Robotics and Control: Theory and Practice
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Lecture - 23
Fundamental of Robot Manipulability

Good Morning. Today we are going to see about the Fundamentals of Robot Manipulability. The outline of today's lecture will be as follows. First, we see the definition of manipulability ellipsoid and the manipulability measure.

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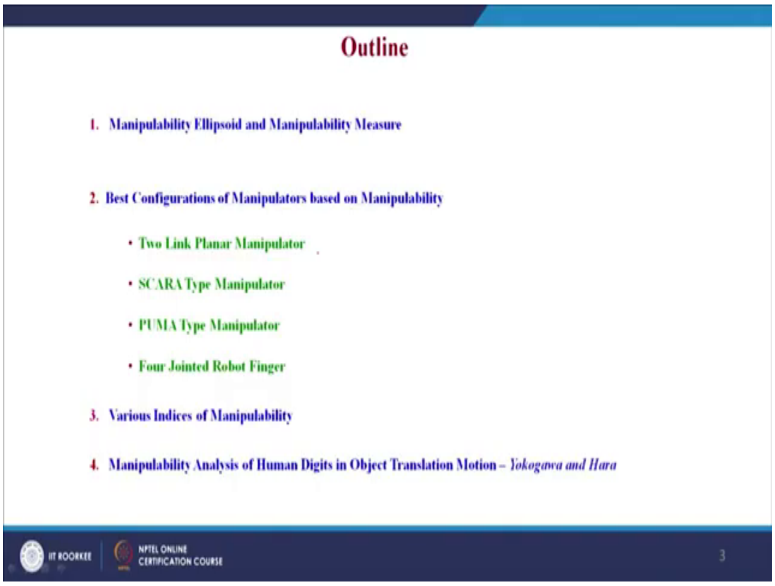

Then we see the best configurations of manipulators based on manipulability index. In that we see the examples: two link planar manipulator, SCARA type manipulator, PUMA manipulator, and the fourth jointed robotic finger. Then we see the various indices of manipulability measure. And then finally, the research work done by Yokogawa and Hara in 2004 for Object Translation Motion the Manipulability Analysis of the Human Thumb and Index Finger.

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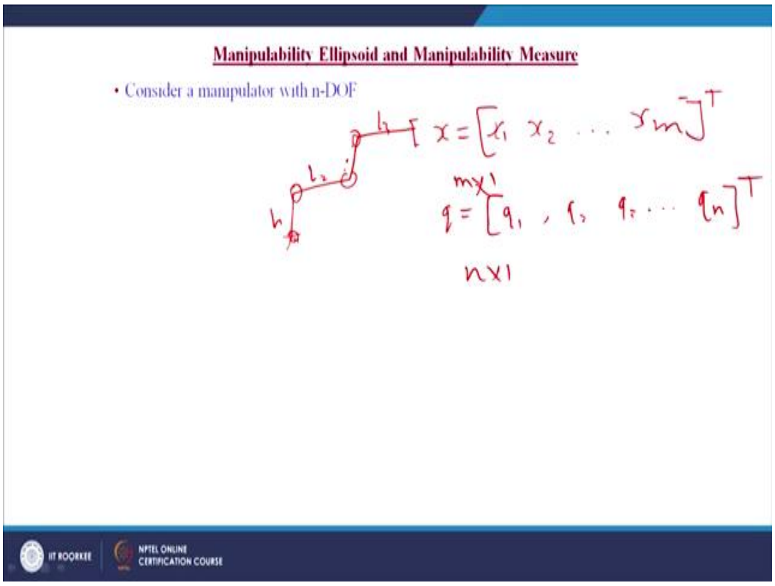

First coming to the manipulability ellipsoid definition: first, we see an n-degrees of freedom manipulator given by its n-degrees of freedom manipulator where the links in say L_1, L_2 and L_3 up to L_n. We can see that the joint angles are q_1, q_2, q_3 and it goes. So, that the end of the vector is given by x, which is equal to x_1, x_2 up to x_m transpose. Similarly, the joint angle vector is given by q equal to q_1, q_2, q_3 up to q_n transpose.

So, the size of the end effector position is $m \times 1$, whereas the size of the joint space is $n \times 1$.

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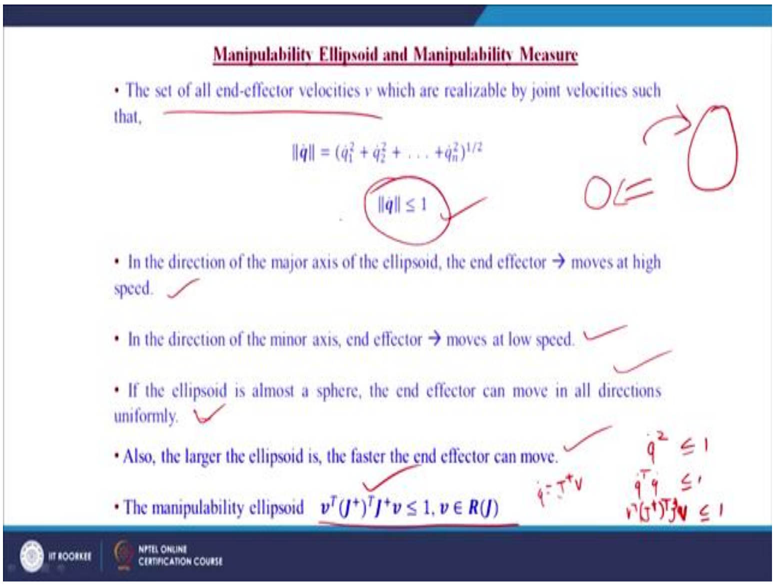

So, for this situation when we see the definition of manipulability ellipsoid it is given by the definition: the set of all end effector velocities v, which are realizable by the joint angular velocity such that; the equilibrium norm of the joint angular velocity is less than equal to 1. That is, the end effector velocity will lead to an ellipsoid, whereas the joint angular velocities are confined by a spear. In the joint space they are spear whereas, in the Cartesian space it is an ellipsoid.

Now in that what is observed or what is inferred is, in the direction of the major axis of the ellipsoid the end effector that is the tip of the manipulator moves at high speed whereas, in the direction of the minor axis of the ellipsoid the end of term moves at lesser speed. If the end effector is almost a spear, if the ellipsoid is almost a spear then the end effector that is a tip of the manipulator can move in all directions uniformly. Also the larger the ellipsoid is the faster the end effector can move. Now, the manipulability ellipsoid is given by

$$v^T (J^+)^T J^+ v \leq 1, v \in R(J)$$

How it is obtained: $\dot{q}^2 \leq 1$ which is $\dot{q}^T \dot{q} \leq 1$, whereas $\dot{q}$ is given by $J^+ v$. So, $\dot{q}^T$ is $v^T (J^+)^T J^+ v$, ok. So, this is less than equal to 1, this is v. So, that is what we have seen here. This is the expression for manipulate ellipsoid which is less than equal to 1. That means, the joint angular velocities are confined to be within spherical shape.

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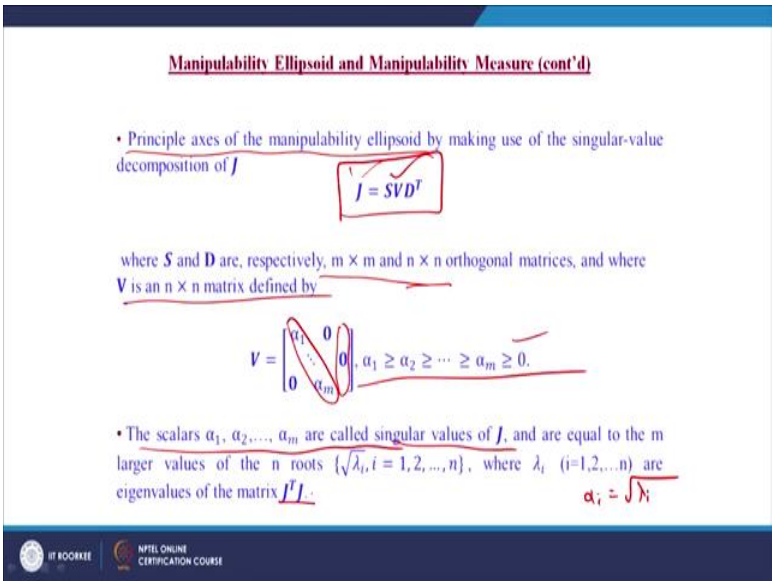

Now, coming to the principal axis of the manipulability ellipsoid:- they are found by decomposing the Jacobian matrix J using singular value decomposition. So, by decomposing we get $J = (SVD)^T$: the Jacobian matrix J is decomposed into 3 matrices S, V and D. Where, S and D are respectively an $n \times m$, $m \times m$ and $n \times n$ orthogonal matrices, whereas the matrix V is an $n \times n$ matrix defined by a diagonal singular values rest all the elements are 0, where the last row is a 0 vector. Where the singular values $\alpha_1 \geq \alpha_2$, which is greater than equal to α_m which is greater than equal to 0.

The scalars α_1, α_2 and up to α_m are called the singular values of the Jacobian J and are equal to the m larger values of the end roots. That is they are equal to $\alpha_i = \sqrt{\lambda_i}$; for λ_i is the scalar which is the eigenvalue of the matrix $J^T J$.

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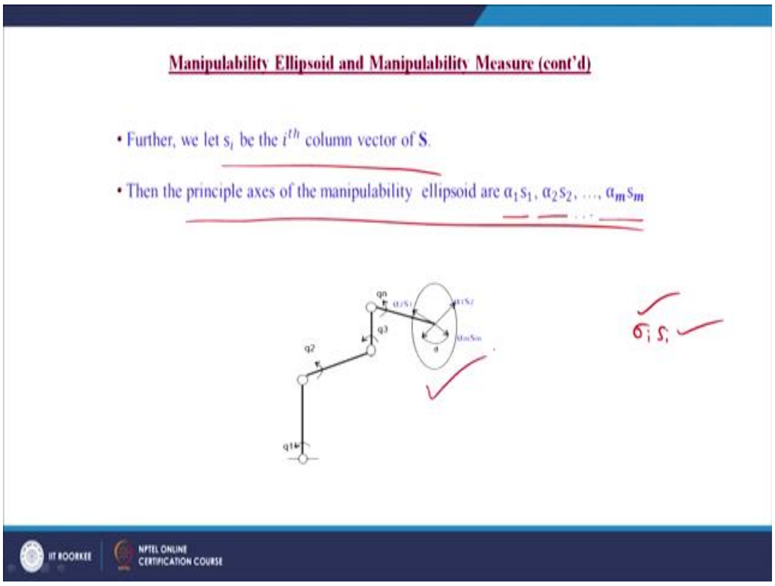

Further, let s_i be the i-th column of the vector s or the matrix s. Then the principal axes of the manipulability ellipsoid is given by $\sigma_i s_i$. Where, σ_i is the radius along the principle axis and s_i is the direction of that principle axis. And hence, with this the principle axis of the entire manipulability ellipsoid is given by $\alpha_1 s_1$, $\alpha_2 s_2$ up to $\alpha_m s_m$ which is what they shown in this schematic.

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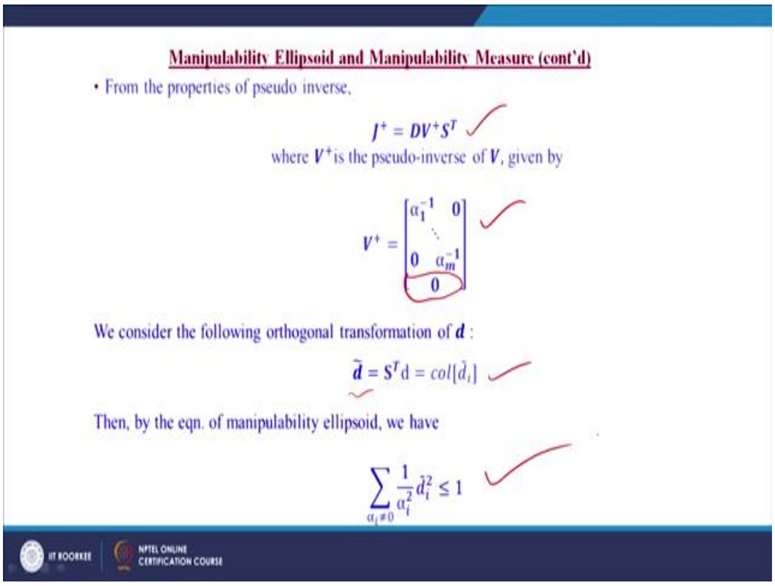

From the properties of pseudo inverse we say that $J^+ = DV^+S^T$, Where, V^+ is the pseudo inverse of V given by this matrix where we have the diagonal elements being the inverse of all the singular values ranging from α_1 to α_m inverse, where the last row is a 0 vector for the V pseudo inverse. We consider the following orthogonal transformation of d such that $\tilde{d} = S^T d$ which is equal to the $col(\tilde{d}_i)$. Then by the equation of the manipulability ellipsoid we have $\sum_{a_i \neq 0} \frac{1}{a_i^2} \tilde{d}_i^{\,2} \leq 1$. This is an extra information we obtain from this theory.

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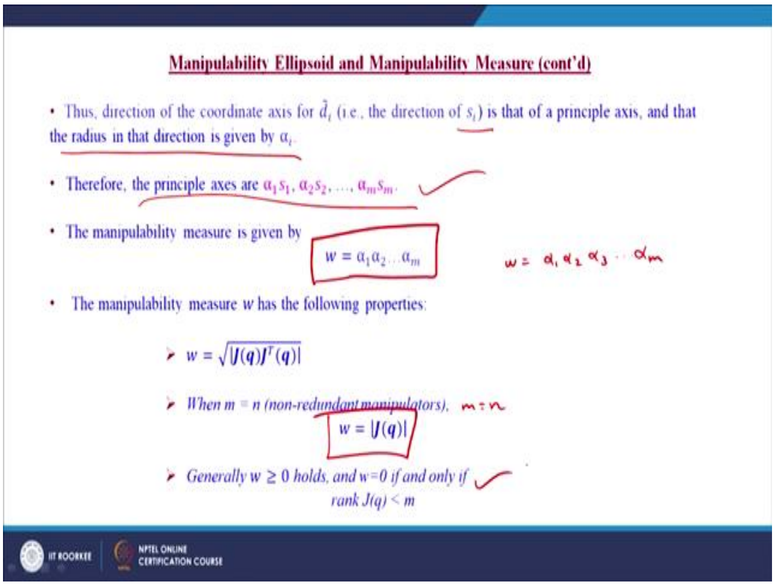

Now, the direction of the principle axes is given by s_i, whereas the radius in that direction is given by α and hence finally, the principle axes again given by $\alpha_1 s_1$, $\alpha_2 s_2$ up to $\alpha_m s_m$. Then the manipulability in terms of the singular values is given by manipulability measure is given by w equal to the product of all the singular values i.e. $w = \alpha_1\alpha_2 \ldots \alpha_m$

The manipulability measure w has the following properties. For the non-redundant manipulator, the manipulator measure is given by the expression $\sqrt{\det(J^T J)}$, of course J is a function of the joint variable q. When it is a non-redundant manipulator; that means, m size of the Cartesian space dimension is equal to the degrees of freedom of the robot manipulator, then we have the manipulability measure given by w equal to determinant of the Jacobian matrix. Generally, w that is a manual to measure greater than equal to 0 holds and if the manipulability measure is 0; that means, the rank of the Jacobian matrix is deficient which leads to the singular configuration.

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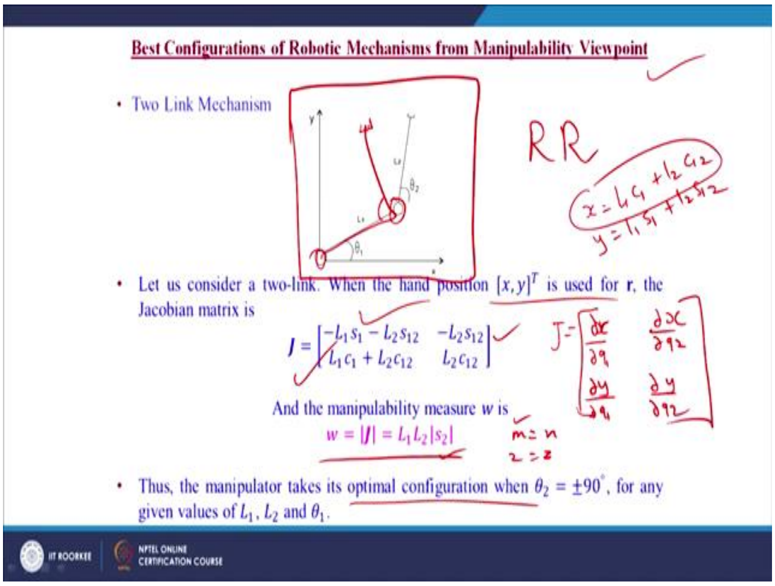

Now coming to the best configurations of robotic mechanisms from the viewpoint of manipulability: first let us consider here two degrees of freedom planar RR manipulator; that is a revolute joint here, revolute joint both the joints are revolute joint and hence joint variable is θ_1 and θ_2 and the link lengths are given here as L_1 and L_2 . So, let us consider this manipulator to link RR planar manipulator when the position is x and y for the end effector, which is used to for R the Jacobian matrix is given by J equal to which is shown in $-L_1 s_1 - L_2 s_{12}$ and the second element of the first row is $-L_2 s_{12}$. How we get this?

This one is obtained by J equal to $\frac{\partial x}{\partial q_1} \quad \frac{\partial x}{\partial q_2}$. Similarly, the second row $\frac{\partial y}{\partial q_1} \quad \frac{\partial y}{\partial q_2}$ this is what. So, if you differentiate the x which is here the x is $L_1 \cos\theta_1 + L_2 \cos\theta_{12}$. Similarly, y equal to $L_1 \sin\theta_1 + L_2 \sin\theta_{12}$. If you differentiate this with respect to joint variable you get this expression similarly you differentiate the x expression with respect to q_2 that minus $L_2 \sin\theta_{12}$ that is what is shown here for the Jacobian matrix. Now the manipulability measure for this planar robot which is a non redundant manipulator, where m equal to n that is the size of the dimension of the Cartesian space is x and y which is equal to the dimension of the joint space which is the degrees of freedom of the robotic system. So, that is why it is an ordered in manipulated in hence the manipulability measure is given by w equal to determinant of J, which is $L_1L_2|\sin\theta_2|$.

So, what is observed here is: the manipulator takes its optimal configuration when θ_2 is equal to $\pm$ 90 degree for any given values of L_1 L_2 and θ_1. That is, when this joint angle is 90 that is this angle is 90, then we have the manipulability measure being very high. What is manipulability? The manipulating ability or the manipulating capability of the manipulator is very high.

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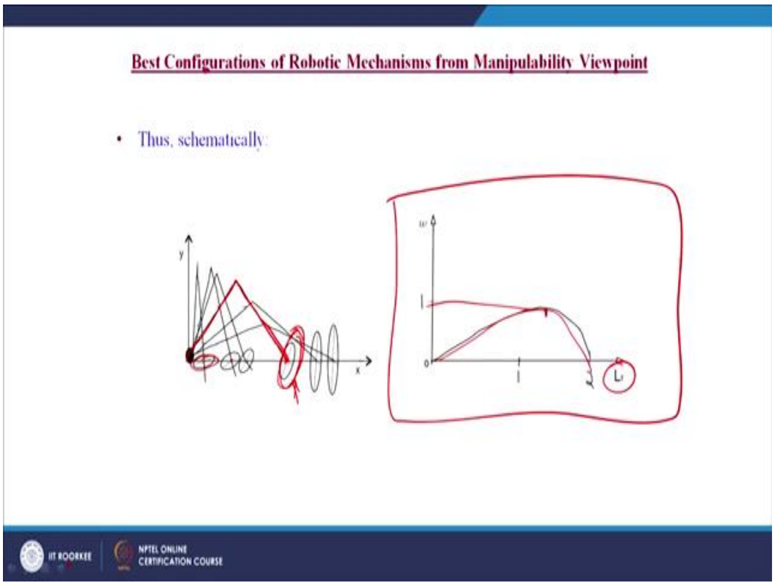

Graphically what we observed is: the manipulability measure value takes from 0 to 1. And it is coming with this profile with this configuration having the ellipsoid being maximum volume, or the area of the ellipse is very high with this corresponding posture of the tip of the manipulator. The x axis is the tip of the manipulator and the y axis is the manipulability measure value. And the configurations corresponding to the tip of the manipulator is shown here at each tip you can see that the posture of the ellipse is varying, also the volume of the ellipse or the area of the ellipses were varying.

So, this shows that in this configuration the ellipse is very high. And also, having larger area ellipse and you can see that this posture and corresponding to this value the manipulability measure is close to 1. That is why this posture is called the optimal posture for this two link planar manipulator.

Similarly, now we go to the next manipulator, ok.

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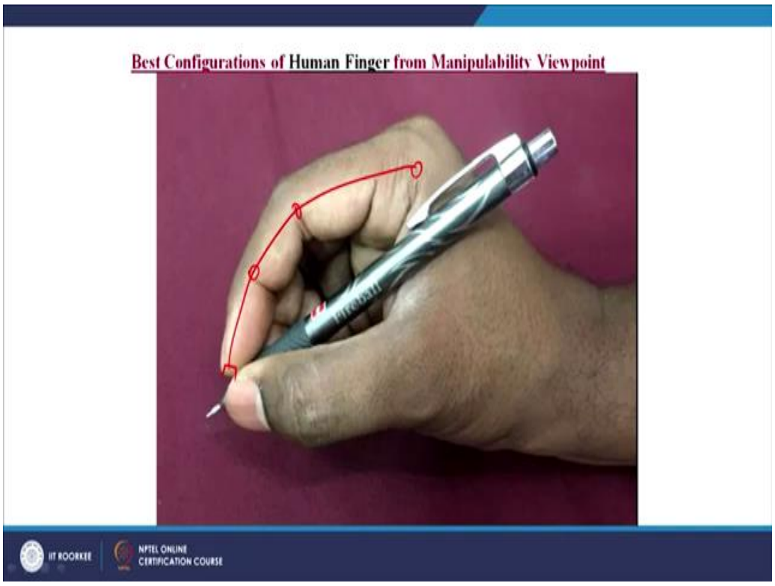

Before that we just to see how the human finger behaves with the manipulability measure concept. So, when we grab a finger; grab a pen as shown in the figure we see that the three linked planar manipulator, where this is the tip. You see that this is the configuration we hold generally for writing posture; where the index finger behaves as a three link planar manipulator will have the optimal configuration here in terms of manipulability measure. We consciously choose this configuration in order to have the writing posture.

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Whereas, we will not choose this configuration which is called an extended configuration, which is almost too close to the singular configuration of the three linked planar manipulator. We will never use this configuration for writing posture.

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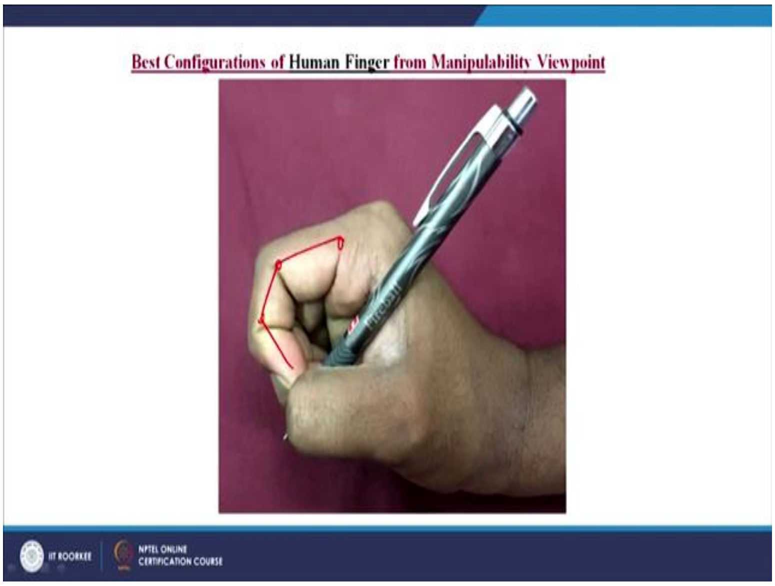

Or this fluxed configuration in writing posture. We never used this configuration rather this configuration, we use this configuration which is the best configuration in the writing posture in terms of manipulability measure.

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Similarly, for the arm that is the arm containing the biceps and the forearm; the upper arm and the forearm being at 90-degree theta to being at 90 degree plus or minus 90 degree. We have the optimal configuration in order to have a writing posture by the arm.

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We never use this configuration either the straight one or it is called a extended or the completely flexed configuration. We never use this configuration or this configuration for the writing posture. This is also optimal configuration, where the humans consciously take while before writing.

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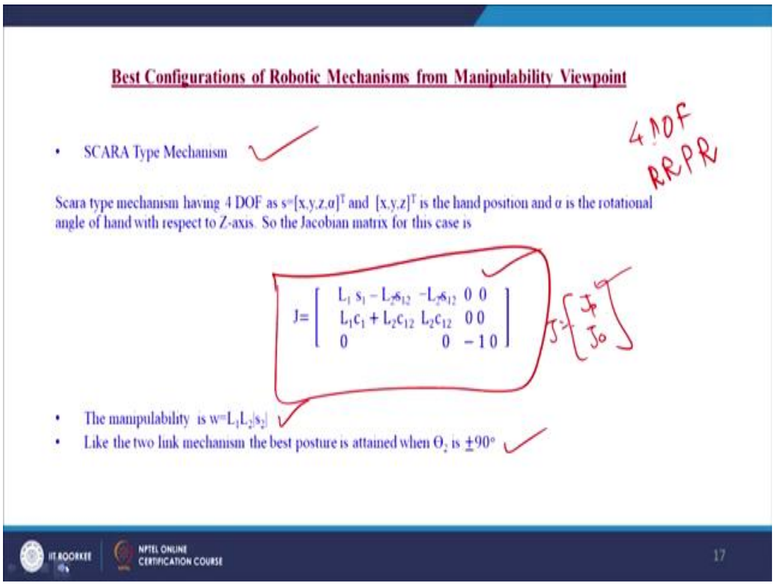

And now coming to the best configuration of the robotic mechanisms, example is the SCARA manipulators. SCARA manipulator is the one which has four degrees of freedom, and the configuration is such that it has first two joints revolute, and the third joint being prismatic, and the fourth being revolute joint. So, the manipulability measure is given by the Jacobian matrix is given by this one which is of size we take only the position of the tip, that is J_p basically it is; where J is constituting J_p and J_o. It can be divided like this that is the first portion of the Jacobian matrix is for linear velocity and the last three portion are meant for orientation.

So, we just take the position part of the Jacobian matrix and the manipulability here is given by again $L_2 L_2 \sin\theta_2$. Again the best configuration of this manipulator is at θ_2 plus or minus 90 degree.

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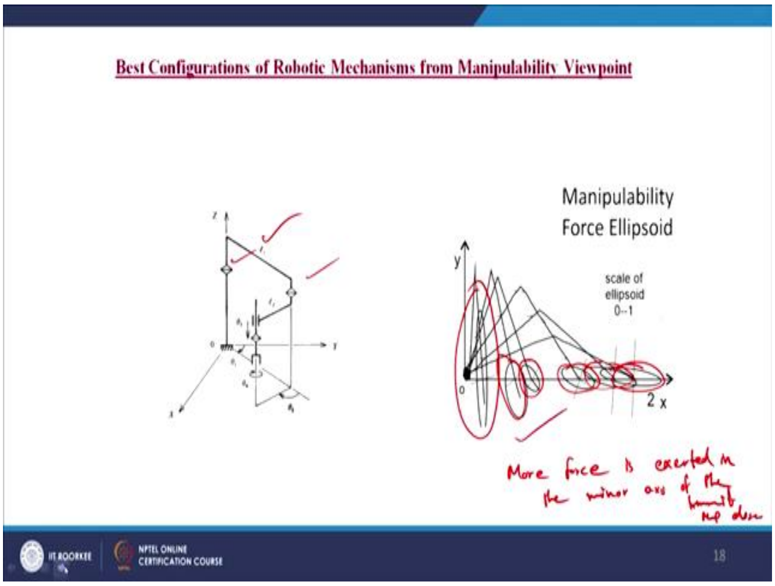

Schematically it is shown here this a SCARA manipulator. The first two joints RR which is same as the planar manipulator. And here you can see that the force manipulability ellipsoids are shown here.

So, force manipulability ellipsoid is such that more force is exerted in the minor axis of the kinematic manipulability ellipse or ellipsoid. That means, force ellipsoids major axis is equal to the minor axis of the kinematic ellipsoid. And the minor axis of the force ellipsoid is equal to the major axis of the kinematic ellipsoid. That is our concept you can understand.

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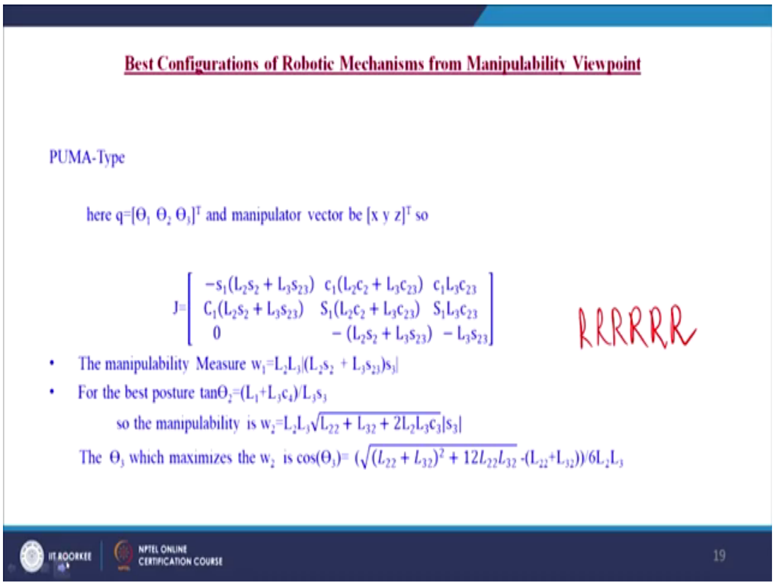

Now, coming to the PUMA type: again, the PUMA type manipulator is the one with the 6 degrees of freedom all being the revolute joints.

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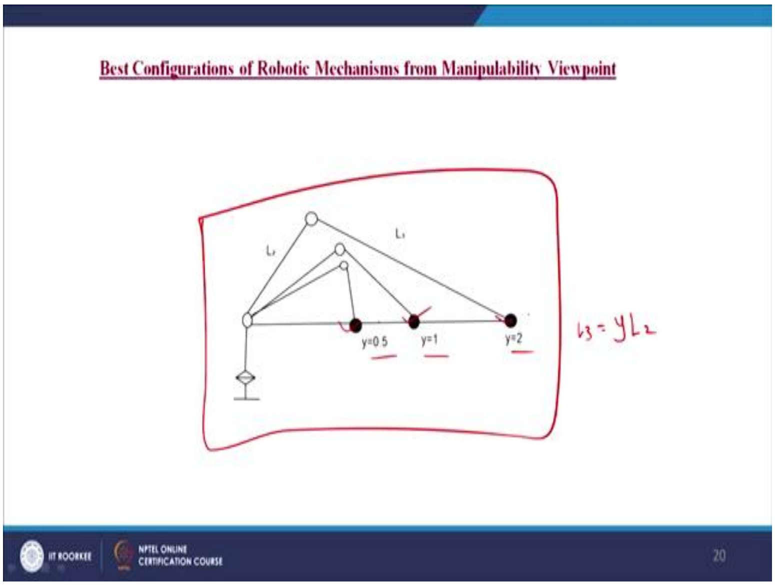

And here you can see that the best configuration of the PUMA manipulator is obtained with this concept, where L_3 is the variable which is given by y into L_2 where y can take 0.51 and 2 for these configurations of L_3. We get the best configuration 1, 2 and 3 configurations based on manipulability measure.

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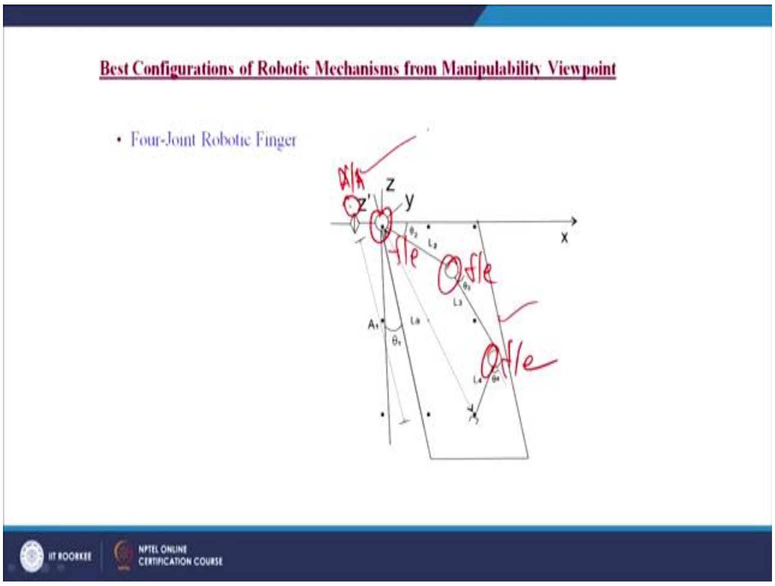

Now coming to the four joint robotic finger. We see that in the joint one which is abduction adduction, and joint two flexion extension, joint three flexion extension, and joint four flexion extension. So, all this friction extension are in a plane and this one is covering the circular trajectory of the tip by this first degrees of freedom.

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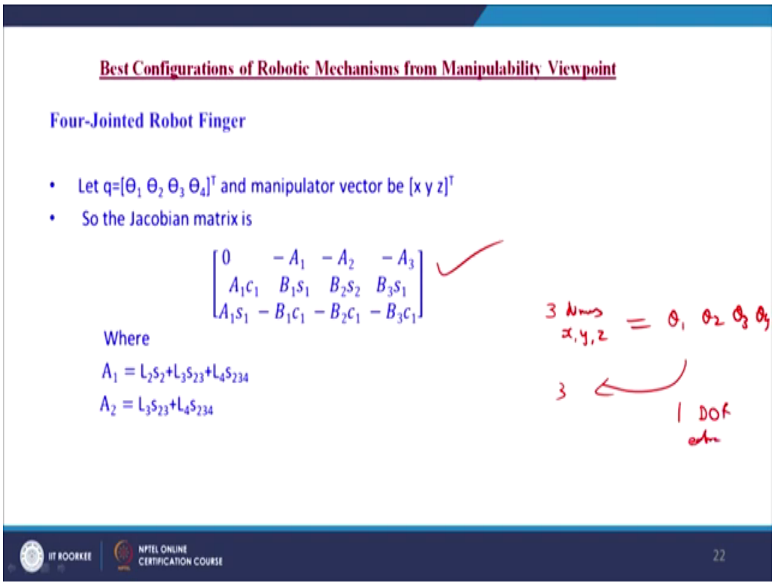

So, in this situation we have the Jacobian matrix given by this expression for that tip or the position of the manipulator. It is a redundant manipulator, because we have three dimensions say x, y and z of the end effect tip will be matched by θ_1, θ_2, θ_3 and θ_4

variables: so 4 to 3. So, 1 degrees of freedom is extra which is why it is called redundant manipulator.

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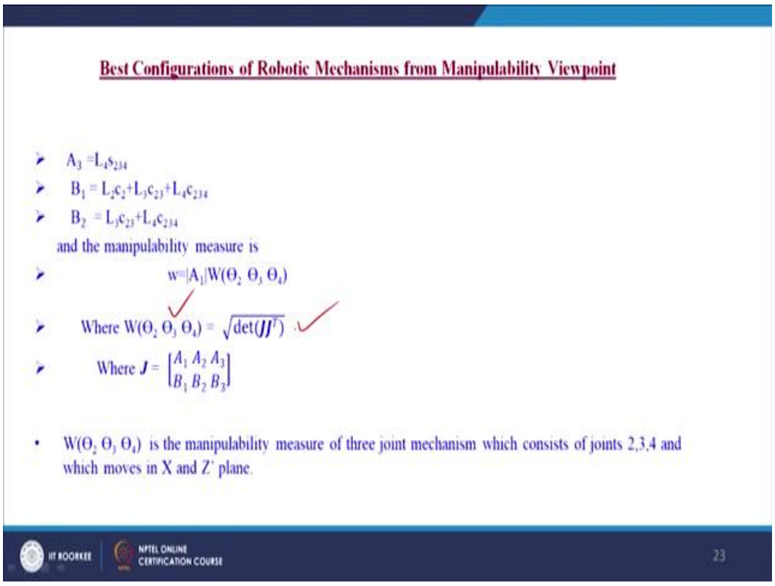

And here the manipulability measure which is a function of $\theta_2\theta_3$ and θ_4 is given by $\sqrt{\det(JJ^T)}$.

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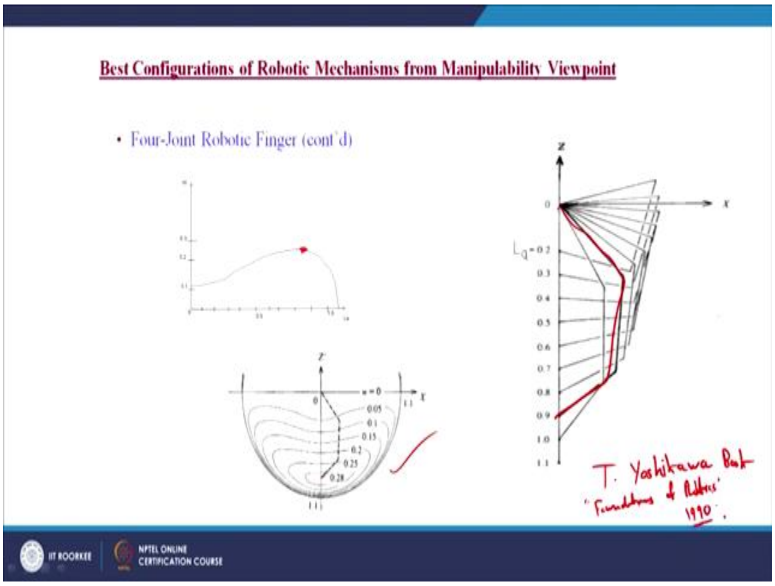

And the best configuration is given by this configuration in terms of the manipulability measure. As you can see this is the posture, where it takes the greater value of manipulability measure which is the value 0.28. And this is the three dimensional view of the ellipsoid these schematics are taken from the reference Tsuneo Yoshikawa book which is called Foundations of Robotics 1990, all right.

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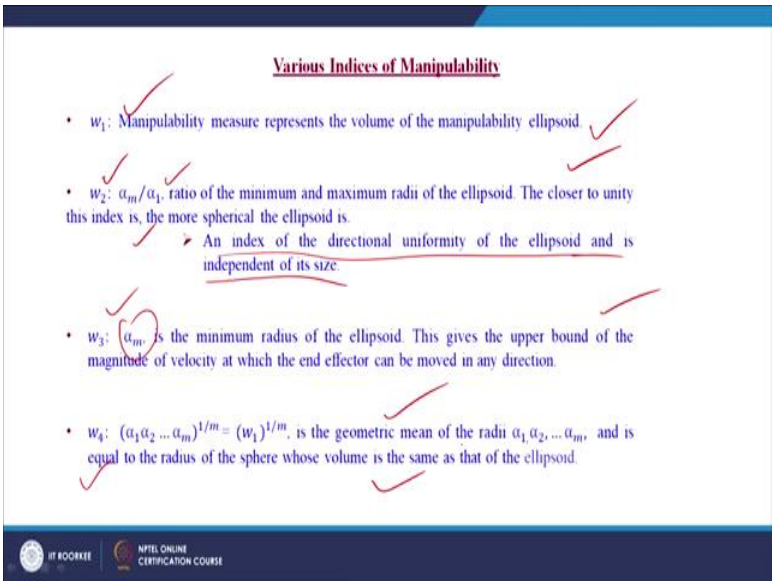

So, now various indices of manipulability we see that: w_1 which is the manipulability measure which represents the volume of the ellipsoid manipulability ellipsoid. And the other index called w_2, which is the ratio of the minimum radius to the maximum radius of the ellipsoid closer to unity this index is. The more spherical the ellipsoid is if the ellipsoid spear then the tip that is end effector can move in all directions uniformly. And if the ellipsoid is a bigger one it can move at a greater speed. An index of this directional uniformity of the ellipsoid and is independent of its size.

And coming to the other index w_3 which is the smallest radius the minimum radius of the ellipsoid, α_m, this gives the upper bound of the magnitude of velocity at which the end effector can know in any direction. So, this is the giving a bound of the end effector tip velocity. And w_4 which is the n-th root of the manipulability measure w_1 which is a volume of the ellipse ellipsoid is a geometric mean of the radii α_1 to α_m and this equal to the radius of the spear whose volume is same as that of the ellipsoid.

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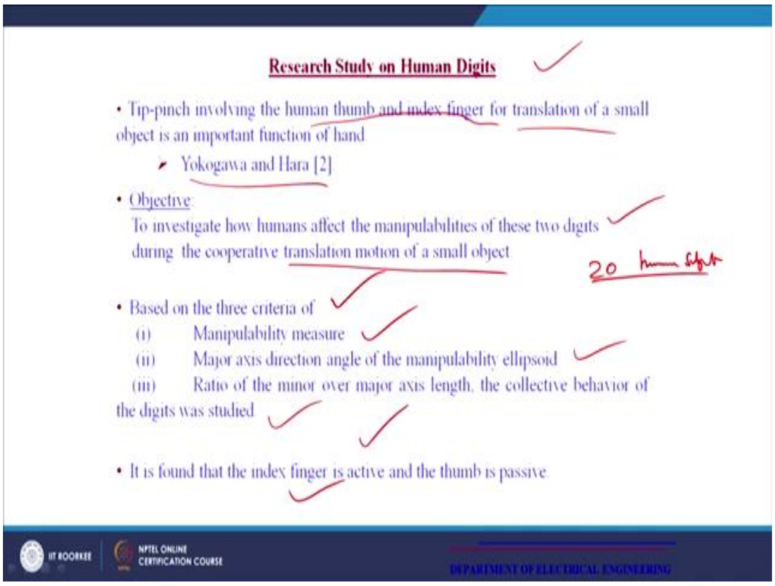

Now, coming to the research study on human digits: first based on the human manipulability measure human study is performed by Yokogawa and Hara in 2004. So, they have performed the tip pinch involving human thumb and index finger for object translation of a fine object.

So, they have performed with an aim to investigate how the human subjects affect the manipulability of these two digits; digits during cooperative translation motion of a small fine object with 20 human subjects. They have performed here strenuous study with 20 human subjects. Based on three criteria of manipulability measure major axis direction angle of the manipulability ellipsoid, and the ratio of the minor over major axis length the collective behavior of the digits was studied.

It was solvent that from the study the thumb was acting passively, whereas the index finger is an active digit.

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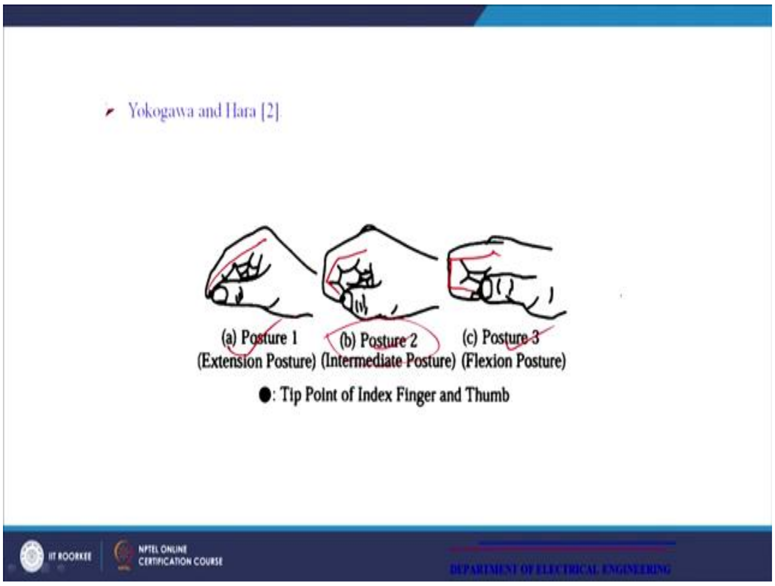

They have considered the three posture: one is the extended posture, and the other one is the flexed the posture intermediate posture, and the last one is the very flexed posture. Precisely extended posture, intermediate posture, and flexed the posture or flexion posture: posture 1 posture 2 and posture 3.

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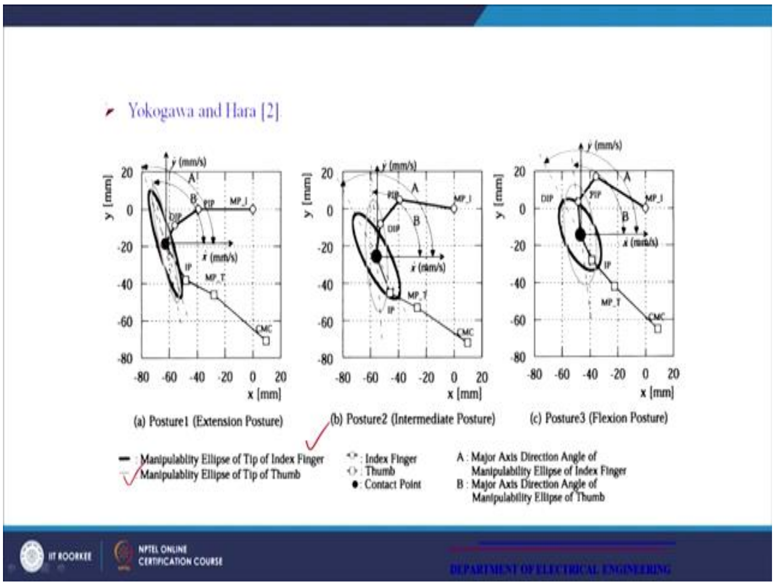

And what they found is the criterion each criterion very significantly from one posture to another posture: posture 1 to posture 2, posture 2 to posture 3 they have a greater variation of the criterion for the active digit. For the index finger it is more compared to that of the thumb.

Here you can see that, the thicker line the darker line is the ellipse of the index finger and the ash color or the grey color is the one for the thumb. You can see that variation of the index finger ellipsoid ellipse is more compared to that of the thumb, in the major axis direction angle criteria.

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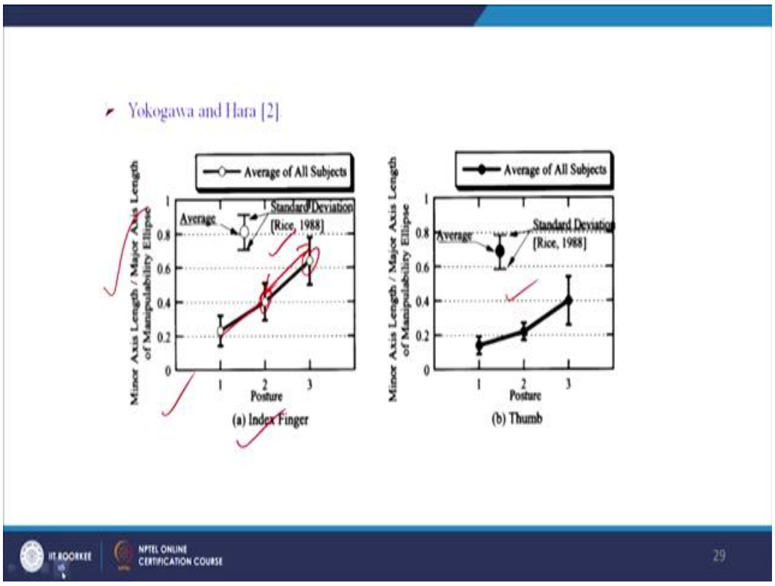

Similarly, they have their ratio criterion where they also have the index finger has greater change. The difference between the mean value of the criterion change, from one posture to another posture is greater for the index finger, whereas it is not

So, great compared to that of the index finger in the thumb case and hence second reinforced statement research observation that the index finger is active.

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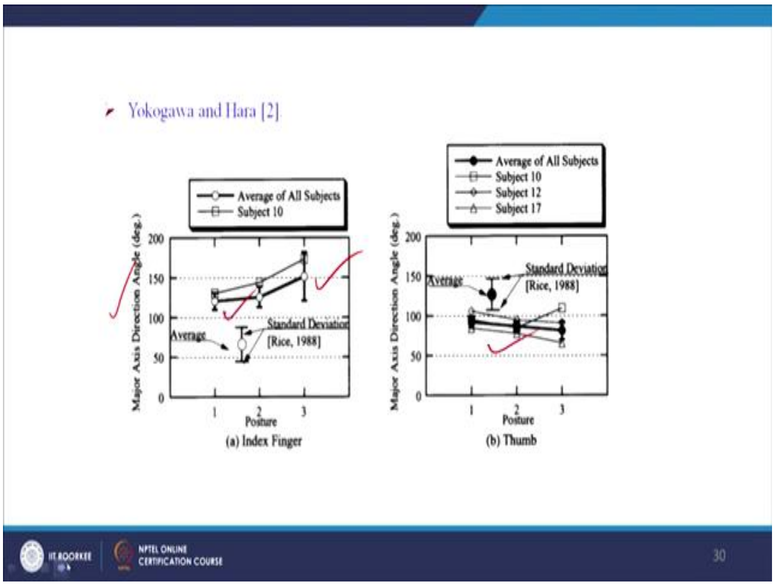

Finally, they taken the variable that is the major axis direction angle also having greater change compared to that of the thumbs variation of this criterion.

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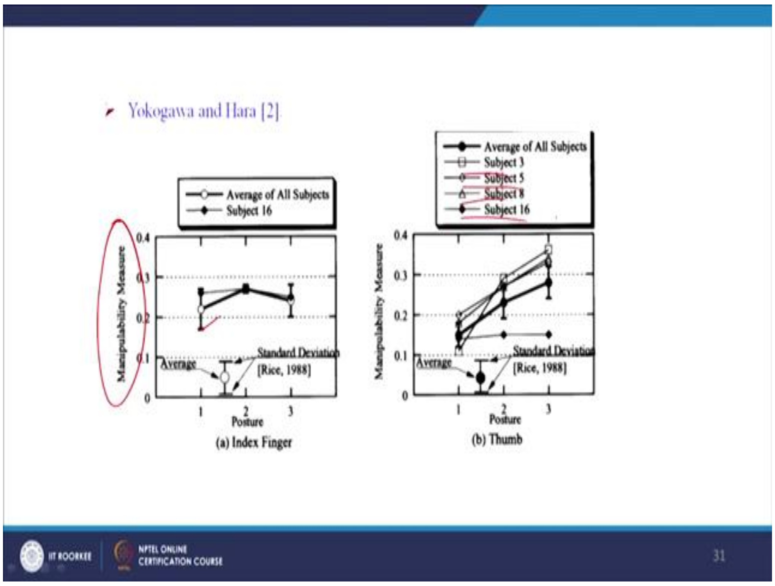

And the manipulability measure which is a third criteria criterion which is also having for the index finger greater one as compared to that of the thumb. But the subjects significantly have some difference compared to all the 20 subjects.

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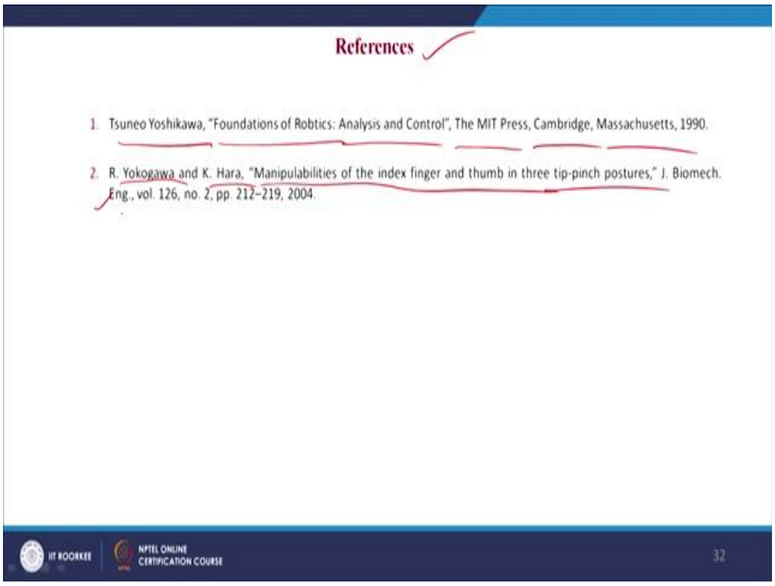

And thank you so much. The references which I have taken for this lecture is from Tsuneo Yoshikawa which is the 'Foundation of Robotics Analysis and Control', the MIT Press Cambridge Massachusetts 1990. And the other research topic is from the reference Yokogawa and Hara topic is 'Manipulability's of the index finger and thumb in three tip-pinch postures'. General of biomechanics, biomechanical engineering and this is the reference precisely given here.

With this, I wind this lecture.

Thank you.