Robotics and Control: Theory and Practice
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Lecture - 07
Direct Kinematics

Hello viewers. In the last lecture we have seen how to use the D-H procedure to fix the coordinate frame at each joint of a robot manipulator and find the parameters of the manipulator at each joint. The four parameters: joint angle, joint distance, link length and link twist angle. So, in this lecture we will see how to; using three examples we will see how to find the Direct Kinematic equation for the for robot manipulators.

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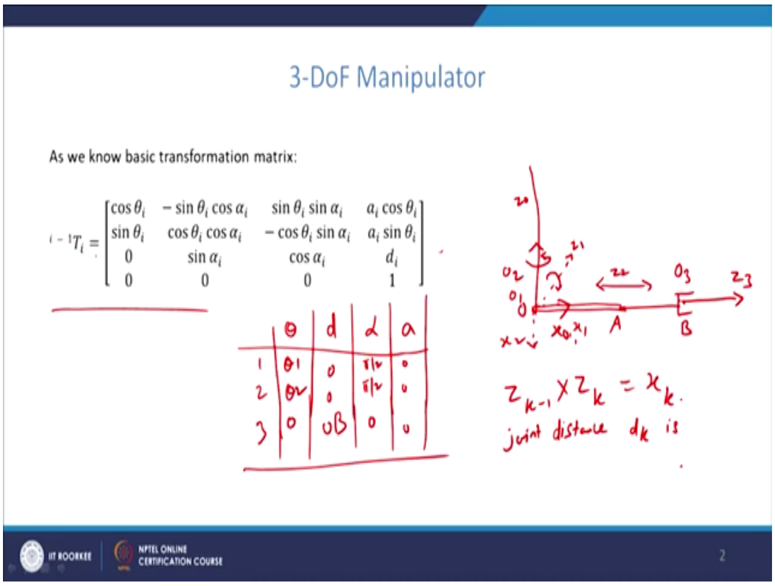

So, as a first example we see a 3 link manipulator in the; as shown in the following picture. So, the manipulator is having the axis of rotation is the vertical axis and one horizontal axis which is perpendicular to the plane of the paper. So, we call this as the Z 0 axis and the perpendicular to that as the Z 1 axis. So, these are the two degrees of freedom situated at the single joint, we call it as O. Then the robot manipulator if it is in this form let us assume that.

So, the robot manipulator it rotates in the horizontal plane with the axis Z 0 and it rotates in the vertical, in the plane of the paper with the axis Z 1; both of them situated at the joint O. And at this joint A there is a prismatic joint, it will slide forward and backward. So, these are the three axis of this particular manipulator. The end effector is the point B, the axis; the approach direction we call it as this axis is Z 2 and the approach direction is Z 3.

So, this is according to the D-H procedure, the axis of rotation and the sliding direction these are the Z axis to be fixed at each joint. Now, the x axis can be fixed by the following method. For the base Z 0 axis we can fix the X 0, the same direction and Y 0 axis is fixed according to the right-hand rule system. The X 1 axis is given by the perpendicular direction to Z 0 and Z 1. In fact, it is Z k minus 1 cross Z k that will give the x k axis. So, x 1 axis will be Z 0 cross Z 1. So, it will coincide in this particular situation with X 0 itself. The same as X 0 direction in this picture, but if it is in a different position in a rotation then x 0 and x 1 will be in a different directions.

Then x 2 direction is Z 1 cross Z 2; Z 1 is this direction, Z 2 is the direction given here, in this direction, then the cross product will give the axis. So, the x 2 axis is pointing downwards in this particular situation. The origin at each joint already explained in the D-H procedure is the intersection of the Z axis. So, all the origins are fixed at the same position here. O is the origin of the base coordinate frame, then O 1 is the origin of the Z x 1 Z 1 y 1 frame, and O 2 is the origin of x 2 y 2 z 2 frame. All of them are situated at the same position O only, here.

So, according to this procedure we can formulate the four parameters of the robot manipulator; the first, second, third joint, there are three joint here theta is the joint angle, d is joint distance, alpha is the twist angle, and a is the link length. So, here theta 1 and theta 2 are the variables, because these two are the revolute joint and the third one is the prismatic joint.

The two z axis are parallel, so the third one is a constant, the joint angle is fixed here; the joint distance all the origins are coinciding at the same position.

So, here the joint distances are all 0; first and second; the third one is o B. Because the end effector position is at B, the origin is here. The joint distance, the definition of the joint distance if we see that; the joint distance d k is the distance between the two origins which is given here O 2 and then O 3 is the origin of the third link here. So, O B is the joint distance here. Then alpha is the twist angle, alpha is the angle between the Z 0 axis and Z 1 axis.

So, that gives the angle to be pi by 2,. And Z 1 axis and Z 2 axis that is again the angle pi by 2. And Z 2 axis and Z 3 axis they are in the same direction that is 0. And if two Z axis are parallel then the link length is the shortest distance between the two Z axis. So, here we can see that because all the origins are fixed at the same joint the link length all of them will be equal to 0. So, this gives the robot parameters according to the D-H algorithm.

The previous lecture we have seen how to construct the coordinate homogeneous transformation: the i eighth coordinate frame with respect to the i ninth one coordinate frame in terms of the homogeneous transformation is given by this 4 by 4 matrix. Where, this theta i d i alpha i a i are the corresponding joint parameters. So, if we take the first joint parameter that is theta 1 and d 1 is 0, alpha 1 is pi by 2 and a 1 is 0. So, if we substitute this values in i equal to 1, we get the matrix 0 T 1. So, we will get the matrix as follows.

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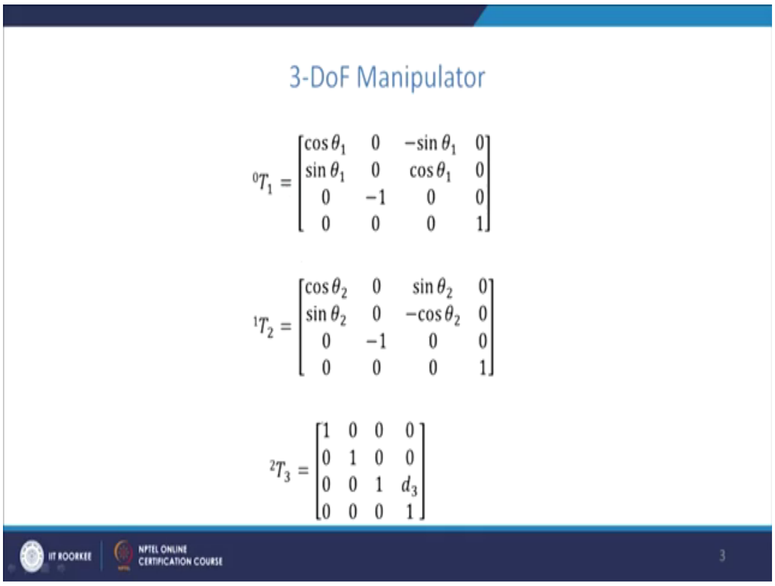

0 T 1 is given by this expression and 1 T 2; for 1 T 2 we should substitute the values of the parameters theta 2 and alpha is pi by 2. If you substitute we get the second 1 T 2 matrix and 2 T 3 is given by the third one. If we substitute d 3 is the distance o b. So, that expression is given by and theta is the angle 0 in this case. If you substitute here, we get the identity matrix for the first 3 by 3 matrix and 0 0 d 3. So, these three matrix give the joint coordinate systems.

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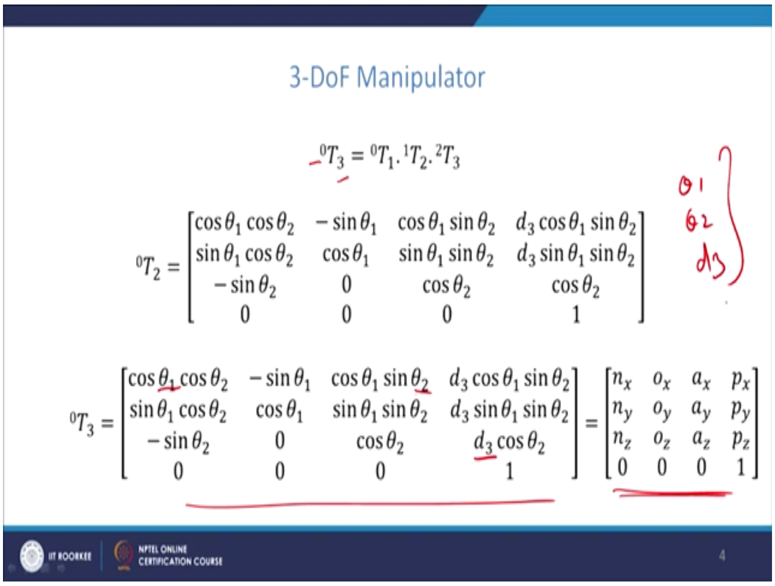

Now, if you multiply all these three we get the 0 T 3 homogeneous transformation, which gives the end effector coordinate frame with respect to the base coordinate frame 0. So, the product finally will give 0 T 3 is given by the expression given here in terms of theta 1, theta 2 and d 3. These are the three variables: theta 1 theta 2 and d 3 variables are there.

And, so for a given end effector position with respect to the base, if 0 T 3 is given by a homogeneous transformation like this, then by comparing each element of the two matrix we will get a system of equations: in theta 1 theta 2 and d 3. So, solving this system of equation we will get the inverse kinematic solution of the problem. So, that we will see in the another lecture how to solve the inverse kinematic problem for a given end effector position. So, now we will see another example for illustrating the D-H parameter in a more detailed way.

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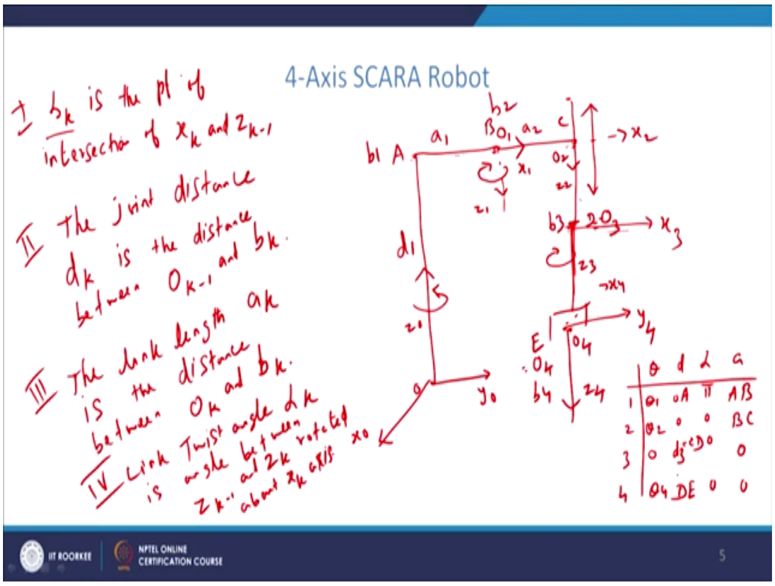

So, here we consider a 4-Axis SCARA Robot manipulator. So, the manipulator which we will consider is the following. We consider the this is revolving about the axis, z axis z 0 axis is a vertical line and so this joint is a fixed one, there is no movement at this joint. We call the base as o and this joint as A where there is no movement it is 90 degree l shaped position.

And at this joint B there is a axis of rotation which is parallel to z 0 axis in the opposite direction. So, the rotation at this joint is performed in this direction; that is opposite to the z 0 direction, the performance of the rotation is done. Then at this joint C there is a sliding movement, it is a prismatic joint. So, this joint will enable the link to move up and down in this direction; vertical direction.

So, this will be according to the D-H algorithm this sliding direction is the z axis. So, this axis is z 1 and this is z 2 axis. Now, at this joint D, this position there is again a rotational motion similar to z 1, they are parallel z 1 and here we call it as z 3 axis; both of them are always parallel and the rotation direction is similar. So finally, the end effector position. So, when there is an end effector the approach direction which is the end effectors z axis we call it as z 4.

So, this robot manipulator it is very clear that the end effector will always point downwards and it will move vertical vertically up and down only. It will not tilt in any other x and y direction only vertically it will, it can go down or up or it will rotate about the z 4 axis. So, now we can fix the coordinate frames at each joint as given in the D-H algorithm. So, z 0 is fixed z 1 is fixed, and the origin is the base here, we can fix the x 0 and y 0 axis arbitrarily according to the right-hand system rule.

So, we can fix arbitrarily x 0 and then accordingly y 0 is fixed, then the z 0 axis and z 1 axis they are always parallel in all positions. The shortest perpendicular distance; the shortest distance line or a line perpendicular to both z 0 and z 1 is A B direction itself. So, that is intersecting z 1 at the point B here. So, this point B will become the origin of a new coordinate frame with z 1 axis. Similarly, z 1 and z 2 they are always parallel. So, the shortest distance line it meets the z 2 axis at the point C, so that will become the origin of the coordinate frame with z 2 axis.

Now, z 2 and z 3 they are the same line, they are in the same direction. So, they are intersecting at all the points. So, we can select a convenient point as the origin. So, the convenient point is D itself this point, so we call this as O 3, the origin is the point D itself. Similarly, z 3 and z 4 are in the same direction, we can select the point the end effector point this is C, D, this is E. The end effector point E as the origin we call it as O 4.

So, first we have fixed the origin at each joint here, then according to the procedure we can fix the x direction. So, z 0 and z 1 are parallel. So, the common perpendicular line is the x 1 axis. So, we can say that the line B C act as the x 1 axis in this particular position. Similarly, z 1 and z 2 are parallel, the line extension of the line B C will act as 2 x 2 axis. And now z 2 and z 3 are in the same direction. So, they are intersecting at all the points, so any line perpendicular to both of them is called the x 3 axis.

So, we can select a convenient direction which is actually parallel to x 2 axis, we can select the line x 3 which is parallel to x 2, as the x 3 axis here. And in the same way the end effector z 4 is already fixed and according to the convention the line joining the two fingers of the end effector is called the sliding direction, that is always called the y direction. The approach direction is always called z direction and the line joining the fingers; two finger of the end effector is called the sliding direction y direction. Then x direction can be fixed accordingly by right hand system.

So, the direction x 4 can be fixed here. So, at each joint now we have fixed the x y z axis and the corresponding origins are also fixed already. Now, we should find the robot parameters, the four parameters of the robot manipulator. For fixing that, there is a point to be fixed, according to the D-H algorithm the point b k is the point of intersection of point of intersection of x k and z k minus 1.

So for example, if we if you take z 0 and x 1: z 0 is this direction x 1 is in this direction, when they where they intersect is at the point A z 0 and x 1. So, that point is called b 1. And x 1 sorry x 2 and z 1 x 2 is this direction z 1 is given in this direction they are intersecting at the point b. So, that point is called b 2. And x 3 and z 2: x 3 is in this direction, z 2 is here they are intersecting at the point d here.

So, this point is called b 3. So, similarly the last point we can call it as b 4; the o 4 is called b 4. So, this point is important to find the joint distance; joint distance d k is the distance between O k minus 1 and b k. Once the point b k is fixed we can we know the origin. So, the first origin is o 0: the base is o we can call it as o 0 and b 1 this distance. So, o A is called the distance d 1 and o 1 and b 2.

So, o 1 is here the point b, and b 2 is also the same point so we can say that d 2 is 0 it is at the point o 1 and b 2 are at the same position; therefore, d 2 is 0. Similarly, o 2 and b 3: o 2 is here b 3 is here so the length C D is called the d 3. So, we can find the joint distance according to this particular rule. The link length that is a k is the distance between O k and b k this two point.

So, if you take for example o 1 and b 1: o 1 is at the point capital B and b 1 is at the point capital A. So, this distance is called a 1 according to the D-H procedure. Similarly o 2 is at the point C and b 2 is at the point capital B. So, the distance between B and C we can call it as a 2. And other values are o 3 and b 3 they are at the same position therefore a 3 values 0 and o 4 and b 4 at the same place, so a 4 will be 0.

So, we get the three this one: d k joint distance and link length a k. Now the twist angle is the angle between the two z directions. So, z 0 is pointing upwards z 1 is pointing downwards. So, the angle between this two is 180 degree. So, the link twist angle is alpha k is angle between Z k minus 1 and Z k rotated about x k axis. So, these are the four point we have to note down. This is the first one, the second, the third point, and the twist angle is a 4th point.

So, if you see z 0 and z 1 and it is revolving around the x 1 axis we get 180 degree that is alpha 1 angle. And similarly z 1 and z 2 this two are parallel, so the angle between them are 0. Similarly z 2 z 3 z 3 z 4 they are in the same direction, therefore the angles are 0. So, what we get is the four parameters of the robot manipulator can be obtained in the following manner.

Here there are four joints: 1, 2, 3, 4; theta, d, alpha, a. So, we see that theta 1 theta 2 and theta 4; these are the revolute joint. 1 2 and 4 they are rotational, the third joint is the prismatic. So, theta 1 is a variable, theta 2 is a variable, theta 4 is a variable, and the d 4, d 3 is the thing. The first one is d 1 that is o A as shown here, d 2 is 0 according to this rule and d 3 is the length c d: c to d that is given by c D and d 4 is given by DE.

This angle is 0, the angle between the third one it is see 0 angle. Now alpha 1, alpha 1 is 180 degree for the first z 0 and z 1 they have 180 degree between them; z 1 and z 2 are 0 they are parallel, z 2 and z 3 in the same direction that angle is 0, and the fourth one z 3 z 4 the same direction. Similarly a 1, AB this point that distance is called a 1, and BC is called a 2 and a 3 a 4 these two are 0 according to this particular rule.

So, we get the four parameters at the four joints. Now, substituting that in the general expression. As we have seen this is the general expression i-th coordinate frame with respect to i minus 1 frame. So, we get 0 T 1 1 T 2 etcetera by substituting the values from this expression.

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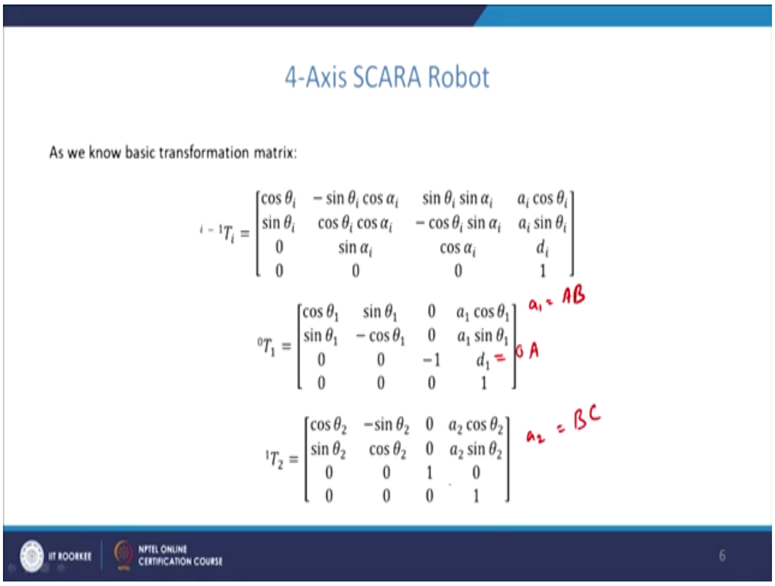

So, what we observe is 0 T 1 when we substitute theta 1, for d 1 we substitute o A that length. This is actually, so first joint this o A. So, the first joint distance d 1 is o A. So, we substitute in the place of d 1 this one. And a 1, in the place of a 1 we substitute the distance AB; whatever is the length AB. Similarly for A 2 we substitute the length BC depending on the size of the robot manipulator.

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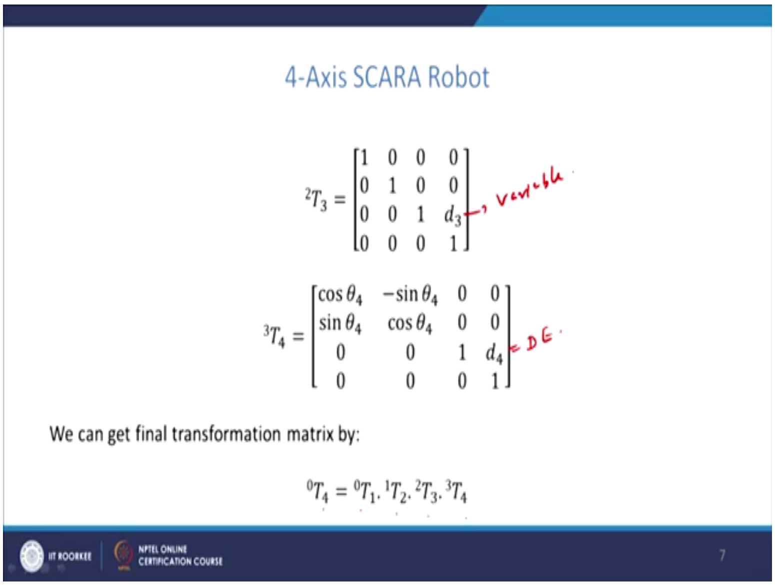

And d 3; d 3 is a variable, because the third joint is a prismatic joint it is varying from the value. That is the lowest value is 0 and the highest value is the length of that particular link. The maximum length of the link can be the highest value for that variable So, now d 4 is the final this thing this is DE, up to the end effector to the previous link. So, these values can be substituted and by multiplying the four matrix: 0 T 1 1 T 2 2 T 3 and 3 T 4 we get the end effector position.

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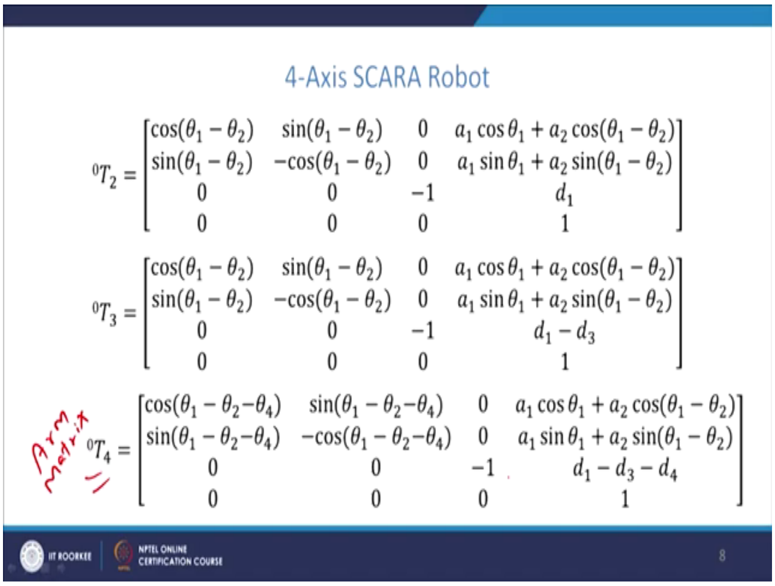

Finally, the product will give the this thing; this is called Arm Matrix : the end effector with respect to the base is given by this one. So, the next lecture we can see the inverse kinematic solution for this type of problem.

So, in this lecture we have seen the two examples for illustrating the direct kinematics of the robot manipulator. In the next lecture we will see more examples and their inverse kinematic solutions.

Thank you.