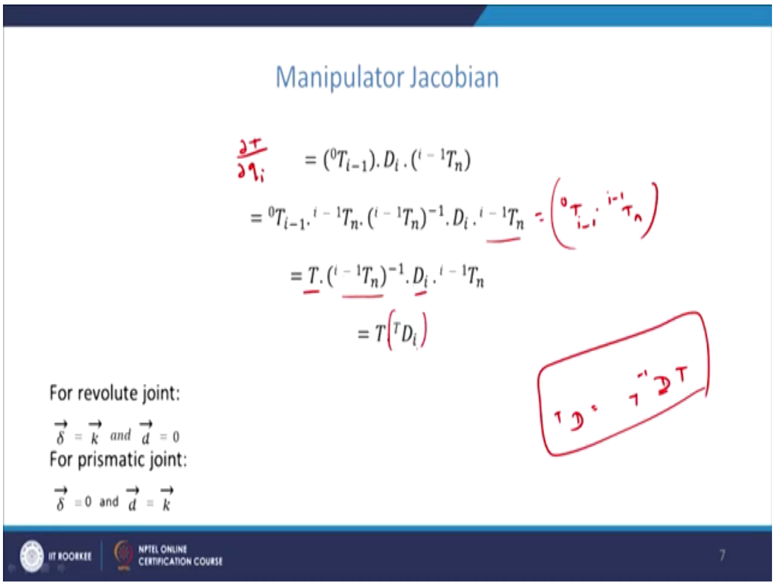

This expression is given by 0 T i minus 1 into the D i matrix multiplied by i minus 1 T n, where D i matrix is given by this two. It is either this one for revolute joint or this one for the prismatic joint depending on the joint nature. So, we can substitute here and now we can; here we have 0 T i minus 1; we can introduce i minus 1; T n and then take its inverse. So, this can be written as 0 T i minus 1 is already there and we multiply with i minus 1, T n and then cancel it with i minus 1 T n inverse.

So, this will become; so these two combined is becoming T here and then i minus 1 T n inverse is there; multiplied by D i multiplied by i minus 1 T n. So, this whole thing can be written as and already we have seen that if for any matrix; T, this D T is equal to T inverse; D T; this formula we have seen.

Now, if the T matrix is replaced by i minus 1 T n; so it is a exactly similar formula, the inverse of the matrix multiplied by the D matrix multiplied by the same T type of matrix. So, what we get is the; in the place of D, we have D i. So, the whole thing can be replaced by D i T, using this formula ok.

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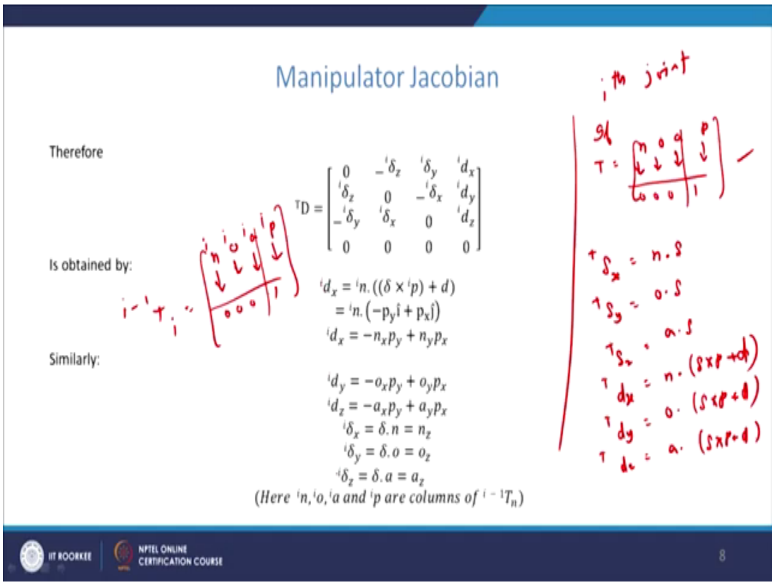

Now, if you recall the formula; D superfix T is given by this expression ah; del z t and del y t etcetera; where in the place of T we are putting i because it is corresponding to the ith joint. The differential changes corresponding to the ith joint is given by this one; so we are indicating this superfix i at every place.

So, we if you recall this formula once again; I will, I will write it here; if T matrix is given by the first column is n matrix; n vector, second column is o and third column is a and fourth is p vector; 0, 0, 0, 1. So, if you have this expression for the T matrix; then we have this formula del x T that is given by n dot del and del y T is given by o dot del and del z T is a dot del; d x T is given by n dot del cross p plus d and d y is o dot del cross p plus d and d z is a dot.

So, this is the general formula for any homogeneous transformation T; we can write this formula. The relation between del that is the D and del the translational and rotational differential changes with respect to the base frame and the right; left hand side are the rotational and translational changes with respect to the current frame; so this relation is more general.

Now, we are talking particularly about the i minus 1 T n; so instead of n, o, a, p; a general notation we can write it as n superfix i, o superfix i, a superfix i and p superfix i. These are the column vectors for this one; so you can just denote it by n superfix i. The differential changes with respect to the ith frame is given by the left hand side.

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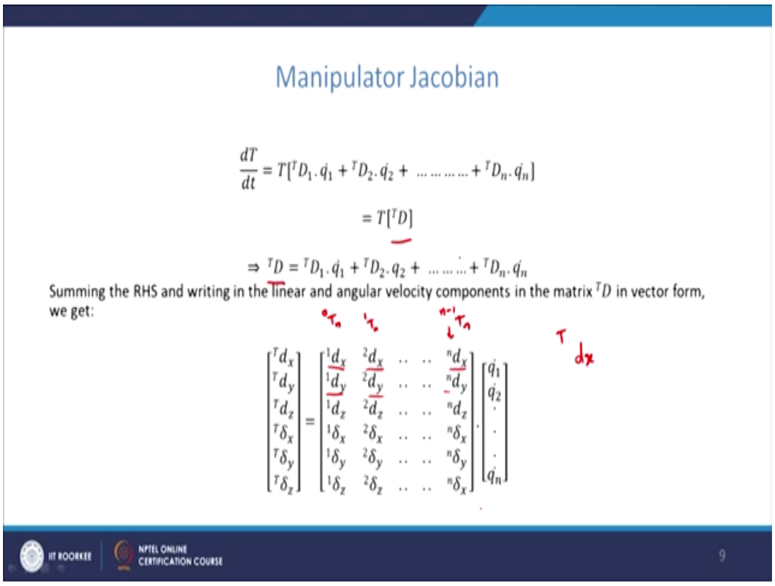

So, now we recall the other formula d T by d small t is also same as T into D superfix T ah; where the D superfix T are the differential changes with respect to the current frame. So, the differential changes with respect to that current frame is given by the differential changes with respect to the first frame; that is D 1 T and the differential changes with respect to the second frame D 2 T etcetera; where the general D i T is given by this particular formula here.

The d x T; the total differential change along the x direction with respect to the T frame; the current frame is given by the differential change with respect to the first frame and multiplied by q 1 dot. The differential change with respect to the second frame multiplied by q 2 dot etcetera; the values of each element is given by the previous slide here.

So, we can easily observe that to calculate the first column here; we should take the matrix 0 T n. Because when we put i is equal to 1; in this we get 0 T 1; to calculate the second column of the Jacobian matrix we should take the matrix 1 T n and the corresponding columns of n, o, a, p should be used for used in this formula as given here. Then the last one is n minus 1 T n should be used for this one; for calculating the nth column of this matrix.

So, this procedure is illustrating how to calculate the Jacobian matrix for a n arm manipulator. So, if for example, if you consider this simple case of two arm manipulator. So, if you take the two arm manipulator for example, here. So, here it is theta 1 and theta 2 and the x axis, y axis.

So, here it the end effector is a single point and the relation can be easily written as L 1 cos theta 1 plus L 2; cos of theta 1 plus theta 2; y is L 1; sine theta 1, L 2 sine; relating the end effector position and the joint angles theta 1 and theta 2 is the relation. If you directly differentiate with respect to T; we will get d x by d t to be equal to the first row of this matrix into theta 1 dot; theta 2 dot. So, here we observe that the x dot, y dot represent the velocity of the point x, y; velocity linear velocity of this point x y in the x and y direction.

Theta 1 and theta 2 dot; they denote the angular velocity of the actuators, the velocity x dot y dot is with respect to the base coordinate frame. Whereas, we are always interested in finding the relation between the velocity of the end effector coordinate frame with respect to itself and the actuator velocities; the joint velocities of the robot manipulator. This procedure which we have explained is for the manipulator Jacobian.

So, directly differentiating the given matrix T may not be very useful in finding the manipulator Jacobian matrices. So, we will illustrate this particular procedure as described here in the next lecture by actually computing all these matrices 0, T 1, 1 T 2 etcetera and then how to write the manipulator Jacobian for a specific example ok.

Thank you.