Robotics and Control: Theory and Practice
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Lecture - 06
Kinematic Module for Robotic Manipulater

Hello viewers, welcome to this lecture on Kinematics Model for Robot Manipulators. So, in this lecture, we will see how coordinate frames can be assigned, at every joint of a robot manipulator. So, in the last lecture and the previous lectures we have seen how the position and orientation of one object with respect to another can be expressed in the form of homogeneous transformations or a 4 by 4 matrix form. So, in this lecture we will see how such coordinate matrices can be assigned at each joint and finally, how to obtain the arm matrix of a robot manipulator.

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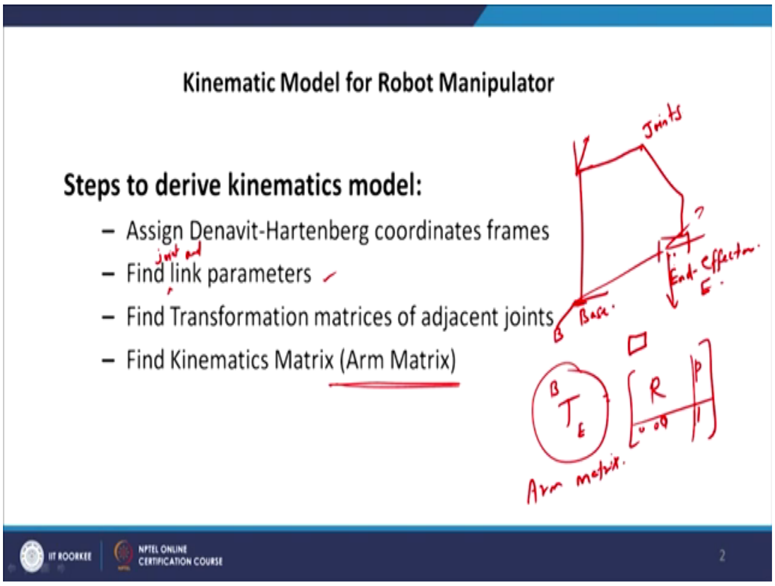

If we consider a robot manipulator, model like this the base and various joints are given and this is the end effector of the robot manipulator. Now, if the robot has to handle an object in its work space, then the robot should know the whereabouts of the objects. In other words it should know the position and orientation of a particular object so, that it can handle the object in a comfortable orientation or position. So, how to obtain the position and orientation of the object with respect to the end effector because end effector has to handle that objec?

In many times, we may be knowing the position and orientation of the object with respect to the base of the manipulator. So, we should be able to convert that information with respect to the end effector of the manipulator. So, in total what we want is we want to find the relation between the base of the manipulator and the end effector of the manipulator, as well as with the object in the environment. So, for doing this, we should know or we should attach various coordinate frames and the position orientation will be expressed in terms of homogeneous transformation, as we have seen in the previous lectures.

So first, if we have a coordinate frame at every joint in a particular manner so, we can and if you call this as the base frame B and this is the end effector frame etcetera. Then if we write B T E that is the end effector frame with respect to base frame this will be a 4 by 4 matrix. In this the first 3 by 3 matrix represent the rotation or the orientation of the end effector with respect to the base coordinate frame.

And the last column, we can call it as the vector P represent the position of the end effector. In other words, the origin of the end effector coordinate frame with respect to base coordinate frame how far it is away from the base coordinate frame that is the position of the end effector. So, if we know the position as well as the orientation of the end effector with respect to base that will give the a picture a correct picture of a robot manipulator. So, 0 0 0 1; because as we have already seen.

Now the steps to find the kinematics equation for a robot manipulator is as follows. we assign the Denavit Hartenberg coordinate frames at the joints of the manipulator. And then, after assigning the coordinate frames, we define the link parameters joint link and joint parameters;

joint and link parameters of the robot manipulator. And then we find the transformation matrices between two adjacent joints. Finally, we find the arm matrix this arm matrix is nothing, but the relation between the end effector and the base of the manipulator this B T E is called the arm matrix.

So, if a manipulator is given a particular structure is given, then we can find a unique the arm matrix which is the relation between the end effector and the base frame by following this 4 steps.

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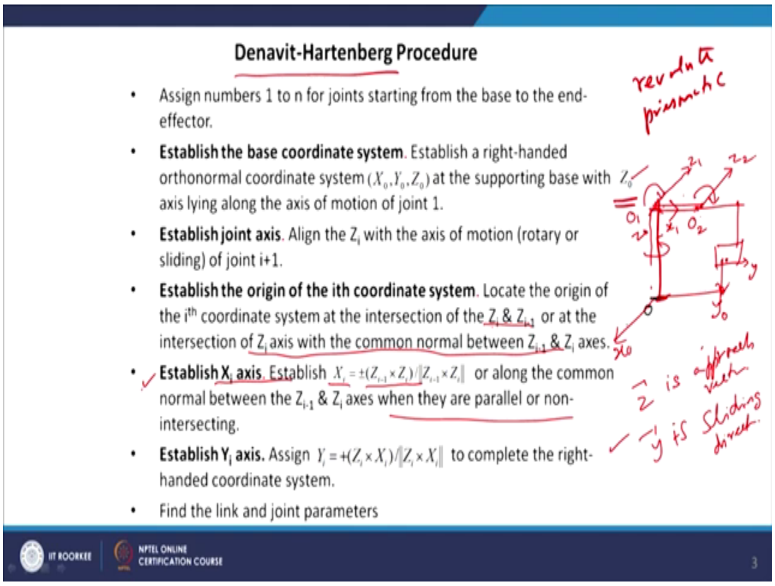

So, first let us see what is the D H procedure; Denavit Hartenberg procedure. So, first we have to assign the numbers for the joints. So, we can call this as the first joint is the base that is 1 then 2, 3, 4; the end effector is 5; in this particular picture we say there are 5 joints. Now for each joint we establish the coordinate system in the following way. First we want to establish a right hand coordinate system at every joint.

So, for the base frame the base of the manipulator, first we assign a coordinate frame. How to assign? First thing is we have to assign the z direction of that coordinate frame. So, every manipulator as we have seen in the introductory lecture, there are two types of joints that is revolute or prismatic. So, the prismatic joint means the sliding joint and the revolute joint is a rotational joint.

So, let us assume that the first link is a revolute joint, the first joint is revolute and the axis of rotation is this one. So, we call this as the z 0 axis as given in this step, second step z 0 is assigned to be the axis of the joint, then we can assign the x 0 y 0 arbitrarily. So, that it forms a right hand system. So, we write x 0 and then y 0 and so, we can observe that x 0, y 0, z 0 forms a right hand system.

Then we go to the next joint. So, how to fix the z axis of the next joint? So, that is also the axis of rotation or the axis of sliding; whether it is rotational joint or prismatic joint. So, let us assume that the second axis is also a rotational joint, where the rotation is performed in this way that is perpendicular to the paper. So, this is the Z 1 axis. So, once we fix the two axis for the two joint the first joint z 0 is fixed and the second joint z 1 is fixed here; then the origin can be fixed suitably for the second joint, the origin is nothing but the intersection of the Z i minus 1 and Z i.

So, they are intersecting at this point. So, we call this as the origin O 1; because it is the first joint has origin O, the second joint we call it as O 1 and etcetera the ith joint the origin is called O i minus 1. So, how the origin is fixed? It is the intersection of Z i and Z i minus 1.

Now, if let us say the third joint is also a revolute joint for example, let us say z 1 and z 2 both are parallel. So, here also there is a rotation about the z 2 axis revolute joint. Then how to fix the origin at this joint? So, it is given by this expression if that z 2 axis and z 1 axis are intersecting then the point of intersection is called the origin, but if they are not intersecting then the common tangent common normal between the 2 z axis where it intersect at z 2 is called the origin.

So, we can say that O 2 is the point of intersection of z 2 and the common normal between z 1 and z 2. So, it is given here the origin of this is fixed in this way. So, if they are not intersecting then Z i with the common normal between Z i and Z i minus 1 will be the origin of the system. So, whether the two X axis are intersecting are not intersecting this procedure can be followed to fix the origin of the coordinate frames.

So, once we fix the Z axis for various coordinate frames coordinate various joints and the origins for those joints; then what we can fix is the X axis establish the X axis at each joint. It is given by the vector perpendicular to both Z i minus 1 and Z i. So, the X i vector is the vector perpendicular to both Z i minus 1 and Z i as given in this, but if the Z i minus 1 and Z i, they are not intersecting each other; then we can take the common normal as the X axis. So, here if they are parallel or non-intersecting then the common normal between this two axis will be the X i axis of the system.

So, once we fix the X i axis and already we fixed the Z i axis, it is easy to fix the Y axis to form a right hand system. So, in this way we can fix all the coordinate frames at every joint of the manipulator. Now, if you have the end effector, the end effector of the robot manipulator is a special joint; here there is neither rotation or translation because it is at the end of the manipulator. So, we have to assign a coordinate frame without considering the revolute or prismatic joint.

So, what we do is, we take the vector perpendicular to the end effector the risk of the manipulator that as the Z axis. And the vector which is joining the 2 fingers of the end effector as the y axis and the x axis can be fixed as a to form a right hand system X, Y, Z should form a right hand system. So, the Z axis at the end effector is called the approach vector. So, Z axis is approach vector and then the Y axis is called sliding vector sliding direction and X axis is fixed according to the right hand system. So, this will complete the fixing of coordinate frame at every joint of a robot manipulator.

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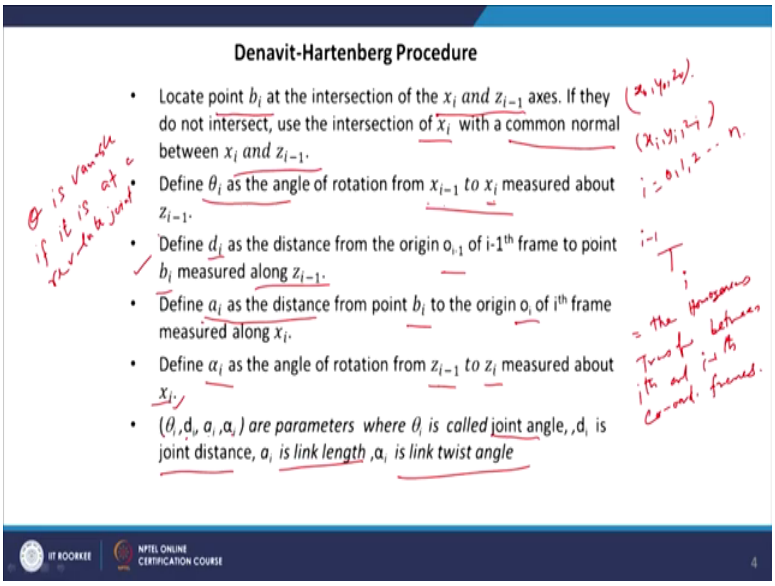

So, using this coordinate systems, next we can find the relation between the i th coordinate system and the i minus 1 th coordinate system so, we want to see. So, now we give the coordinate system as x 0 y 0 z 0 is the base coordinate system; x i, y i, z i is the i th coordinate system. i is equal to we can have 0, 1, 2, 3 up to n; n'th coordinate system is the end effector coordinate system. So, by D H procedure, we have already fixed this coordinate system.

Now we want to find the relation between the i minus 1 th system with respect the ith system with respect to i minus 1 th coordinate system. So, i minus 1 T i it represent the homogeneous transformation between i th and i minus 1 th coordinate frames. So, the joint coordinate frames how they are related that is what we want to see. So, for doing that we have to define the parameters at every joint. So, the there are 4 parameters, theta i d i alpha i and a i this four parameters to be fixed at every joint.

So, theta i is called the joint angle and d i is called joint distance, a i is called the link length and alpha i is called the link twist angle. So, these 4 parameters some of them may be 0 and uh. So, we can fix the 4 parameters at every joint by following these definitions. So, here first thing is we have to locate the point b i, which is the intersection of the axis x i and z i minus 1; because we have already fixed all this coordinate axis. The intersection of x i and z i minus 1 is called b i point, if they are not intersecting then the common normal between x i and z i minus 1 with x i wherever it is intersecting that is called the point b i.

Then define the angle theta i the joint angle theta i is defined as the angle between x i minus 1 and x i axis this two axis which is measured about the z i minus 1 axis. So, if you have z i minus 1 axis so, for example, if we have in this particular picture, we have z 0 axis this way and x 0 axis is here; now we can fix by following this fixing of X i axis this one it is the vector perpendicular to z 0 as well as z 1.

So, we can say that this is the x 1 axis z 0 and z 1 the perpendicular line is either in this direction or in the opposite direction. So, we can select anyone of the direction, but it is a convention that we select the direction which is pointing towards the end effector of the manipulator. So, we call x 1 axis to be this line. Now, the theta 1, how to define theta 1? It is the angle between x 0 and x 1, if we put i equal to 1.

So, x 0 here and then x 1 the angle between them measured about z 0 axis. So, how this x 0 it rotates about the z 0 axis so, that x 0 and x 1 are parallel. So, that angle is called the theta 1 axi angle. Similarly all other angles, if we take x 1 and x 2 the angle between x 1 and x 2 measured about the z 1 axis will give theta 2 value so etcetera. If it is a revolute joint, then the theta is variable if it is a revolute joint ok, if it is at a revolute joint.

Now, the joint distance d i it is measured from o i minus 1 of the i th frame to the point d i. So, already we have defined what is d i. So, if you want to calculate d 1 the joint distance d 1, we have to see o suffix 0 that is the basic base coordinate frames origin and the point b i b 1. So, the distance between them measured along z 0 axis, will give the value d 1 similarly d 2 d 3 all can be measured using this expression.

And the link length a i it is a distance between the point b i and the origin o i. And similarly, the twist angle alpha i is the angle between the axis z i minus 1 and z i measured about the vector x i. So, this 4 definitions for the parameters of a robot manipulator at every joint is given by this procedure. So, once we have this definition then we can construct the matrix the relation between one coordinate frame with the other coordinate; that is i th coordinate frame with respect to i minus 1 coordinate frame. So, we can see from this picture, at every joint we are fixing a coordinate frame suitably x 0, y 0, z 0; x 1, y 1, z 1 etcetera. Now, how to relate one coordinate frame with the other? That is to be seen in the in this expression.

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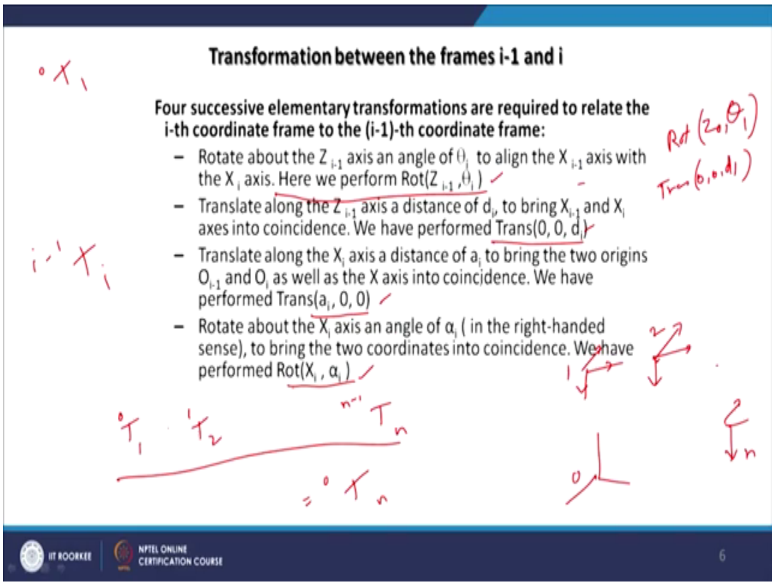

So, if we have the base coordinate frame like this and another coordinate frame to be like this and the third joint has a coordinate frame this way and the end effector coordinate frame is this way. So, we can have the 0 th frame and then first frame second frame etcetera the nth frame we want to find the relation between the first frame with respect to the 0 th frame and then we want to find the second frame with respect to the first frame. Similarly, we want to find the n th frame with respect to the previous frame n minus 1 th frame.

So, if we have all this relations 4 by 4 matrices, then if you multiply all this. So, we can see that the relation between the n th frame with respect to the 0 th frame can be obtained. As we have seen previously, we can multiply successively if you are doing a homogeneous transformation successively with respect to the current coordinate frame we should multiply in the right hand side. So, by that logic we can multiply all this matrix one by one and then finally, we get the relation between the end effector and the base as 0 T n.

But how to find this 0 T 1, 1 T 2 or in general how to find i minus 1 T i for the i th frame with respect to i minus 1 th frame. So, for that we use the 4 parameter definition as given here. So, we can see that for example, theta i if you take this one; if you take if you take the base frame z x 0, y 0, z 0 and x 1, y 1, z 1. If you make a rotation by an angle theta 1 about the z axis, then we can see that x 0 and x 1 will become collinear they become parallel.

So, if you perform a rotation here, we can see that you perform the rotation about z i minus 1 axis by an angle theta 1 what we see is the x 0 axis will become parallel to x 1 axis. Similarly, if you perform rotation about T z 1 by an angle theta 2 then x 1 axis will become parallel to x 2 axis etcetera. So, first we concentrate on let us say this 0 th axis and the first axis x 0, y 0, z 0 and x 1, y 1, z 1. So, first if we make a rotation about z 0 by an angle theta 1 the x 0 will coincide with not coincide it will become parallel to x 1.

Now, the second step if we make a translation of the distance d 1 along the z direction. So, translation 0 0 d 1 along the z direction if we they take. So, if we take along the z direction, d 1 distance then we can see that this origin will coincide with this origin and the x 0 axis x 0 axis already has become parallel to x 1 it will exactly coincide with the x 1 axis so that is the performance here. We make a translation of 0 0 d 1 so that x 0 axis will become it coincides with x 1 axis.

Now, if we make again a translation of a 1 0, 0 that is along the x direction a 1 0 0 then we can see that the origins also will coincide the origins o 0 and o 1 will also coincide. And if we make a rotation about the X axis by an angle alpha i, then the z axis which are different; in this case you can see z 0 is pointing in this direction z 1 is pointing perpendicular to the paper. So, they will also coincide if you make the twist angle alpha i rotation about the x axis.

So, we see that, after making this 4 rotation and translation 2 rotation and 2 translations, the base coordinate frame x 0, y 0, z 0 completely coincide with x 1, y 1, z 1 coordinate system. So that means, we have obtained 0 T 1 the first coordinate system with respect to the 0 th system after making this 4 operations. How we have made this four operation? First rotation, then translation again a translation and then rotation.

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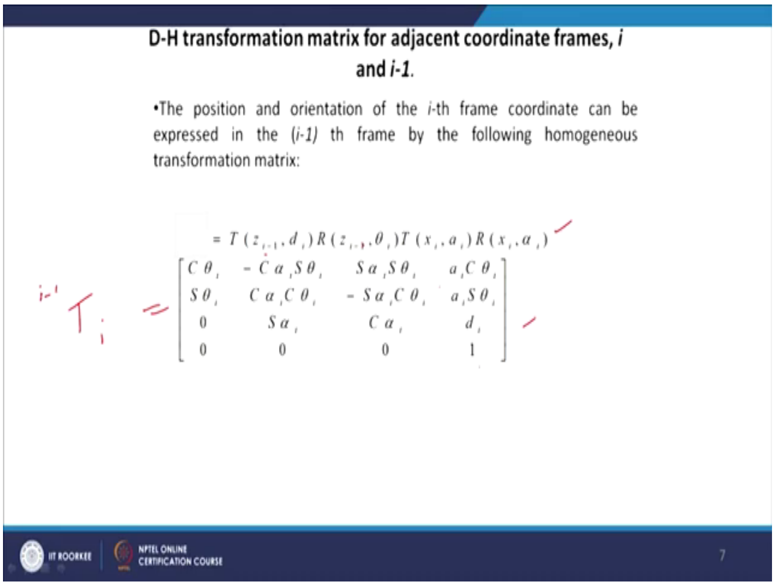

If we multiply four of this operations like this one by one whatever we have performed then we get a matrix this expression. So, this expression contains theta i and d i alpha i a i all the 4 parameters are involved. So, this gives the relation i minus 1 T i ok; so, it should be expressed like this the relation between i minus 1 T i is given by this 1. So, this we will explain using a concrete example in the next lecture to obtain the arm matrix of a robot manipulator.

Thank you.