Robotics and Control: Theory and Practice
Prof. Felix Orlando
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture – 34
Higher Order Sliding Mode Control

Good morning, today we have the lecture on Higher Order Sliding Mode Control.

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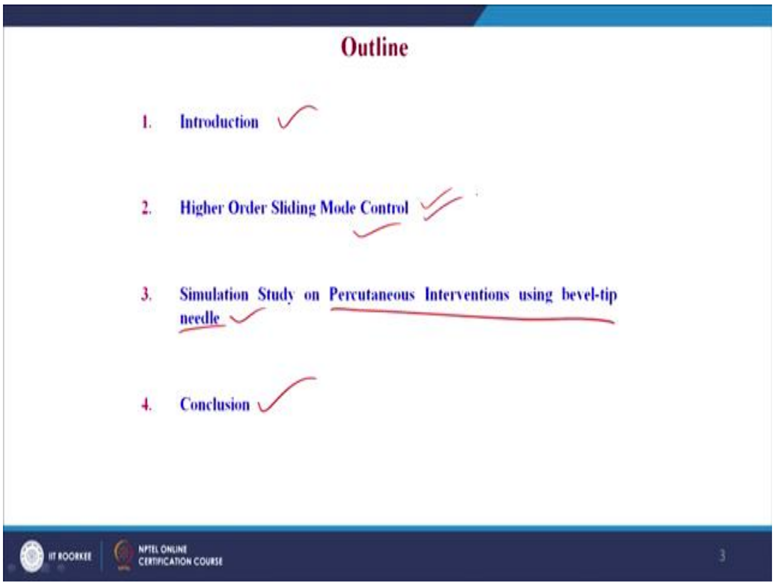

The organization of today's lecture will be as follows; first, we have the introduction to higher order sliding mode control; then we see the convergence of distance and time. So, that the state trajectory convergence to the equilibrium point in finite time will be having a proof in this lecture. And then we implement the higher order sliding mode control on Percutaneous Interventions using bevel tip needle.

In that research study, we implement the higher order sliding mode control and see the performance how it is getting converged in finite time and finally, we conclude todays lecture based on the sliding mode performance on the percutaneous interventions.

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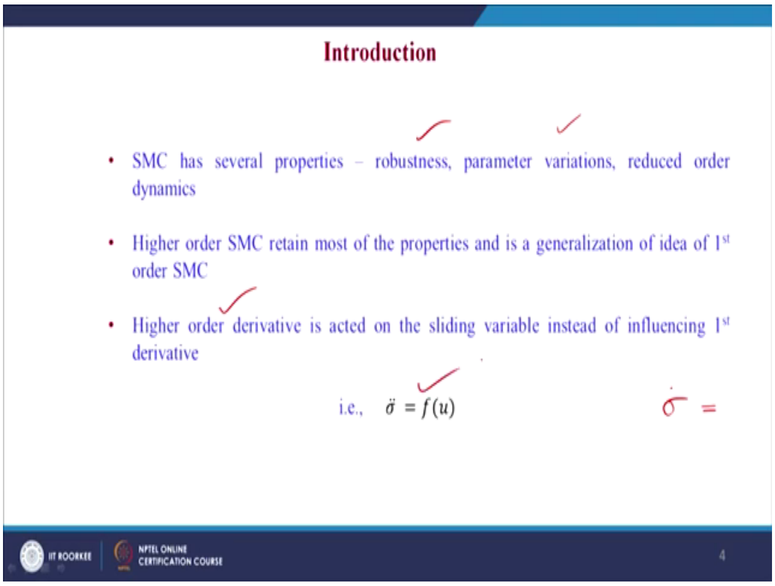

Coming to the introduction; sliding mode control has several properties, finite time convergence, robustness, parameter variations, reduced order dynamics. The higher order sliding mode control retain most of these properties and is a generalization of the first order sliding mode control. Here in this higher order sliding mode control, higher order derivative is acted on the sliding variable σ instead of influencing first order dynamics.

That is in the conventional sliding mode control; we have seen $\dot{z}$ that is the $\dot{\sigma}$ is derived from the σ variable.

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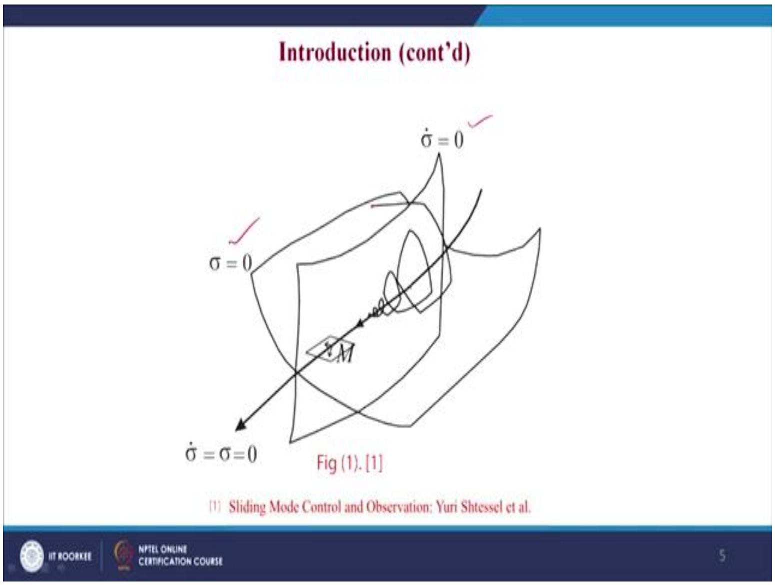

But here we go for $\ddot{\sigma}$ that you see higher derivative is involved in the dynamics of the sliding variable. Now, coming to the introduction, continued with the introduction; so we have the sliding line is in such a way that we have two surfaces in perpendicular to each other; that is one surface is or one plane is $\sigma = 0$ and $\dot{\sigma} = 0$ is another plane.

And the intersection of these two planes will have a line and that is the sliding surface you can say; that is the sliding domain where the trajectory of the states should converge that is the line of interest where the trajectory of the states must sit on it and get into the equilibrium point that is $\dot{\sigma} = 0$. And, how this trajectory is getting converged to the equilibrium point is through multiple twisting of the state trajectory.

And each time we see that the twisting distance as well as the time to reach the intersecting line is reduced and that is the proof, we are going to see because we have taken from two planes; one is corresponding to $\dot{\sigma} = 0$ and another one is $\sigma = 0$. When these two planes intersect; we have a line of intersection that is the line of interest which is where the trajectory of the states will get reached there and slide it to the equilibrium point.

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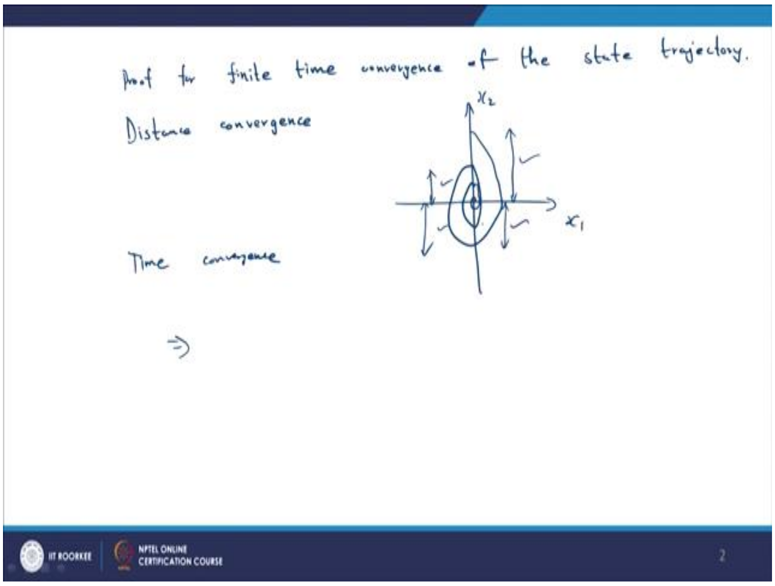

So, now we see the proof for finite time convergence of the state trajectory. So, first we see distance proof like distance convergence; convergence that is we have the state trajectory x_1, x_2 such that accordingly; so that this distance and this distance, when you compare; this distance is lesser than the starting distance, then this distance and this distance we compare and then this distance is lesser than this distance.

So, that this proves that the trajectory of the state is getting converged to the equilibrium point in finite time. So, how much time is taken is the second proof we are going to see through time convergence. So, eventually what we prove through this higher order sliding mode controller is; we have finite time convergence of the state variable to the equilibrium point which was not possible through the conventional sliding mode controller.

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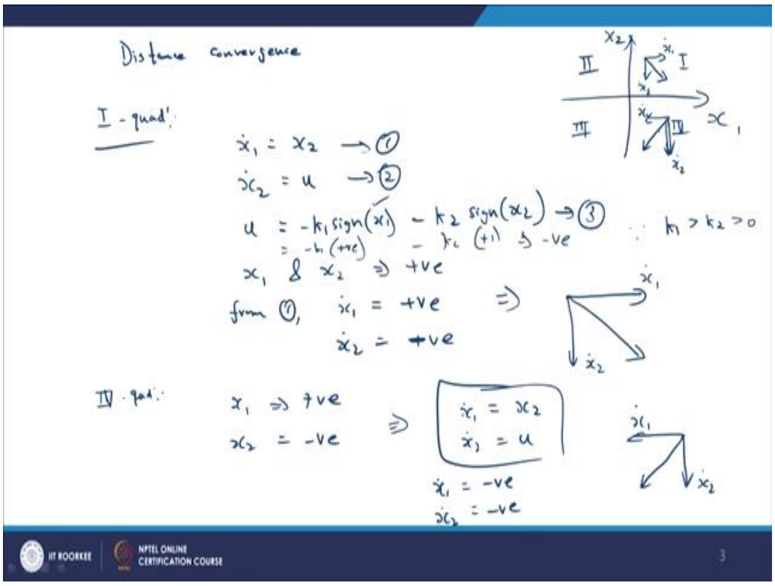

So, now we see the proof of distance convergence. So, we take here the quadrants x_1, x_2; this is the I quadrant, IV quadrant, II quadrant, III quadrant. So, in the I quadrant when you take the I quadrant with the state model

$$\dot{x}_1 = x_2 \qquad (1)$$

$$\dot{x}_2 = u \qquad (2)$$

where the controller is coming in the state equation which is equation 1 and the $\dot{x}_2$ is equation 2; then we have the control law for this higher order sliding mode controller is taken to be

$$u = -k_1 sign(x_1) - k_2 sign(x_2) \qquad (3)$$

So, the I quadrant both x_1 and x_2 are positive. So, from 1 we have $\dot{x}_1$ equal to positive and $\dot{x}_2$ is also positive; so $\dot{x}_1$ positive implies, this is the direction of $\dot{x}_1$ and $\dot{x}_2$; here if you see i just made a mistake sorry that if you see that this is basically the $\dot{x}_2$; it turns out to be negative because when we substitute x_1 and the x_2 positive in the u equation 3; what we have is $-k_1$ into positive that is 1; $-k_2$ into plus 1, so eventually it is negative since k_1 is greater than k_2 which is positive values it is a constant nonzero values.

So, we have $\dot{x}_1$ trajectory being this one and $\dot{x}_2$ trajectory is negative; thus the resultant is this direction. So, in the I quadrant what we have is $\dot{x}_1, \dot{x}_2$; so we have the direction of the state trajectory is in this way. Similarly, in II quadrant; what we have is x_1 sorry we can take the IV quadrant because we come in this direction become we take I quadrant I and then quadrant IV.

So, the quadrant IV; what we have is x_1 is positive and x_2 is negative which leads to the resultant in such a way that $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$ which is a state variable; when you substitute here what we obtain is $\dot{x}_1$ is negative and $\dot{x}_2$ is given by negative. So, we have both coming out to be that is $\dot{x}_1$ direction is this one and $\dot{x}_2$ direction is down. So, the resultant is this one; that is the direction of the state variable is this one. So, I put here; this is $\dot{x}_1$ and this is $\dot{x}_2$, so the resultant is this one.

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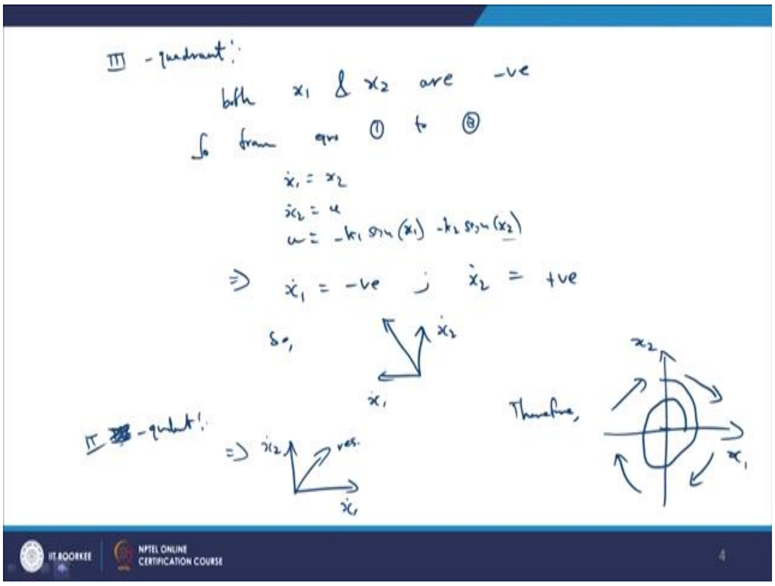

Next, we go to the III quadrant; what we have is both x_1 and x_2 are negative. So, from equations 1 to 3 which is $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$ and u equal to

$$u = -k_1 sign(x_1) - k_2 sign(x_2)$$

We have the direction that is $\dot{x}_1$ equal to negative, similarly $\dot{x}_2$ is equal to positive. So, we have this direction $\dot{x}_2$ being positive and the $\dot{x}_1$ be negative; so the resultant is in this that is a state trajectory direction is in this way.

And similarly in the IV quadrant; we have the direction of $\dot{x}_1$ is in this way positive and the $\dot{x}_2$ in the; II quadrant, this one IV quadrant it is in the II quadrant and we have both $\dot{x}_1$and $\dot{x}_2$ are positive; so the resultant is this one that is the resultant which is a state trajectories direction. Therefore, we have the trajectory of the state variable coming out to be this direction.

Then this direction then this direction then finally, this direction which shows that comes this way and goes this way, then goes this way goes this way. So, we need to prove that these trajectories of the state variables will converge to the equilibrium point in finite time. So, the distance must be now observed; whether it is each time getting reduced.

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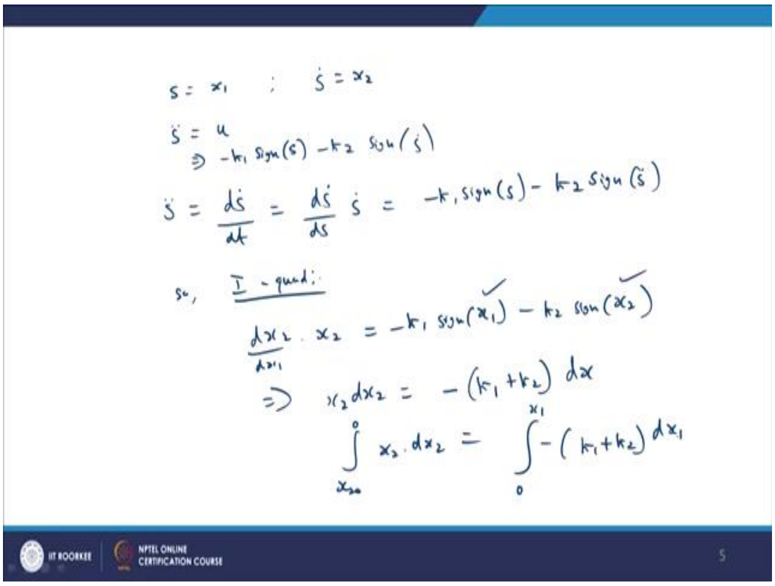

So, what we do here is we take the situation in such a way that $s = x_1$ and $\dot{s} = x_2$. So, $\ddot{s} = u$ which implies $-k_1 sign(x_1)$ which is s in this case $-k_2 sign(\dot{s})$. So, $\ddot{s}$ is in general given by

$$\ddot{s} = \frac{d\dot{s}}{dt}$$

which is equal to

$$\ddot{s} = \frac{d\dot{s}}{ds}\dot{s} = -k_1 sign(s) - k_2 sign(\dot{s})$$

So, in the I quadrant; what we prove is, we have because $s = x_1$ and the $\dot{s} = x_2$; we have

$$\frac{dx_2}{dx_1}x_2 = -k_1 sign(x_1) - k_2 sign(x_2)$$

which can be written like this,

$$x_2 dx_2 = -(k_1 + k_2)dx_1$$

depending on the sign of x_1 and x_2 in the I quadrant; both are going to be positive and hence signum will give plus 1 values for that.

So, integrating both sides we get

$$\int_{x_{20}}^{0} x_2 dx_2 = \int_{0}^{x_1} -(k_1 + k_2)dx_1$$

which leads to $-\left(\frac{x_{20}^2}{2}\right) = -(k_1 + k_2)x_1$

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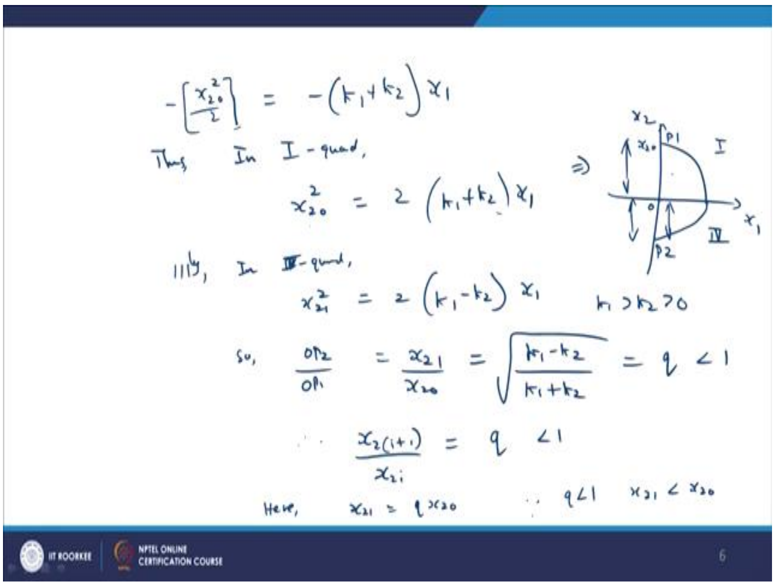

This is basically

$$x_{20}^2 = 2(k_1 + k_2)x_1.$$

which is schematically, it is given by x_1 this is x_2; so here it is 1. So, here we start from x_{20} and this; the O value; now say this is P; so, the I quadrant this is the situation.

In the IV quadrant which is going to happen like this which is see this is P_1 and this point is P_1 and this is origin; so it is O. So, for the; I quadrant, the left hand side has the limits from x_{20} to x_0 whereas, the right hand one has the limits varying from 0 to x 1. So, now what we have observed is

$$x_{20}^2 = 2(k_1 + k_2)x_1$$

Similarly, in the II quadrant; we have in the similar procedure we have

$$x_{21}^2 = 2(k_1 - k_2)x_1$$

So, basically it is not II quadrant, it is basically this quadrant which is basically the IV quadrant; I am just making the mistake sorry. So, this is basically the IV quadrant we have this relationship because $k_1 > k_2 > 0$.

So, the distance ratio $\frac{oP_2}{oP_1}$, we have $\frac{x_{21}}{x_{20}}$ which is equal to $\frac{x_{21}}{x_{20}} = \sqrt{\frac{k_1 - k_2}{k_1 + k_2}}$ which is equal

$$\frac{x_{21}}{x_{20}} = \sqrt{\frac{k_1 - k_2}{k_1 + k_2}} = q < 1$$

to a variable q; it is a constant which is less than 1. Therefore, we generalize;

$$\frac{x_{2(i+1)}}{x_{2i}} = q < 1.$$

So, here between x_{20} and x_{21}; we have the relation $x_{21} = qx_{20}$. Since $q < 1$; $x_{21} < x_{20}$.

So, we prove that; we are proving that this distance that is the distance traveled from this to this, by the state trajectory is greater than the distance traveled from this point to this point P_2; from the x_1 axis is greater. So, the distance OP_2 is lesser than distance OP_1 that is proved here; likewise we can prove that accordingly from each quadrant to quadrant.

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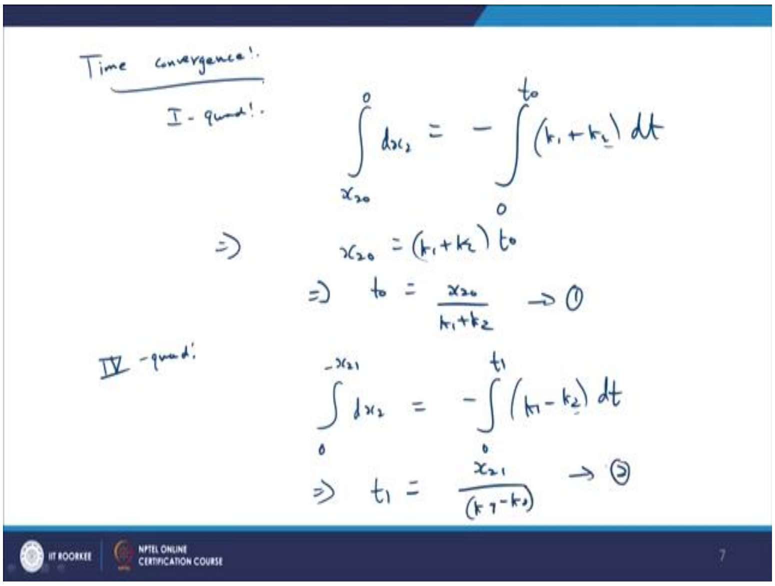

Now, coming to the time convergence; we have proved now in the state trajectory, the distance is converged whereas the time convergence is another parameter to be proved in order to reinforce this statement that it is converging in the finite time. So, now coming to the I quadrant; what we have is from

$$\int_{x_{20}}^{0} dx_2 = \int_{0}^{t_0} -(k_1 + k_2)dt$$

which implies

$$t_0 = \frac{x_{20}}{(k_1 + k_2)} \qquad (1)$$

In quadrant 4,

$$\int_{0}^{-x_{21}} dx_2 = \int_{0}^{t_0} -(k_1 - k_2)dt$$

because of the signs of the variables x_1 and x_2; we have this one which further giving the value of

$$t_1 = \frac{x_{21}}{(k_1 - k_2)} \qquad (2)$$

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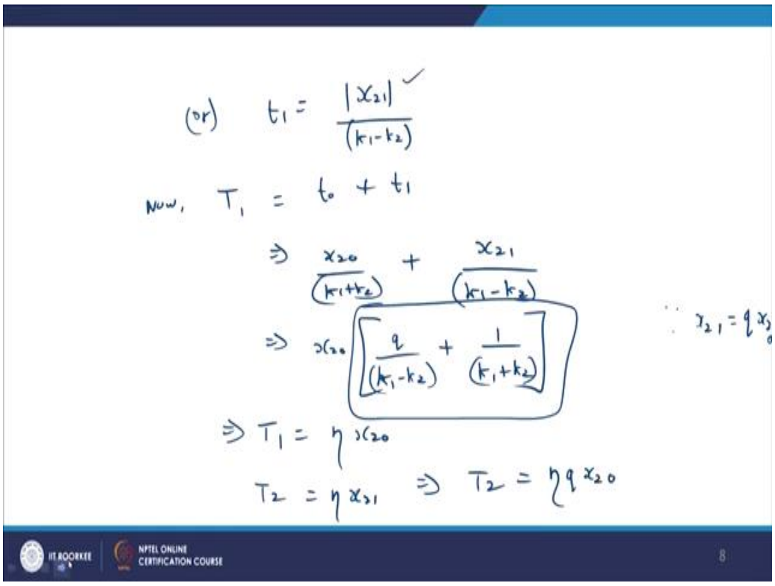

See which is equation 2 or we can take

$$t_1 = \frac{|x_{21}|}{(k_1 - k_2)}$$

But whatever being x_{21}; negative or positive; it will give the positive value only; that is why we taken the; we were taken the absolute value. Now, the total time $T_1 = t_0 + t_1$ which is given by

$$\frac{x_{20}}{(k_1 + k_2)} + \frac{x_{21}}{(k_1 - k_2)}$$

where we take x_{20} outside. Why? Since the $x_{21} = qx_{20}$.

So, we can take x_{20} common; so

$$x_{20}\left[\frac{q}{(k_1 - k_2)} + \frac{1}{(k_1 + k_2)}\right]$$

which implies $T_1 = \eta x_{20}$ this full part is a constant. Similarly, $T_2 = \eta x_{21}$; which implies $T_2 = \eta q x_{20}$ which can be generalized in this way that $T_{i+1} = \eta q^i |x_{20}|$ which implies

$T_{i+1} = \frac{\eta |x_{20}|}{1-q}$ which implies convergence of time.

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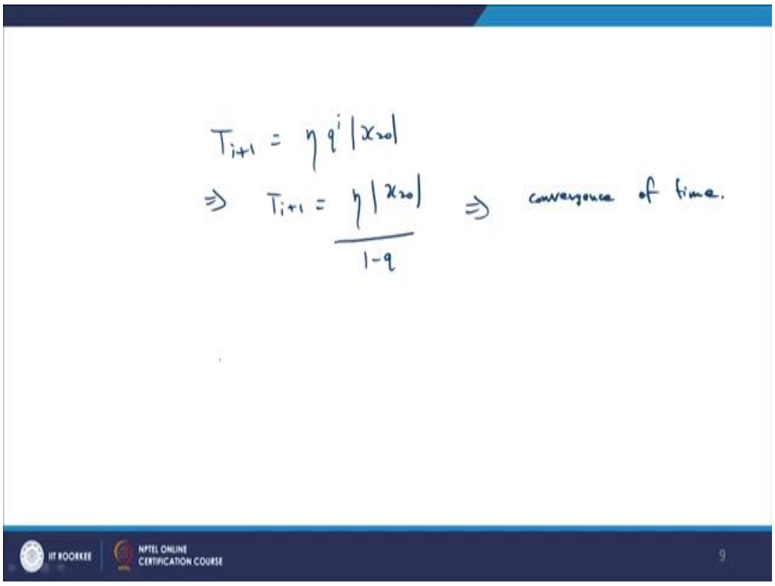

Thus we have proved that both convergence of distance as well as convergence of time for the state variable trajectory in this higher order sliding mode which proves finite time convergence.

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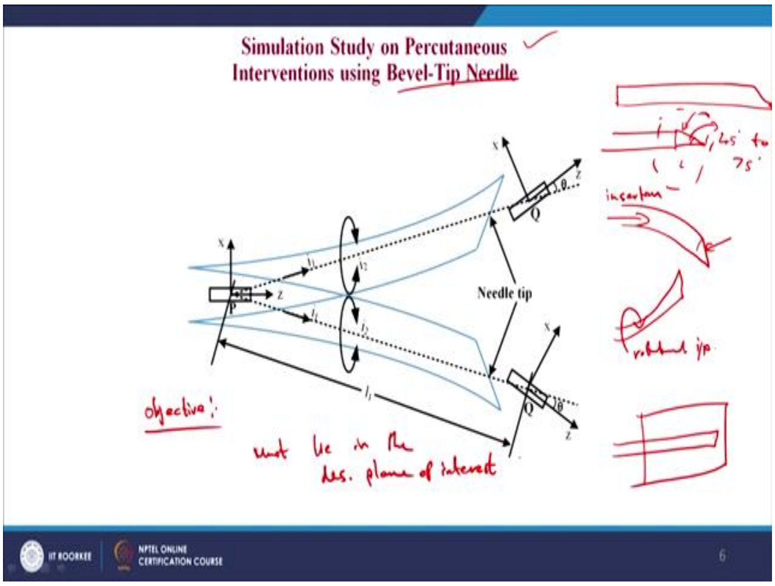

So, now we come to the implementation of this higher order sliding mode control strategy on the research study; on our research study on Percutaneous Cancerous Interventions using bevel tip needle. So, what is a bevel tip needle? So, this is the bevel tip needle where the tip is having this shape; bevel shape, the bevel angle is given by this one which can vary from say 45 degrees to 75 degrees. So, that once the tissue reaction is happening here; once it is this needle is inserted into the tissue domain, the tissue reaction force happens on the beveled surface on the surface of the needle tip. So, the needle will undergo this type of bending because of the tissue reaction force on the bevel surface.

Then the needles can be spinned along the needle axis in order to have the direction change either upper direction or lower direction depending on the spinning of this needling surface or around the needle axis; the needle must be rotated in order to have the direction of curvature. So, here we have two inputs; one is the insertion input, another one is the rotation input the rotation input for the needling system.

So, the objective of this research study is to keep this needling system in the plane of interest; desired plane of interest. So, this is the plane of interest, once a needle is inserting; inserted in the tissue region, the needle must be lying in the desired plane of interest; it must not be diverted away from this need the plane of interest; so that is the objective.

So, must the decide objective; objective is the needle must lie in the desire plane of interest while maneuvering inside the tissue region; this is the important objective of this research study.

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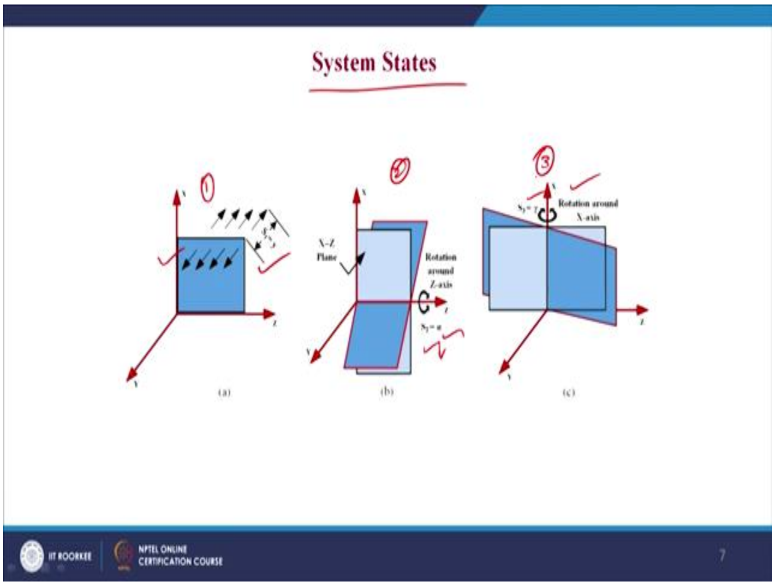

And we have taken or considered the system states; in the needling system states in order to satisfy the objective or the system states or the distance away from the plane that is shown in the figure, in the blue plane is the plane of interest and the distance away from the plane is first state which must be controlled so, that a distance should be minimized or made it to be 0.

Finally, the state variable must lie on the plane which implies this state variable $S_1 = y = 0$. Another state variable second state variable is the first state variable, the second state variable is the rotation around the z axis which you say alpha must be minimized; if it is rotated around this axis, then the out of the plane of the needle happens.

So, the angle around the z axis α must be minimized that is the second state variable must be converged to 0; this variable. Similarly, the third state variable which must be converged to 0 in the finite time is the rotation around the x axis, which is γ. So, the three state variables are distance away from the plane of interest and rotations around z axis α and the rotation around the x axis γ; these are the three state variables that must be converged to 0 in the finite time.

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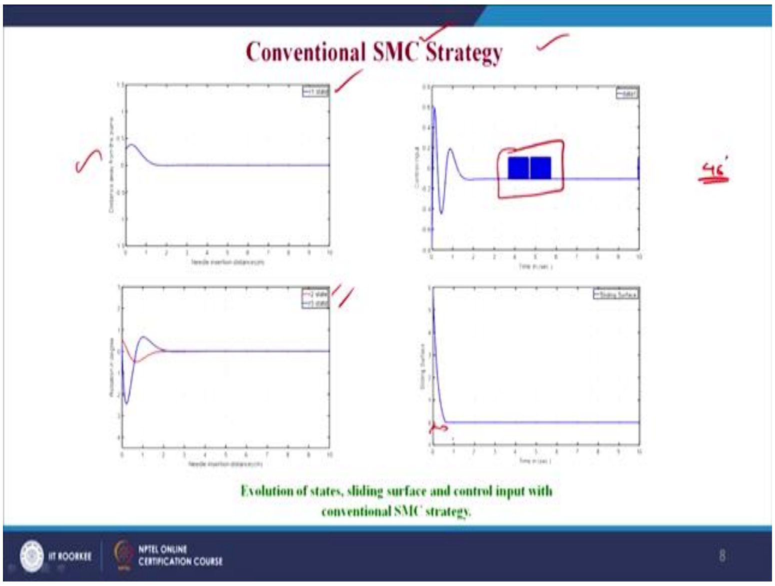

So, the results we have obtained in implementing the higher order; sliding mode control strategy or as follows. First, we have implemented conventional sliding mode strategy on our system model; kinematic model based on the bicycle modeling of the passive needle which is with the bevel tip of 45 degrees; bevel angle.

So, the state first is getting converged on the second states that is the angle; the first is the distance state shown separately and the angle states S 1 and S 2 are shown here where we denote them by r_1, r_2 and r_3 and the controller is having the chattering high frequency control input; which is this chattering effect that can be shown. And we see that a sliding variable is getting converged in finite time; oh sorry you have seen here with the conventional sliding mode strategy.

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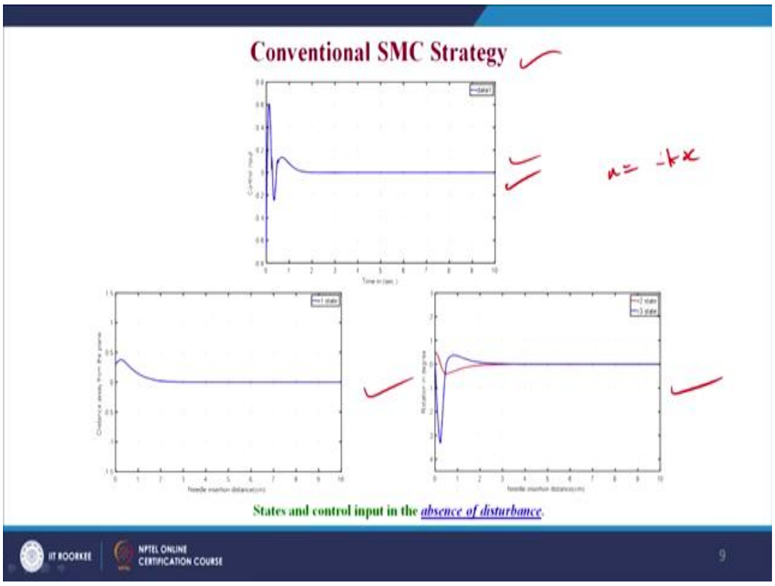

So, here we see the conventional sliding more strategy without the presence of disturbance. So, this can be the same system performance the evolution of the states and the control input without a chattering can be observed by a simple state feedback controller $u = -kx$ can also be used to observe this because there is no distance disturbance in this control problem.

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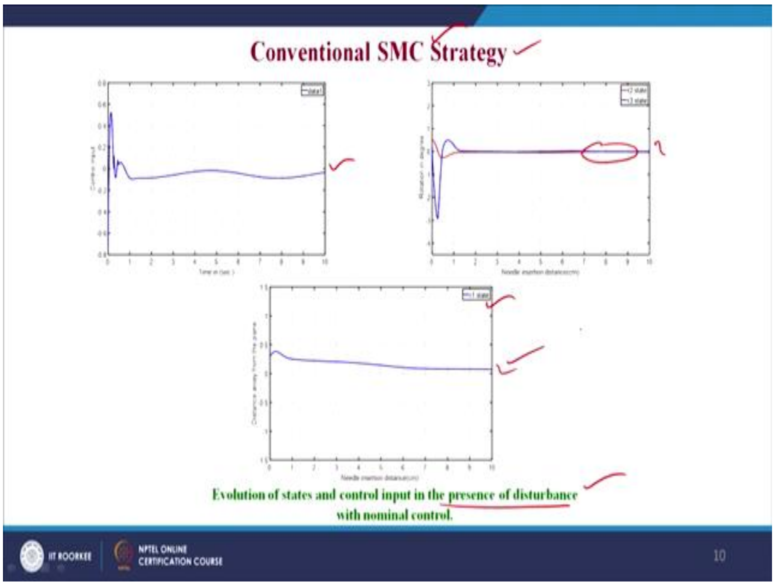

So, when we give disturbance; what happens with the conventional sliding mode control strategy. The control input is feasible without any chattering; the states you see the convergence is suffering. Thus distance state is not getting converged; likewise, when we zoom this we have the other states angle states are also not converging.

So, when the disturbance is given with the conventional sliding mode controller strategy; we have the evolution of the states and control input in such a way that the states are not getting converged at all in finite time.

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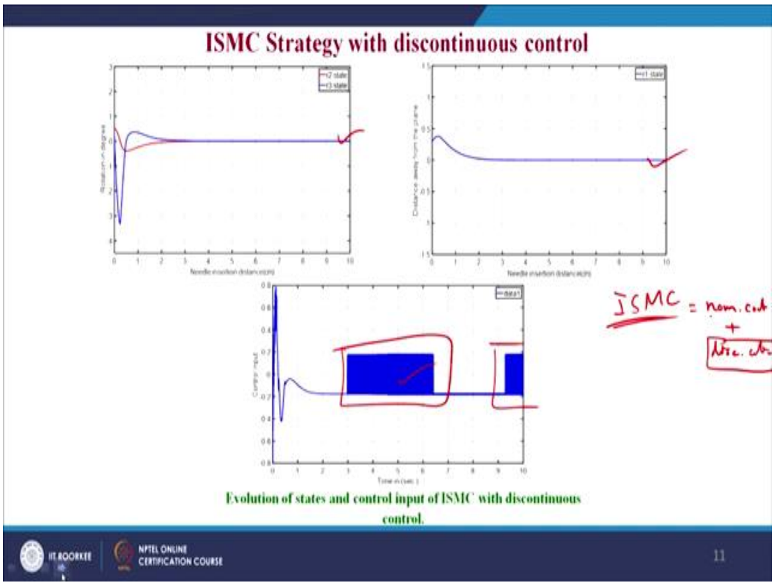

So, we go for the strategy called integral sliding mode control strategy. So, the integral control mode sliding strategy is with the nominal control plus discontinuous control. When we have the discontinuous control with the interval sliding mode control strategy, we have the high frequency components existing in the control input.

So, even though the states are converged in finite time; finite time convergence is observed with the integral sliding mode strategy. But the controller is having a chattering problem because of the high frequency component here which is not feasibly acceptable in the clinical scenario of percutaneous needling interventions.

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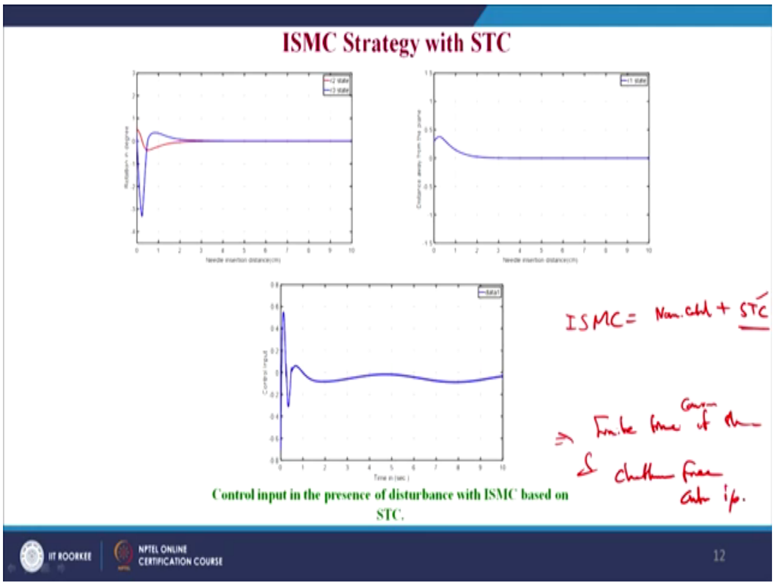

So, the discontinuous part in the integral sliding mode controller; this is nominal control plus super twisting controller is used; instead of the discontinuous part of the integral sliding mode control, we have used the super twisting mode control where this controller replacing the discontinuous part in the integral sliding mode control gives finite time convergence of state variables and chattering free control input; chattering free control input which is desirable in the clinical scenario.

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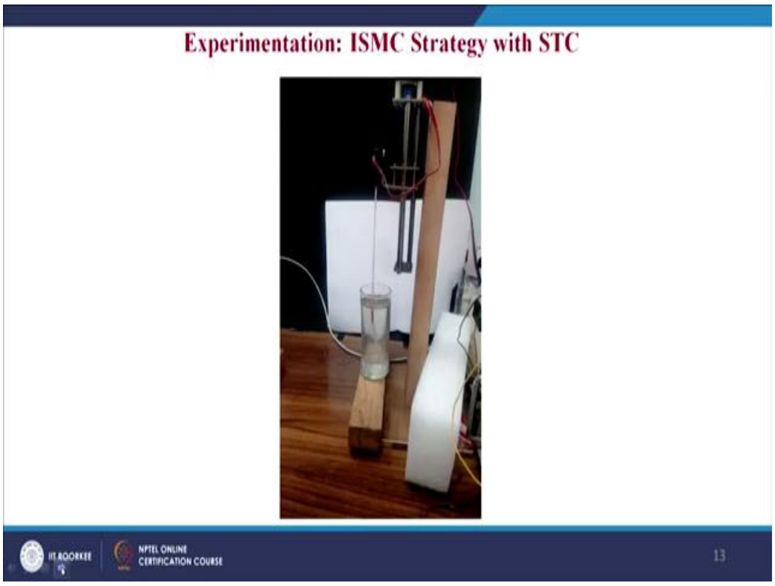

And the experimentation is shown here with this strategy implemented on the real system we have observed that the needling system can be inserted with the bevel tip, so that the angle diversion from the plane that is the line of insertion is shown here.

So, that this video confirms that due to integral sliding mode control with super twisting algorithm the needle lies within the desired plane of interest with the bevel angle coming into picture because of the tissue reaction force.

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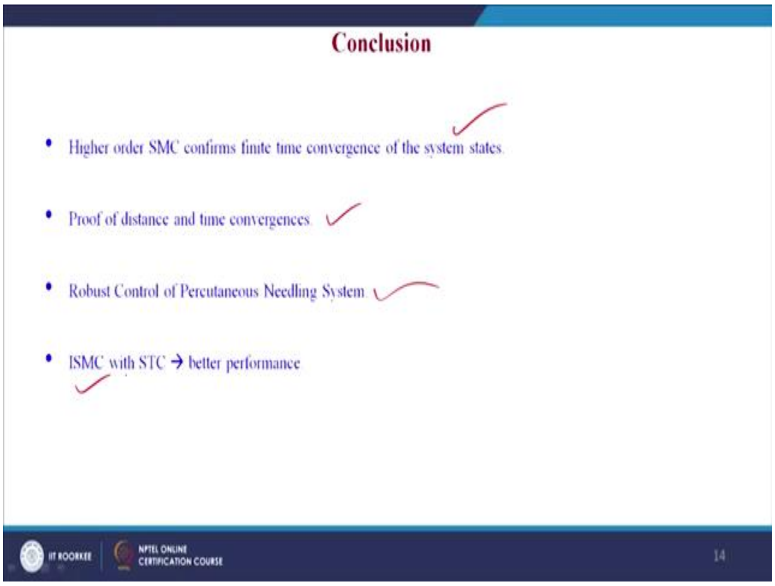

Thus we conclude that higher order sliding mode control confirms finite time convergence of the system states and we have also by this lecture we proved that both time convergence and the distance convergence in order to reinforce the statement given for the higher order slide one control in terms of convergence.

And we have performed this robust control strategy on the percutaneous needling system for cancerous treatment. And, we observed that the integral slightly more controller with the discontinuance part being replaced by the super twisting control strategy gives a very better performance when compared to the other strategies. With this we wind up.

Thank you so much.