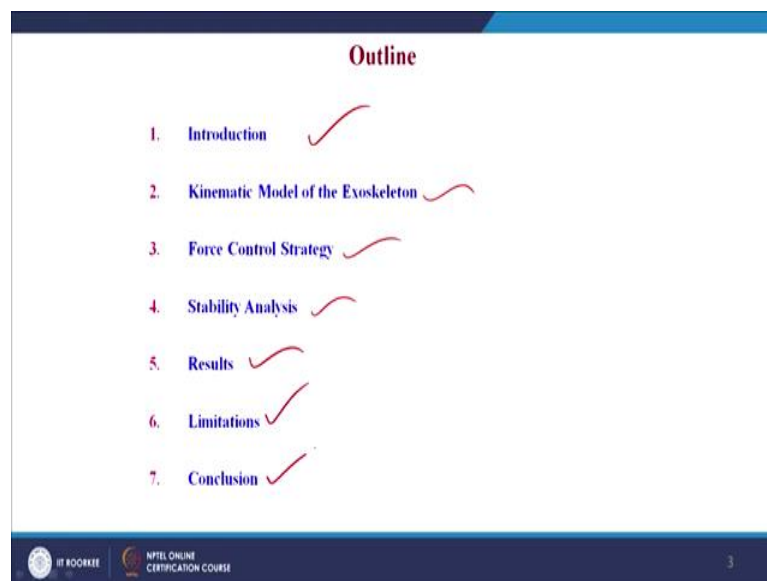


Robotics and Control: Theory and Practice
Prof. Felix Orlando
Department of Electrical Engineering
Indian Institute of Technology, Roorkee

Lecture – 28
Force Control of an Index Finger Exoskeleton

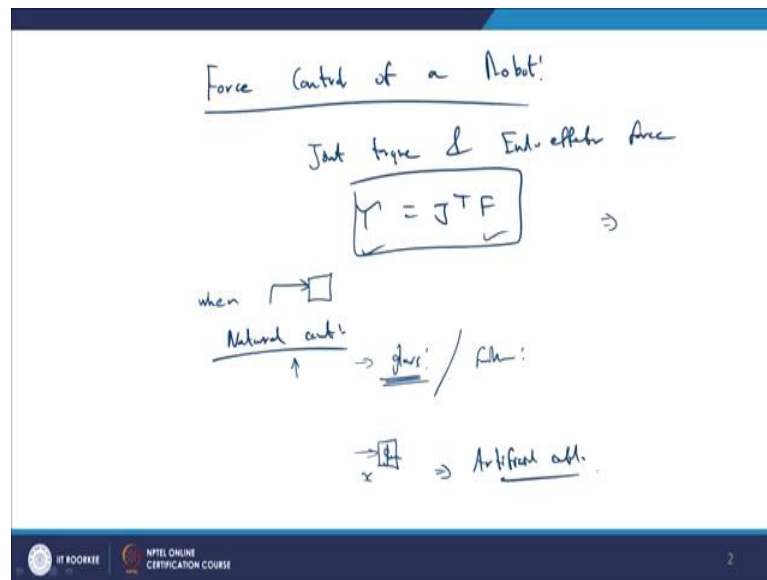
Good morning today we see the lecture on the Force Control of an Index Finger Exoskeleton. The organization of today's lecture will be as follows.

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First we see the introduction to force control of manipulators, then we see the kinematic model of the proposed index finger exoskeleton. Then we see the proposed force control strategy, then we see the stability analysis based on Lyapunov, then we see the results from both simulation as well as the experiment. Then we finally go through the limitations associated with this research study and finally we conclude.

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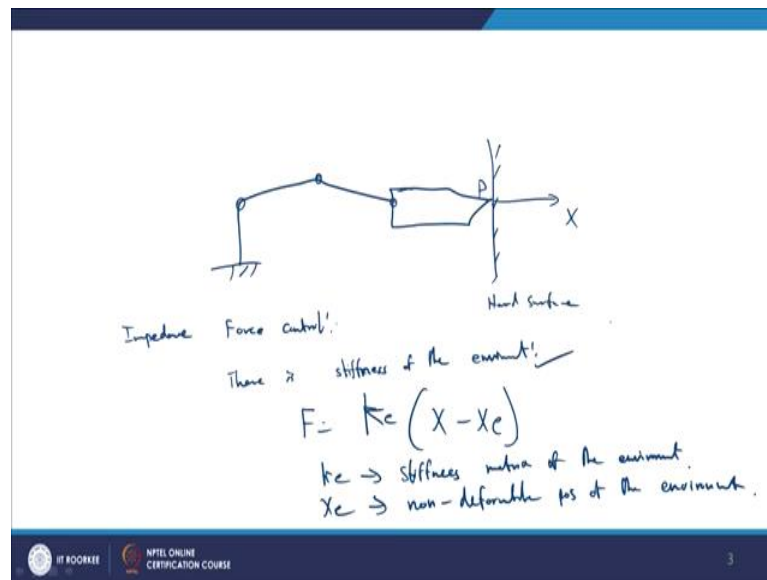
Coming to the introduction. So, force control; so force control of a robotic system. So, before that we should know how the joint torque and the end effectors force. Force are related that is by the static relationship $\tau = J^T F$ we can have this relationship between the joint torque and the end effector force through the Jacobian transpose relationship.

The Jacobian matrix is basically the derivative of derivative matrix. So, that gives the joint torque provided the end effected force is given. Then we see that when we have the robotic system in contact with a glass surface, in order to clean the glass surface instead of position control, because the robot is all already in contact with the glass surface. So, here position control will not be helpful rather force control will be helpful.

And now coming to the artificial or natural constraint the nature will constraint is that. The magnitude of the like for example we cannot apply more force on the glass, because if we apply more force on the glass the glass gets broken. So, that the control becomes failure. So, the contact force must be applied in such a way that that depends on the stiffness of the environment. So, the stiffness of the environment gives you the natural constraint. But at the same time in order to have the glass to be cleaned the direction towards the glass should be controlled.

For example, this is the x direction towards the glass the manipulator has to move in the x direction towards the glass the direction towards the x must be controlled. So, this implies a artificial constraint.

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So, now let us consider a manipulator, where the tool is in contact with the environment which is basically the hard surface X direction, point P is the point of contact and thus the manipulator with its end effector having the tool. So now, when we see through the impedance force control, we observe that there is stiffness of the environment. Through this control what we observe is we have the stiffness of the environment which is used to control the tip on the surface through the force control.

So, here we can see that $F = K_e (X - X_e)$ which is the force acting on the surface by the tool. Where K_e is the stiffness matrix of the environment; stiffness matrix of the environment and X_e is the non-deformable position of the environment and we consider X_d be the desired position trajectory of the end effector and X being the actual position trajectory of the end effector tool.

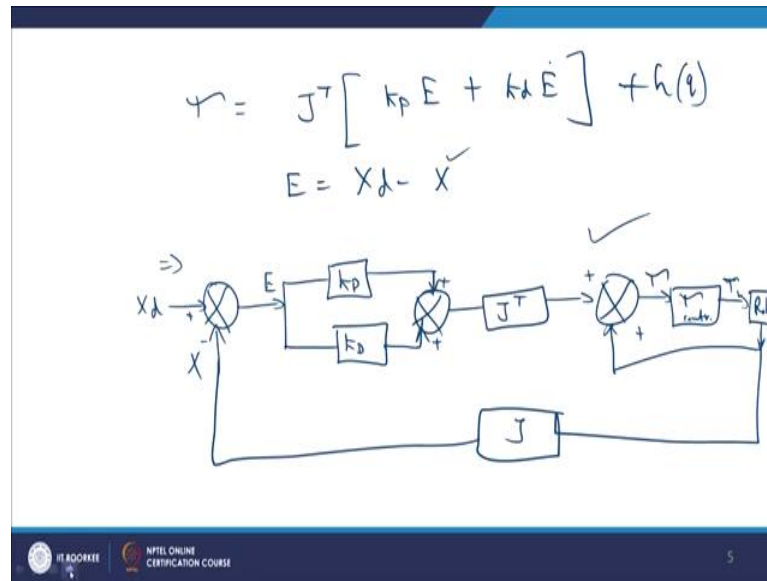
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$X_d \Rightarrow$ des. pos. traj of End. Eff.
 $X \Rightarrow$ Act. pos. traj of End. Eff.
 $\tau \propto \delta q$
 $\tau = J^T K_e J \delta q \Rightarrow \tau = J^T F$
 $\therefore k_q = J^T K_e J$
 $\tau = k_q \delta q$

It means the tool which is in contact with the deformable environment. Then we can have the relationship between the joint torque and the infinitesimal variation of the joint angles that is δq . By the relationship which is $\tau = J^T K_e J \delta q$, that is a relationship that gives between the joint torque and the infinitesimal variation in the joint angle. This is obtained from the relationship $\tau = J^T F$ what we have seen in the first slide that is the static relationship that gives the relationship between the joint torque on the end effector tip force.

Where the Jacobian matrix is here generally 6 cross n degrees of freedom or size matrix which is a Jacobian matrix and the q is the generalized coordinate of the robotic system. Which is a joint angle independent coordinate system and the stiffness matrix here is k_q which is $J^T K_e J$ is the stiffness matrix which is this portion. So, thus τ can be written as $k_q \delta q$.

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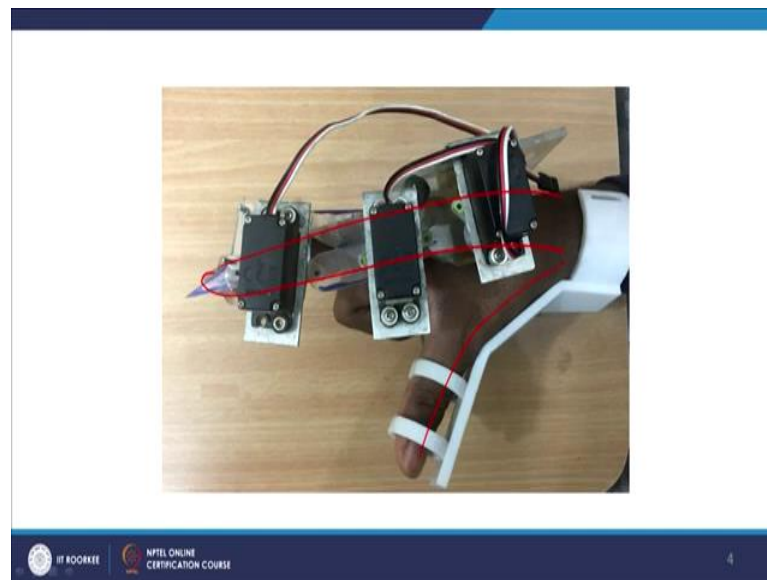


So, when we have the impedance control implemented on the manipulator, the control law will take the form $\tau = J^T [k_p E + k_d \dot{E}] + G(q)$. Where the error E is given by

$E = X_d - X$, where X_d represents the desired trajectory of the end effector and X is the actual trajectory of the end effector through the deformable environment.

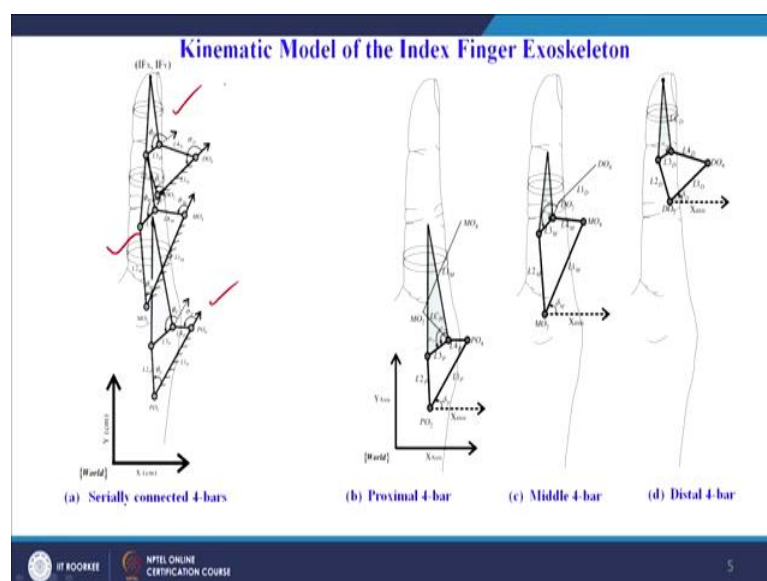
Thus the block diagram pertaining to this control law is given by desire X actual k_p, k_d multiplied by the J^T summed up here to give the joint torque, that goes to torque controller from that we get the torque actual. That goes to the robotic system that gives the joint angle that is fed back here just also taken back to compute the Jacobian then it is fed back as an actual trajectory. So, this is the control strategy block diagram for impedance force control of a generalized to n degrees of freedom manipulator.

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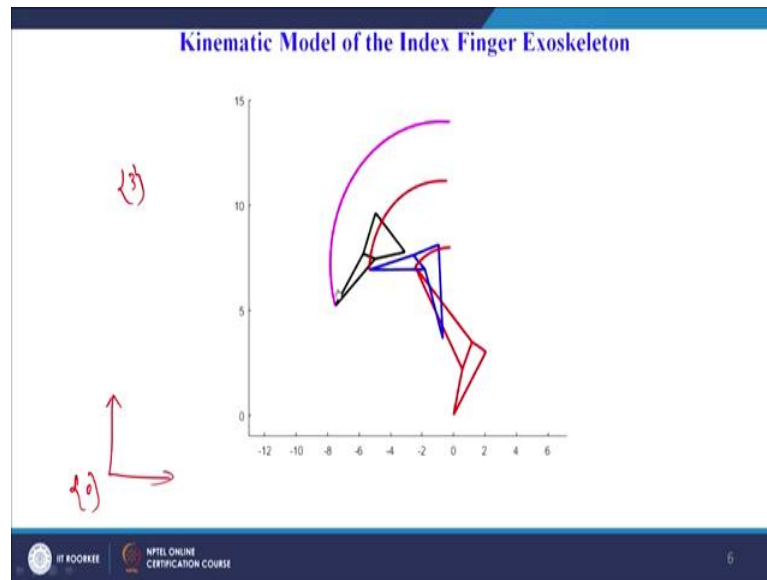
Now, we come to our exoskeleton. So, this exoskeleton is basically the thumb portion is fixed. Whereas, the index finger exoskeleton is a one own by the subject human subject. It has three degrees of freedom as I mentioned in the previous lectures, it has three degrees of freedom; one for the flexion extension motion of the MCP joint and the second is a flexion extension motion of the PIP joint and finally the third one is the flexion extension motion of the distal interphalangeal joint of the index finger. So, this in this figure can provide the motion in three degrees of freedom fashion of the flexion extension movement of the index finger joints independently.

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The kinematic model of the index finger exoskeleton shown here with the perspective that 3 robotic 4 bar mechanisms are serially connected. First one for the proximal phalanx, the second one 4 bar for the middle phalanx and the third 4 bar for the distal phalanx connected serially. So, that they form the index finger exoskeleton for the moment of the index finger.

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The kinematic modeling is observed in such a way that we have simulated. So, that the serial attachment by the former kinematic equation is providing the moment in such a way that this type of simulative trajectory can be possible. As you can see that the magenta line shows the trajectory of the end effector and the red line shows the trajectory of the middle phalanx 4 bar and also the second red line shows the territory of the proximal phalanx of the index finger part of the exoskeleton.

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Kinematic Model of the Index Finger Exoskeleton (cont'd)

$$\begin{aligned}
 X_{IF} = & \cos(\delta_M) \times (L1_M + L4_M \times \cos(\sigma_M) + LC_{MG} \times \cos(\phi_M - \alpha_{MG})) - \sin(\delta_M) \times (L4_M \times \sin(\sigma_M) \\
 & + LC_{MG} \times \sin(\phi_M - \alpha_{MG})) + \cos(\delta_P) \times (L1_P + L4_P \times \cos(\sigma_P) + LC_{PG} \times \cos(\phi_P - \alpha_{PG})) \\
 & - \sin(\delta_P) \times (L4_P \times \sin(\sigma_P) + LC_{PG} \times \sin(\phi_P - \alpha_{PG})) + \cos(\delta_D) \times (L1_D + L4_D \times \cos(\sigma_D) \\
 & + LC_{DG} \times \cos(\phi_D - \alpha_{DG})) - \sin(\delta_D) \times (L4_D \times \sin(\sigma_D) + LC_{DG} \times \sin(\phi_D - \alpha_{DG})) \\
 Y_{IF} = & \sin(\delta_M) \times (L1_M + L4_M \times \cos(\sigma_M) + LC_{MG} \times \cos(\phi_M - \alpha_{MG})) + \cos(\delta_M) \times (L4_M \times \sin(\sigma_M) \\
 & + LC_{MG} \times \sin(\phi_M - \alpha_{MG})) + \sin(\delta_P) \times (L1_P + L4_P \times \cos(\sigma_P) + LC_{PG} \times \cos(\phi_P - \alpha_{PG})) \\
 & + \cos(\delta_P) \times (L4_P \times \sin(\sigma_P) + LC_{PG} \times \sin(\phi_P - \alpha_{PG})) + \sin(\delta_D) \times (L1_D + L4_D \times \cos(\sigma_D) \\
 & + LC_{DG} \times \cos(\phi_D - \alpha_{DG})) + \cos(\delta_D) \times (L4_D \times \sin(\sigma_D) + LC_{DG} \times \sin(\phi_D - \alpha_{DG}))
 \end{aligned}$$

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
Then we come to see the kinematic equations the former kinematic equation defining the end effector tip that is the index finger tip positions X_{IF} and Y_{IF} ; the X_{IF} and Y_{IF} which is the index finger tip trajectory equation. That is the former kinematic equation which is a function of the joint variable θ_p which is for the proximal phalanx joint angle and θ_M which is the middle phalanx joint angle and θ_D which is the generalized coordinate of the distal phalanx 4 bar.

This is this complex equation is obtained through closed loop equation, that starts from the frame which is basically it starts from the base frame and it ended up in the tip frame which is here we started here. But the closed loop equation then go reach here then we reach this coupler, from the coupler we move on to the line which is the base of the middle phalanx 4 bar from that we reach the tip which is a coupler tip of the middle phalanx, from there we reach the distal phalanx tip. That is how through closed loop equation we have obtained the tip position of the index finger exoskeleton.

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Force Control Strategy

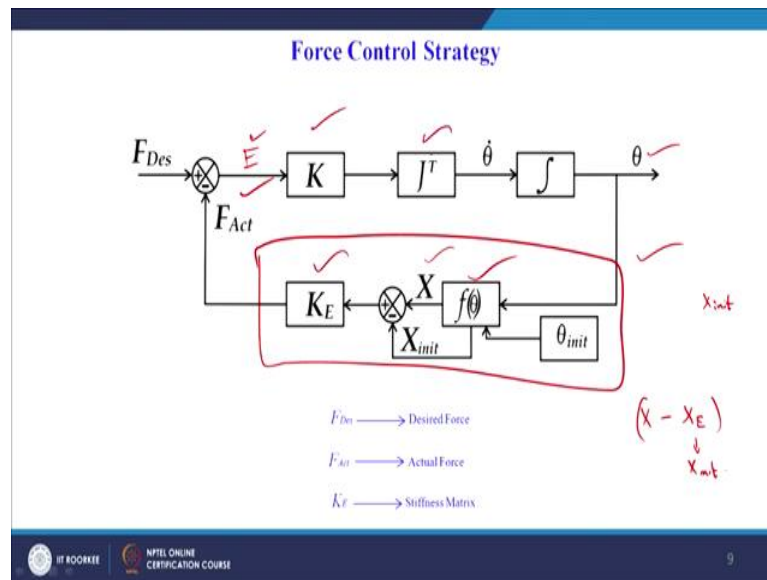
- ❖ To derive the force control law based on the Jacobian transpose method, ideal dynamics of index finger exoskeleton is assumed.
- ❖ Hence, the joint torque is given by
$$\tau = \dot{\theta}$$
- ❖ Thus the update law ✓
$$\dot{\theta} = J^T K E$$
- ❖ Force KE to regulated the tip of the exoskeleton towards the desired force.

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Now, coming to the force control strategy associated or proposed in this study is that transpose of the Jacobian method is utilized. So, that the ideal dynamics of the index finger exoskeleton is assumed. Hence the joint torque by the ideal dynamics is given by tau equal to theta dot, with the update law being $\tau = \dot{\theta} = J^T KE$. Thus the force which is given by the product of K with the error is used as a regulating force for the tip towards the desired force.

The term K multiplied by E is the one which regulates the tip towards the desired given force, with the idea of ideal dynamics of the exoskeleton which leads to the joint torque being $\dot{\theta}$.

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So, the block diagram associated with the proposed control strategy of the force is given by, given the desired force may be a scalar or a desired trajectory it works for both. So, we considered first with the scalar that is a set point, then we can also implement a desired trajectory. So, given the desired force it is multiplied with the gain matrix, so that Jacobian transpose comes into picture.

So, that we see here this is E multiplied by K multiplied by the J^T which is the Jacobian transpose that leads to the $\dot{\theta}$. So, that the joint velocity is integrated by Runge Kutta approach which is the fourth order method to get the joint variable theta. So, that through forward kinematics we obtain the actual position of the end effector of the index finger exoskeleton with the deformed initial position being also obtained with the initial angle given to the power kinematic model $F(\theta)$.

Then that product with the summation X and \dot{X} my X_{init} minus with the product that the K_E , which is a stiffness matrix of the environment will give the force actual that is the actual force because this is a modeling portion. Because we do not have the sensor it is through simulation this approach can be used because the model which is the force for the kinematic model can be utilized to obtain the force actual through the stiffness matrix and the distance or the discrepancy between the actual position of the index finger tip as well as the undeformed position of the index finger tip that is X_E is here is X_{init} , through that we obtain the F actual.

So, the difference or the discrepancy between desired and the actual force value will give the error, again that is multiplied with the control gain. Then it is multiplied further with the Jacobian transpose in order to obtain the data velocity which is given by this expression this update control law.

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Stability Analysis

The Lyapunov function is taken as,

$$V(E) = \frac{1}{2} E^T K E \quad V(E) > 0 \quad \forall E \neq 0 \quad V(0) = 0$$

By Differentiating,

$$\begin{aligned} \dot{V} &= E^T K \dot{E} \\ &= E^T K (\dot{F}_{des} - \dot{F}) \\ &= E^T K \dot{F}_{des} - E^T K K_E (\dot{X} - \dot{X}_{init}) \\ &= E^T K \dot{F}_{des} - E^T K \dot{K}_E \dot{X} + E^T K K_E \dot{X}_{init} \end{aligned}$$

Where,

$$F = K_E (X - X_{init}) \text{ and } \dot{F} = K_E (\dot{X} - \dot{X}_{init})$$

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So, now coming to the stability analysis the Lyapunov function stability analysis is proved here, in order to show that the convergence error convergences happened asymptotically.

So, the function V the Lyapunov candidate is taken as a function of error which is given by $V(E) = \frac{1}{2} E^T K E$

Where $V(E)$ is positive definite and we have is $V(0) = 0$ in the beginning and by differentiating we get $\dot{V} = E^T K \dot{F}_{des} - E^T K K_E (\dot{X} - \dot{X}_{init})$.

Where K is the control gain and K_E is the stiffness, where F is given by $K(X - X_{init})$ and $\dot{F} = K_E (\dot{X} - \dot{X}_{init})$.

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Stability Analysis (cont'd)

Again,

$$\dot{V} = E^T K \dot{F}_{des} - E^T K K_E J(\theta) \dot{\theta} + E^T K K_E J(\theta) \dot{\theta}_{init}$$

Substituting the joint velocity control law

$$\dot{\theta} = -E J^T$$

$$\dot{V} = E^T K \dot{F}_{des} - E^T K K_E J(\theta) J^T(\theta) K E + E^T K K_E J(\theta_{init}) J^T(\theta_{init}) K E$$

Assuming, $\dot{F}_{des} = 0$, $\dot{X}_{init} = 0$,

So, from the above assumption, $\dot{V} = -E^T K K_E J(\theta) J^T(\theta) K E$

Therefore, $V > 0$ and $\dot{V} < 0$.

The error converges to zero so, the system is asymptotically stable.

Again we have

$$\dot{V} = E^T K \dot{F}_{des} - E^T K K_E J(\theta) \dot{\theta} + E^T K K_E J(\theta) \dot{\theta}_{init}$$

Because $\dot{X} = J\dot{\theta}$ that is why we replace the $\dot{X} = J\dot{\theta}$. Similarly, with the second term also with the third term also we have replaced the tip velocity by the Jacobian multiplied by the joint angular velocity. Substituting the joint velocity, the control law we have obtained

$$\dot{V} = E^T K \dot{F}_{des} - E^T K K_E J(\theta) J^T(\theta) K E + E^T K K_E J(\theta_{init}) J^T(\theta_{init}) K E$$

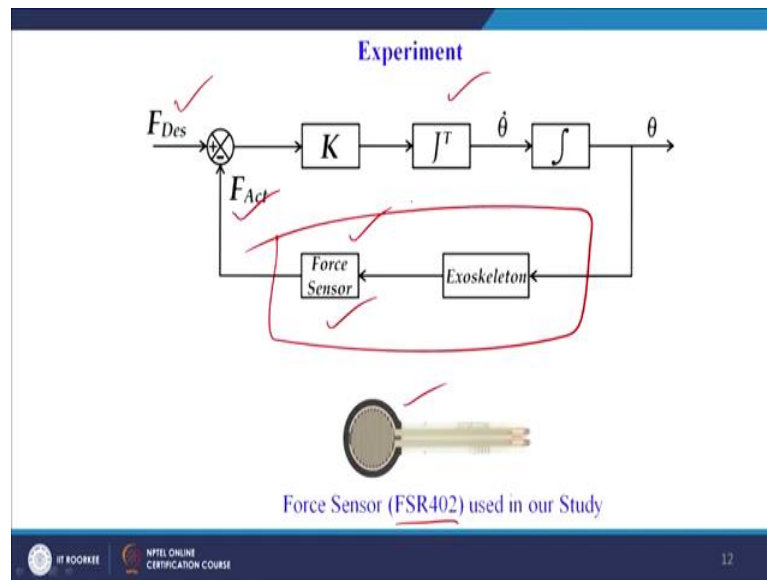
So, here we obtain we substitute the control law which is $\dot{\theta} = J^T(\theta) K E$. So, this controller as I show here this control law $J^T(\theta) K E$ is implemented here in order to obtain this equation. Assuming now \dot{F}_{des} because F is a desired velocity which we desired force. So, the velocity is 0 here and initially they are 0 \dot{F}_{des} and \dot{x}_{init} or 0.

So, from the above assumption we have

$$\dot{V} = -E^T K K_E J(\theta) J^T(\theta) K E$$

which is less than equal to 0. Thus we prove that for the positive definite Lyapunov candidate V greater than 0, we have \dot{V} less than 0. Thus implies that the error converges to 0 asymptotically which implies that the system is stable asymptotically.

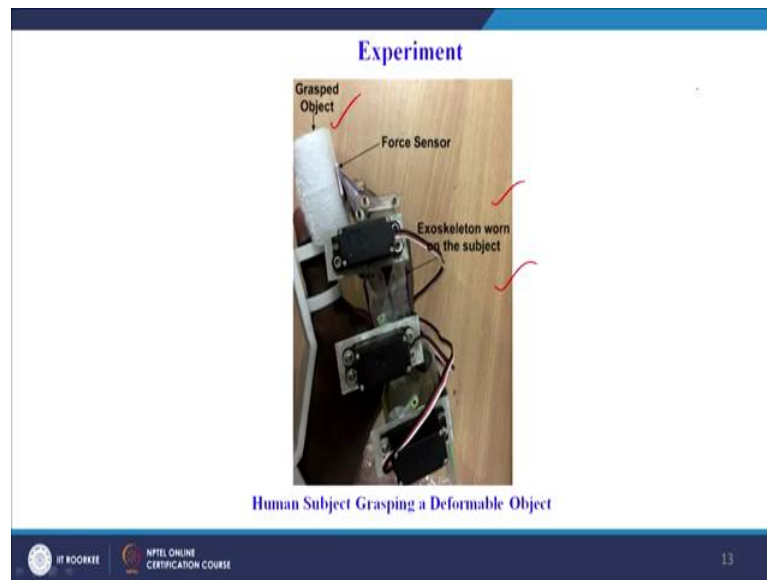
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So, we have performed the experiment by having the desired force given, with the actual force obtained through these sensor feedbacks. Where the sensor is a force sensor which is basically FSR 402, sensor which is giving the force value based on the resistance value change this is the force sensor which is attached to the body surface.

So, that the finger pad will be pressing it with the desired force applied by the exoskeleton master. Here the human finger acts as a slave. So, the block diagram for the experimental study is given us where the model portion has been replaced by the actual exoskeleton with the actual force getting obtained from the force sensor attached to the desired deformable object to be grasped.

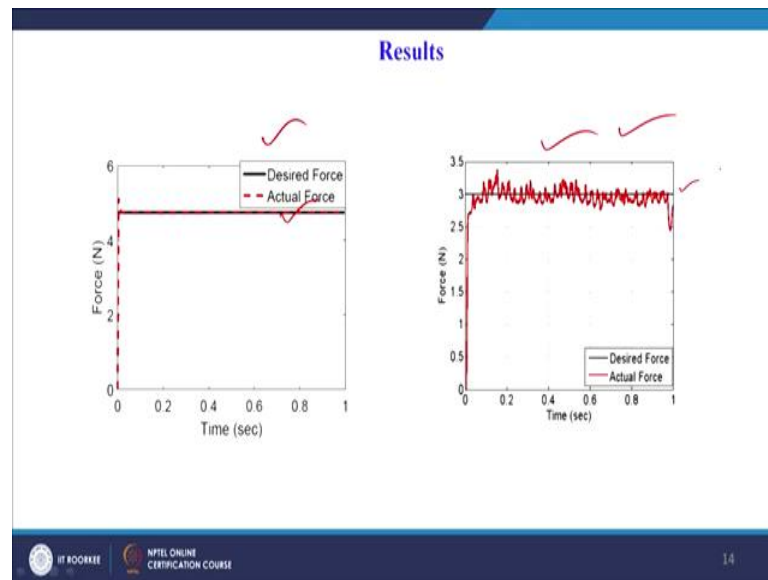
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Schematic shows that experimental model or experimental exoskeleton worn on the human subject grasping a deformable object. Where the force sensor is attached to the surface lateral surface of the body of interest to be grabbed and the exoskeleton master may the slave human subject finger in order to grab the object with the desired force of interest.

So, this is the experimental set up showing how the exoskeleton is helpful in or aiding the patient or the human subject in order to grab an object with their desired force. Because the exoskeleton acts here as physiotherapist in order to make the impaired or the non grasping hand to grasp an object with the desired force of interest.

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We see the results here from the simulation study also through the experimental study. Even the force of closely to 5 Newton the desired trajectory is obtained so fast in order to trace on the system stays here in the convergent position for a time of one second and similarly this is observed with the sensory force feedback trajectory and hence we have noise associated with this sensory feedback. So, given the 3 Newton desired force we have obtained the actual trajectory coming and setting settling in the desired setting point of 3 Newton.

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Results

Time domain specification for simulation

S. No.	Parameters	Values
1.	Rise time (sec)	0.0037
2.	Peak time (sec)	0.0050
3.	Settling time (sec)	0.0121
4.	Maximum overshoot (%)	8.2053
5.	Peak value (N)	5.1263

Time domain specification for experiment

S. No.	Parameters	Values
1.	Rise time (sec)	0.0046
2.	Peak time (sec)	0.1531
3.	Settling time (sec)	0.9972
4.	Maximum overshoot (%)	16.1111
5.	Peak value (N)	3.3768

Then the time domain specifications for the simulation and the experimental study are as follows, the rise time being 0.037 second and the peak time being 0.05 second and settling time is 0.01 second the maximum peak overshoot is 8.2 percentage and the peak value of the forces 5.12 in the simulation. And similarly for the experiment the peak value is 3.37 Newton and the maximum peak overshoot is 16 percentages, where the settling time is 0.9 approximately 1 second.

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The slide is titled "Limitations" in blue. It contains three bullet points with handwritten checkmarks and a graph. The first bullet point is "❖ Only static force analysis, Dynamics?" with a checkmark. The second is "❖ Robustness?" with a checkmark. The third is "❖ Statistical analysis with rehabilitation paradigms?" with a checkmark. To the right of the first two bullet points, there is a handwritten equation: $\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q)$. Below this, there is a boxed equation: $\tau = \dot{q}$. To the right of the boxed equation, there is a graph showing error e on the vertical axis and time t on the horizontal axis. The graph shows a curve starting at a positive value and decaying exponentially towards zero.

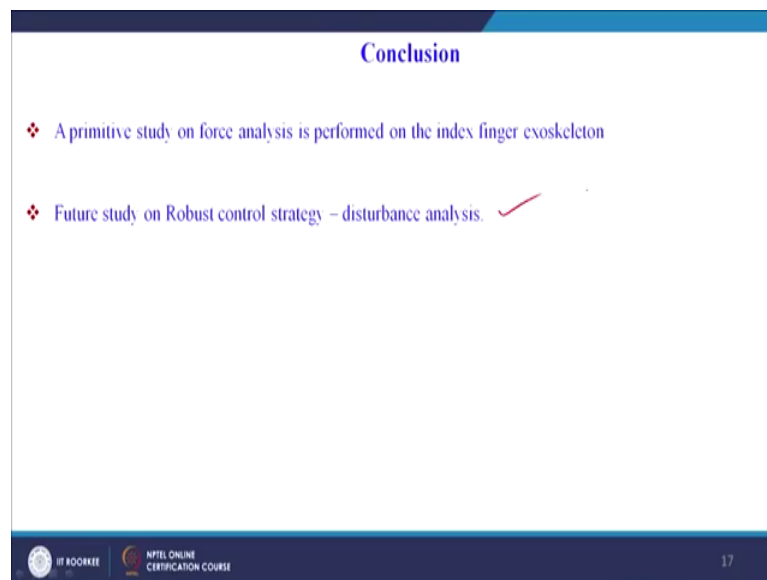
Now, coming to the overall limitations associated with our external study is as follows, there are as follows one first one is we have considered only the static force analysis. But actually we need to consider the dynamics because the dynamics is given by highly coupled equation which is τ equal to mass plus, because we need to consider the inertia matrix and the Coriolis matrix and the gravity term. In order to obtain the joint torque, but what we have considered through ideal dynamic is \dot{q} .

So, with this we have defined our system control strategy, whereas actually the coupled highly coupled dynamic equation must be considered with the inertia matrix Coriolis matrix and the graph gravity term coming into picture, that must be giving the real study. And we have confirmed in our study that the error gets converged to 0 asymptotically. As time turns to 0 the error becomes 0 that is what we have observed through the Lyapunov stability.

But practically what is observed what is required from the study is robustness. Once the disturbance is given because the human subject where we target the usage of this exoskeleton designed will be for the patients who have not used or who are not grabbed an object due to impairment of their hand will have a rigidity. So, that rigidity will makes will provide more impedance while grasping an object. So, that impedance will be the disturbance to be considered here and how the control strategy is going to be robust towards the disturbance applied.

So, that must be the future study immediate study for our such work and also we must be doing statistical analysis with the current rehabilitation paradigms. And coming to the conclusion here we have started this lecture with the basics of force control of n degrees of freedom robot manipulator. Then we have implemented our proposed control strategy of force with the concept of ideal dynamics or with the assumption of ideal dynamics of the proposed exoskeleton, in order to perform the grasping task. And both simulation and experimental study has been performed in order to see how the grasping has been done with the desired force input.

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And our future study will be focusing fully on the robust control strategy of the exoskeleton in grasping an object with both multi step input and also the time varying continuous or discontinuous input variations of the desired force along with the disturbance analysis.

Thank you so much.