Robotics and Control: Theory and Practice
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Lecture –21
Redundancy Resolution of Human Fingers in Cooperative Object Translation -I

Good morning, today the topic of my lecture will be on the Redundancy Resolution of Human Fingers in object Translation Motion. This lecture has been divided into two parts.

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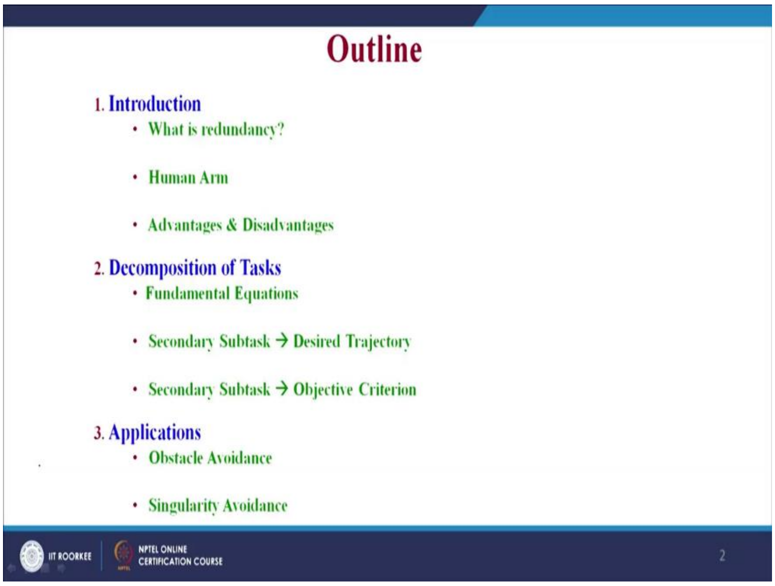

The first one is the introduction to redundant manipulators. The outline of my lecture will be as follows; first, we have the introduction in that will be seeing what is redundancy and how the redundancy is resolved or realized using the human arm and then will be seeing the advantages and disadvantages of redundancy.

Then we have the task decomposition approach, where we will be seeing the fundamental equations pertaining to that. Then will be seeing the secondary sub task where it is given as the desired trajectory and the objective criterion to be instantaneously optimized. Then finally, we will be seeing the applications of redundancy in obstacle avoidance methods and in singularity avoidance.

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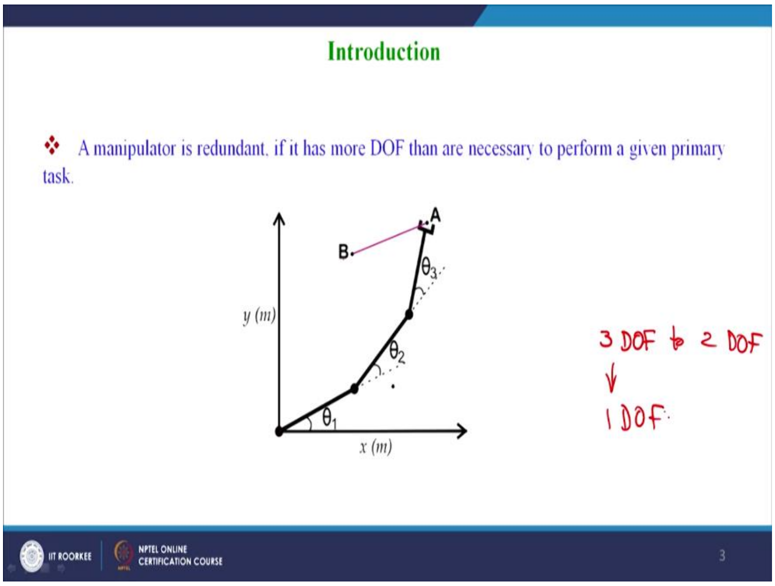

Coming to the introduction, a manipulator is said to be redundant if it has more degrees of freedom than it is necessary to perform a given task. For example, if you see the given manipulator here, it has 3 degrees of freedom, where we have the first joint their revolute Joint, the second is also revolute joint, and the third is also the revolute Join. Where the independent coordinates are θ_1, θ_2 and θ_3, here there is a task to track the trajectory between the point A and B.

This trajectory in the Cartesian space is a two-dimensional trajectory, whereas the degrees of freedom of the manipulator is 3. Thus there is a matching, or there is a mapping from 3 degrees of freedom joints space to the 2 degrees of freedom Cartesian space and there is a mapping from 3 to 2 degrees of freedom.

Hence there is one degrees of freedom extra in the joint space and hence this manipulator is a redundant manipulator, because there is a matching or mapping from 3 degrees of freedom to 2 degrees of freedom. And hence we have here 1 degree of freedom extra in the joint space which is acting as a redundant part.

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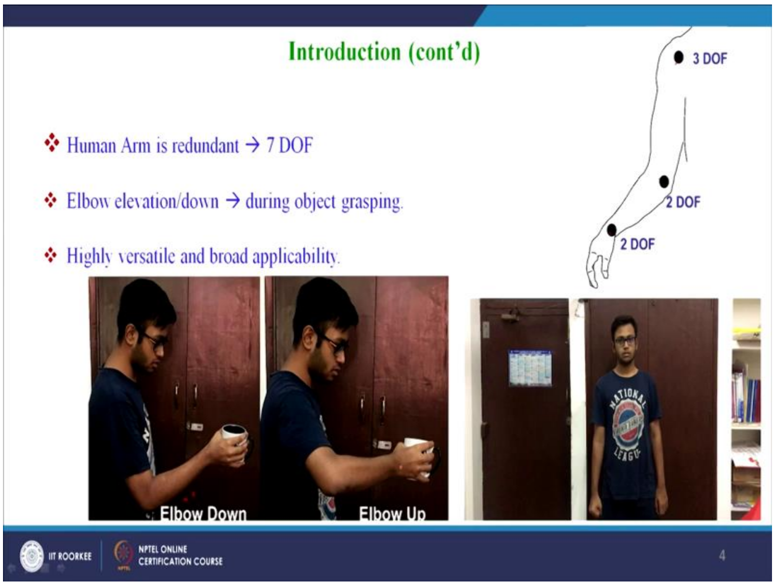

Now, coming to the realization of redundancy by the human arm; the human arm is redundant because it has 7 degrees of freedom mapping to the 6 degrees of freedom Cartesian space. What are the 6 degrees of freedom the Cartesian space? Yes, it has x, y, z positioning in the xyz coordinates and orientation around the $x - axis, y - axis\ and\ z - axis$ that is α, β, γ and totally the Cartesian space has 6 degrees of freedom which is x, y, z and α, β and γ that is roll, pitch, and yaw angle.

Whereas the 7 degrees of freedom in the human arm are, 3 in the shoulder Joint, 2 in the elbow joint and 2 in the wrist joint. The 3 is roll, pitch, and yaw in the shoulder and 2 that is flexion extension and pronation supination, here also we have 2 degrees of freedom flexion extension and another one in the wrist. Thus totally we have 7 degrees of freedom for the human arm. 3 in the shoulder, 2 in the elbow and 2 in the wrist. This 7 degrees of freedom mapping to 6 degrees of freedom in the Cartesian space and hence we have 1 degrees of freedom extra for the human arm.

Thus the 1 degrees of freedom is redundant for the human arm and hence the human arm is versatile and as broad applicability. Thus for example, if you want to grasp an object, that object will be having 2 tasks. One is the positioning of the hand and another one is the orientation of the hand. So, for the positioning we need 3 degrees of freedom and for the orientation we need 3 degrees of freedom.

Thus we have one more degrees of freedom by the human arm which is redundant. For example, in the grasping case we have the redundant configuration by the elbow elevation angle. As shown in the figure here we have elbow down with that posture we can grasp an object. Similarly, we can also graph the same positioning of the object by elbow rising up. Thus we have the elbow elevation angle coming up into the picture.

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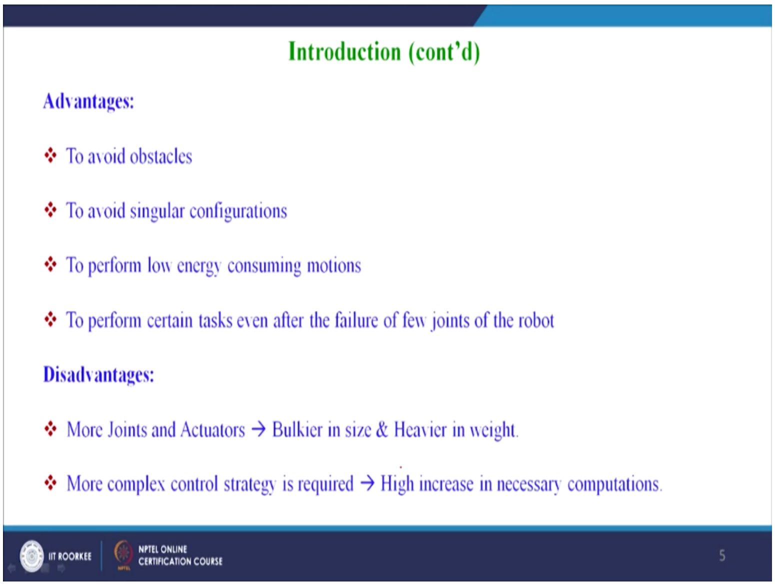

Next coming to the advantages and disadvantages of the redundancy. The advantages obviously it is advantageous to avoid obstacles to avoid singular configurations of the robotic manipulators and to perform low energy consuming motions. And finally if some of the joints are they are not functioning due to the failure of the system, but still the task can be achieved by the remaining joints.

But coming to the disadvantages, the system becomes bulkier in size and heavier in weight, because of more joints and actuators. Likewise, it requires more complex control strategy, because of the heaviness in the necessary computations.

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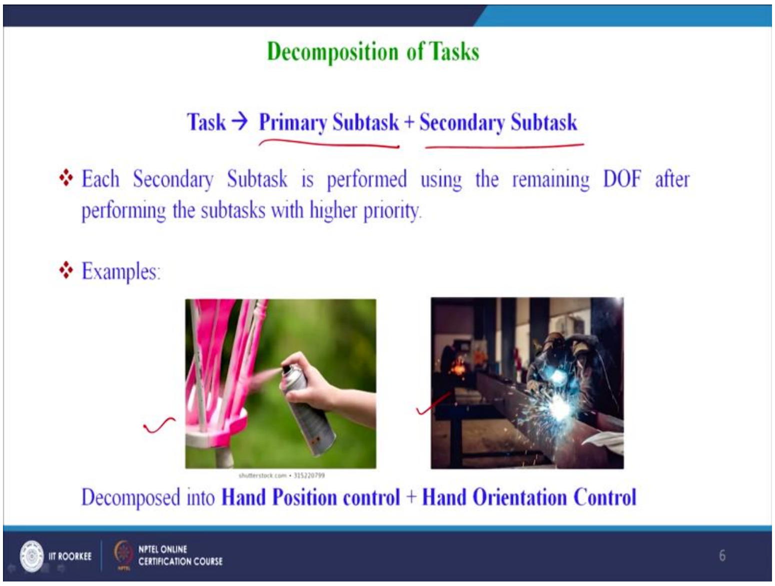

Now, coming to the metrology which is the decomposition of tasks, thus the task given to a manipulator for the redundant manipulators is decomposed into primary sub task and secondary sub task. Where the primary sub task is the major task with higher priority and the secondary subtasks are performed after performing the primary sub task.

For example, in the case of spray painting and welding task we have 2 tasks, the task of spray painting and welding can be decomposed into 2 tasks or subtasks named hand position control and hand orientation control, where the hand position control is the one task with higher priority.

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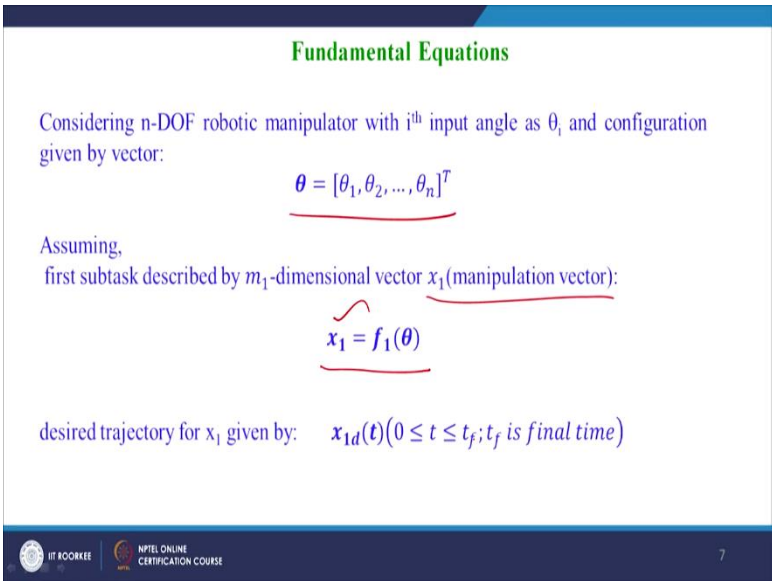

Now, coming to the basic equations of redundant manipulators. Consider an n degrees of freedom which has n joint angles given by the generalized coordinate vector that is θ equal to θ_1 to θ_n, where the i[th] joint is given by θ_i. And assuming that the first task is given by the manipulation vector, which is $x_1 = f_1(\theta)$. It is the power kinematic equation, where x is a Cartesian position of the end effectors which is equal to the function of the joint coordinates.

And in this case consider the desired trajectory for x_1 which is represented by $x_{1d}(t)$. Where t varies from 0 to t_f with the time step, where the time step is the sampling period in this simulation, where t_f is the final time.

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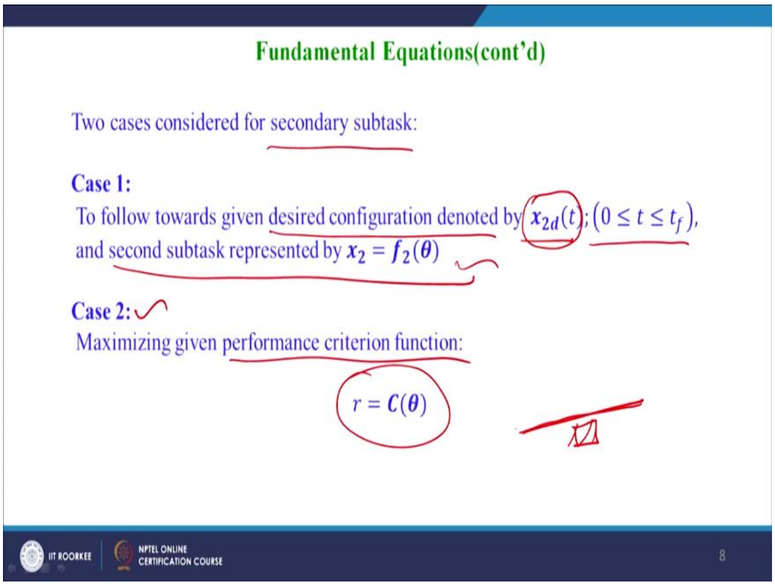

And for the secondary case; for the secondary subtasks there are 2 cases. First one is the case one which is where the secondary sub task is given as a desired configuration represented by $x_{2d}(t)$, where t obviously varies from 0 to t_f, where t_f is a final time and the secondary sub task is represented by $x_2 = f_2(\theta)$.

Whereas, a primary sub task we have seen in the previous slide it is represented by $x_1 = f_1(\theta)$, here for the secondary subtask it is represented by $x_2 = f_2(\theta)$. Here for the case 1 the secondary sub task is given by x_{2d} that is a desired configuration it has to reach.

Similarly, the case 2 for secondary sub task is the performance criterion, where this performance criterion is to be maximized instantaneously to achieve the redundancy based configuration. For example, you are tracking the trajectory in that situation there is an obstacle you are tracking here, you need to avoid this obstacle while tracing this same trajectory. Similarly, this objective criterion is used to avoid singular configuration while tracing this given Cartesian trajectory.

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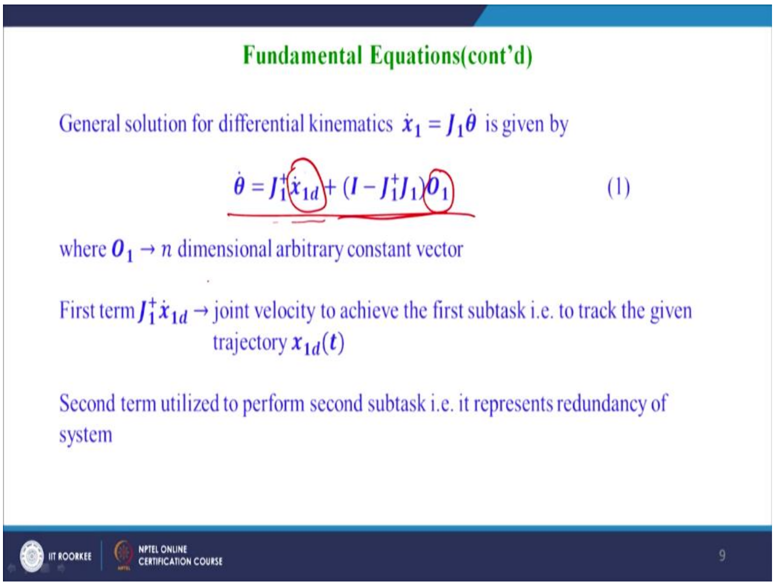

Continuing the fundamental equations, the generalized solution for the differential kinematics which is $\dot{x}_1 = J_1\dot{\theta}$ is given by this expression, which is

$$\dot{\theta} = J_1^+ x_{1d} + (I - J_1^+ J_1)0_1$$

Where the first term that is $J_1^+ x_{1d}$ is the generalized solution of the joint velocity to obtained the primary sub task, to track the given desired trajectory $x_{1d}(t)$.

Similarly, the second term on this equation 1 is to perform the excessive operation due to the redundant degrees of freedom available. The excessive task will be avoiding singularity avoiding obstacles and also to avoid or to keep the joint angles within the mechanical joint angles limit. And the vector 0_1 is the n-dimensional arbitrary constant vector, we are going to see that vector in the next coming slides.

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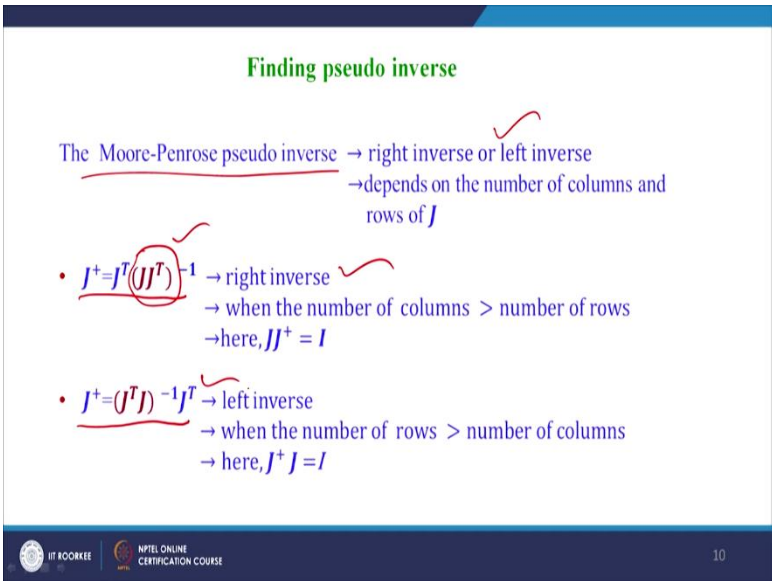

And now coming to the Moore Penrose, because we have seen in the previous slide the generalized expression equation 1 is given by in terms of pseudo inverse and thus the pseudo inverse is how to find or how to compute the pseudo inverse. The Moore Penrose pseudo inverse is computed in 2 ways one by the right inverse and the other one by the left inverse. The right inverse is given by the expression

$$J^+ = J^T(JJ^T)^{-1}$$

This we have the right hand side getting inversed and hence it is right inverse. And this when we will use right inverse depends on the number of rows and columns of the Jacobian matrix J. When the number of columns is greater than the number of rows; that is the degrees of freedom in the joint space is greater than the degrees of freedom in the Cartesian space we use the right inverse metrology. Here $JJ^+ = I$, hence this right inverse.

Similarly left inverse is the situation when the number of rows is greater than the number of columns of the Jacobian matrix J and thus the expression is J pseudo inverse for the left inverse is

$$J^+ = (J^TJ)^{-1}J^T$$

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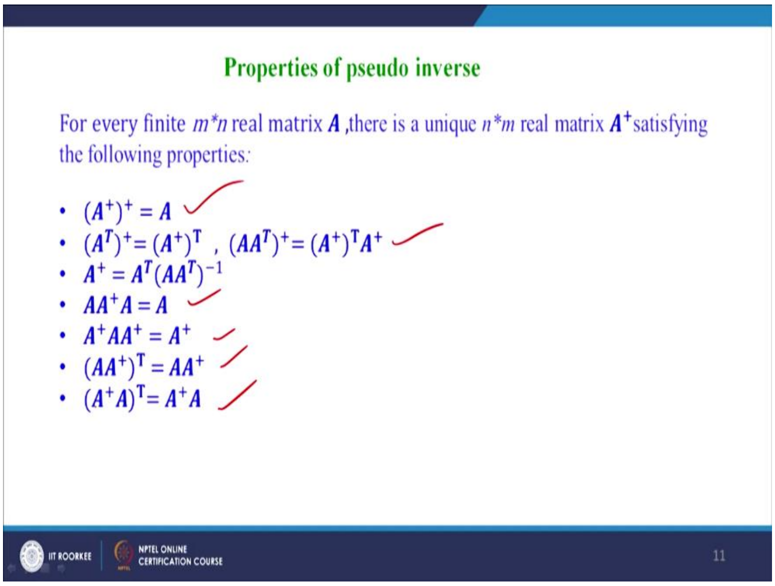

Now, coming to the quick properties of the pseudo inverse, these are the properties of pseudo inverse where it shows I read out some of them, $(A^+)^+ = A$; A pseudo inverse whole pseudo inverse equal to A and the right inverse is given. As we have seen in the previous slide $AA^+ = A^T A$.

Finally, we have one more property which is $(A^+A)^T = A^+A$, Coming to the proof for this expression which is this proof we will come back after this derivation.

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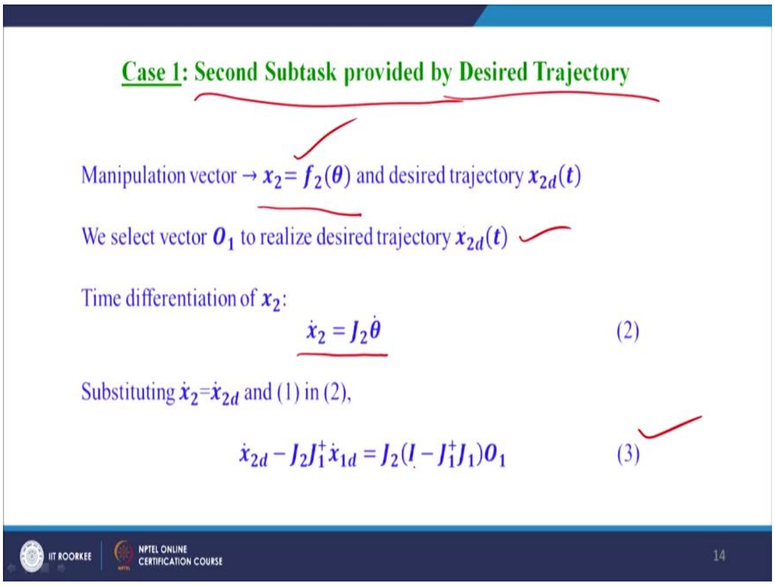

So, now coming to the 2 cases of a secondary subtask; the first one secondary subtask is provided by the desired trajectory. What is the case 2? The secondary subtask is provided by the criterion function which has to be maximized instantaneously. The first case is secondary subtask provided by the desired trajectory. Here the manipulation vector is x_2 equal to $f_2(\theta)$ and the desired trajectory is given by x_{2d}, we select the vector O_1 in the previous expression of the generalized joint velocity vector the O_1 is selected to realize the desired trajectory $x_2(t)$.

The time derivative of this equation manipulation vector equation is given by $\dot{x}_2 = J_2\dot{\theta}$ Substituting $\dot{x}_2$ equal to $\dot{x}_{2d}$. In this equation we have the third equation coming out to be

$$\dot{x}_{2d} - J_2 J_1^{+} \dot{x}_{1d} = J_2(I - J_1^{+} J_1)0_1$$

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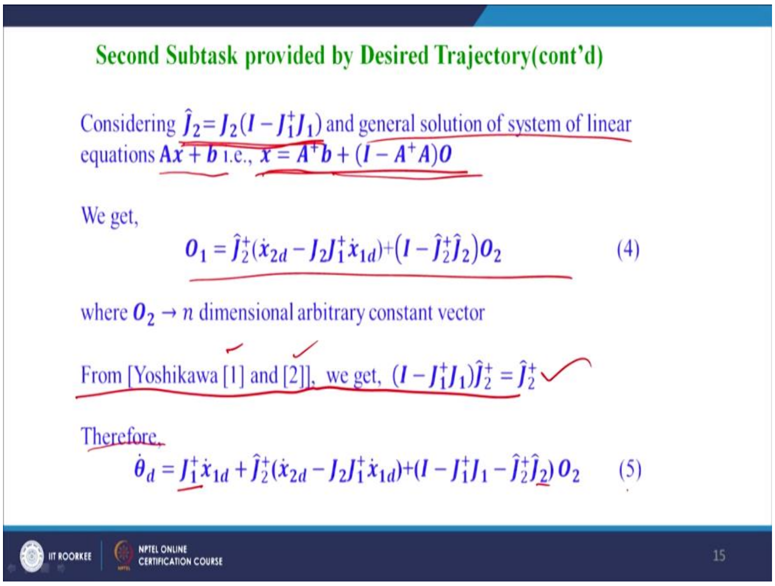

Now, considering this expression which is $\hat{J}_2 = J_2(I - J_1^{+} J_1)$

and the generalized the solution of the system of linear equations which is $Ax + b$ that is the generalized solution for this system is $x = A^{+}b + (I - A^{+}A)0$.

Considering these two equations we get 0_1 in terms of 0_2 as expression or equation number 4, where 0_2 is the n dimensional arbitrary constant vector and from the reference of Yoshikawa and Klein which is given in the references number 1 and 2. We get

$$(I - J_1^+ J_1)\hat{J}_2^+ = \hat{J}_2^+$$

Therefore, the generalized velocity will be obtained in terms of J_1 and J_2 as equation number 5.

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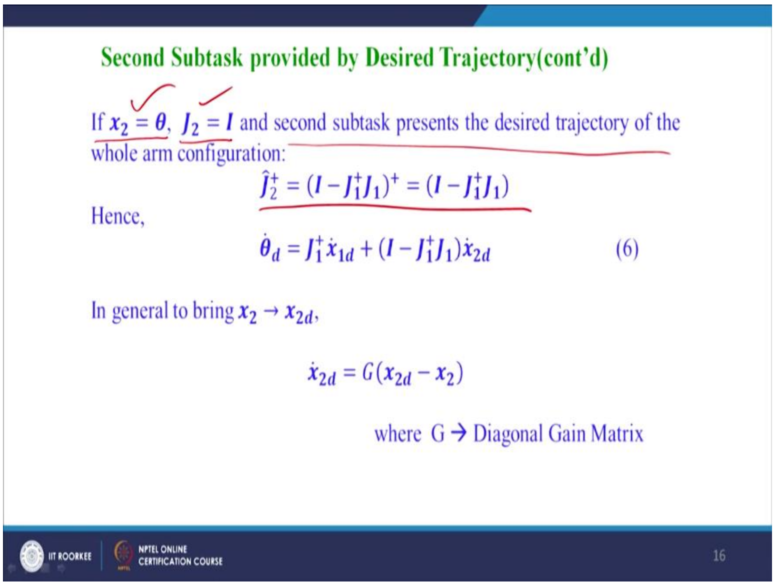

If $x_2 = \theta$ and $J_2 = I$, $x_2 = \theta$ denotes the secondary subtask presence the desired trajectory of the whole arm configuration. With these 2 conditions the $\hat{J}_2$ expression can be turned into $\hat{J}_2^+ = (I - J_1^+ J_1)^+$. The proof of this will quickly see right now.

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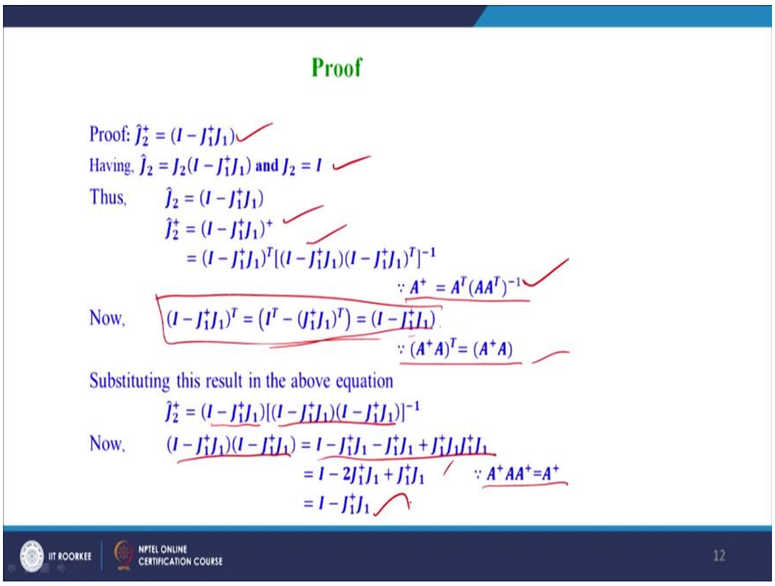

This is the proof where $\hat{J}_2$ is $(I - J_1^+ J_1)$. Having the conditions that $\hat{J}_2 = J_2(I - J_1^+ J_1)$ and $J_2 = I$. Having these two conditions the first equation will turn out into $\hat{J}_2 = (I - J_1^+ J_1)$. Thus $\hat{J}_2 = (I - J_1^+ J_1)^+$.

Now, you expand this equation in terms of a right inverse which is given $A^+ = A^T(AA^T)^{-1}$ taking this and substitute this expression here we have the expression for $(I - J_1^+ J_1)^T$ is turning out to be this 1. Substituting this result in the above equation of

$$\hat{J}_2^+ = (I - J_1^+ J_1)[(I - J_1^+ J_1)(I - J_1^+ J_1)]^{-1}$$

Now, we have this term which is the inner term of this generalized inverse has been expanded to this expression which is $I - J_1^+ J_1 - J_1^+ J_1 + J_1^+ J_1 J_1^+ J_1$

Simplified that by this property of a pseudo inverse, we simplified this expression into

$$I - J_1^+ J_1$$

which is $\hat{J}_2$ coming out in terms of this.

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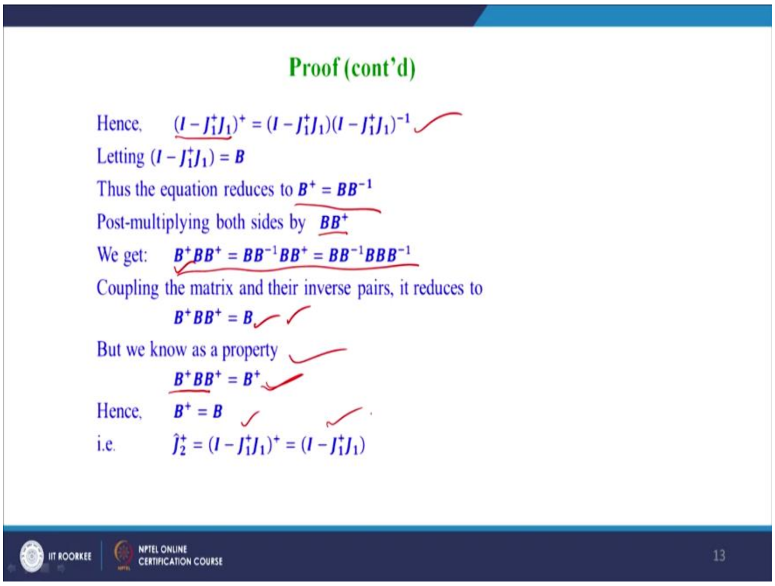

Hence we have this condition from the previous expressions that is

$$(I - J_1^+ J_1)^+ = (I - J_1^+ J_1)(I - J_1^+ J_1)^{-1}$$

Let B is given by the expression which is inside this bracket $(I - J_1^+ J_1)$, thus the equation reduces to $B^+ = BB^{-1}$ post multiplying the expression B^{-1} equal to this by both sides by BB^+.

We have this expression coming out and coupling the matrix and the inverse pairs it reduces to this equation is reduced to B. That is $B^+ B B^+ = B^+$

Thus B equal to B^+ that is $\hat{J}_2^+ = (I - J_1^+ J_1)^+ = (I - J_1^+ J_1)$.

That is $B^+ = B$ as per the given proof.

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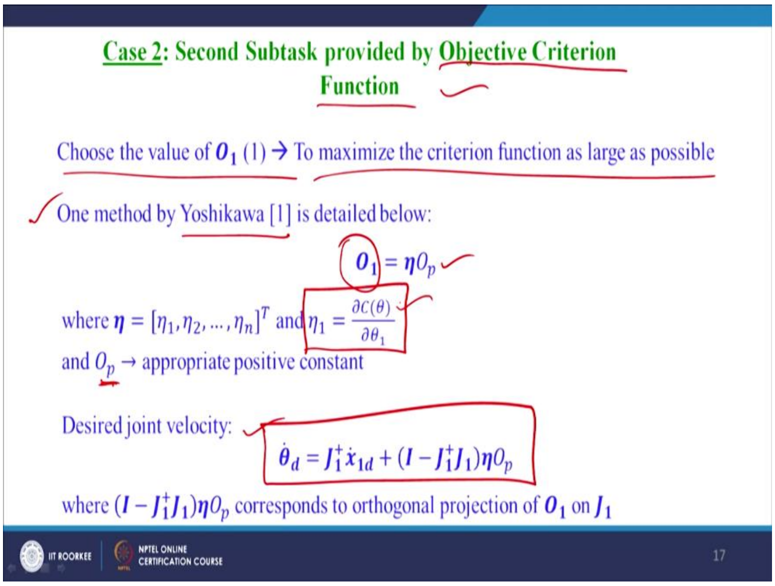

Now, coming to the continuation of case 2 of the secondary subtask where the Objective Criterion Function is used to be to be optimized instantaneously. Choose the value of 0_1 of equation 1; that is to maximize the criterion function as large as possible.

One method is proposed by Yoshikawa and it is detailed below, which is given as $0_1 = \eta 0_p$. The vector 0_1 is given by the vector product of the vector with the scalar constant. Where η is n dimensional vector given by η_1 to η_n, where every η_1 or η_I is given by the expression η_I equal to partial derivative of the function $c(\theta)$ with respect to the generalized coordinate θ and θ_n or 0_p is the scalar constant which is generally positive.

Thus the desired velocity is given by the expression

$$\dot{\theta}_d = J_1^{+}\dot{x}_{1d} + (I - J_1^{+}J_1)\,\eta 0_p$$

where the second term $(I - J_1^{+}J_1)\,\eta 0_p$ corresponds to the orthogonal projection of 0_1 on the Jacobian J_1.

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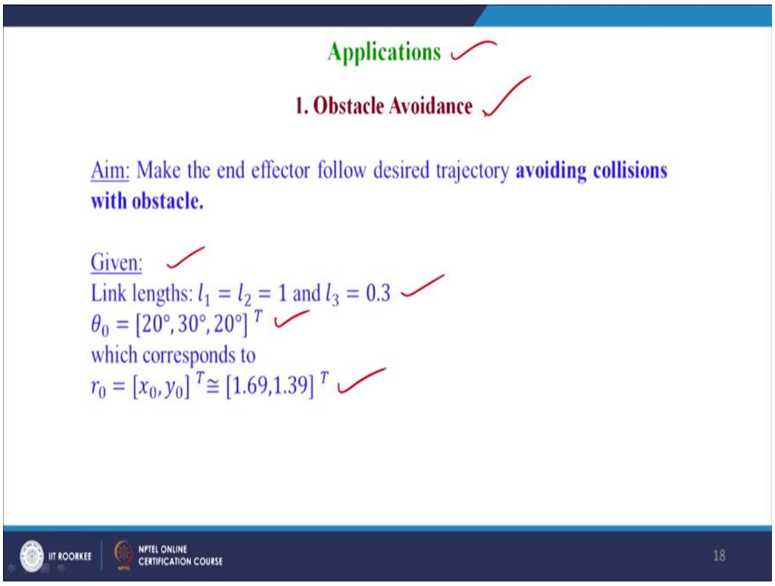

Now, coming to the applications which is the final part of this lecture, application of redundancy. In order to avoid the robot manipulator linkage from the undesired regions of the joint space, for that particular exploitation of redundancy will be seeing 2 or 3 conditions or examples. First one is obstacle avoidance, here the aim our objective is to make the end effector follow the desired trajectory by avoiding collisions with the obstacles. The given parameters are the link lengths and the initial joint configuration which corresponds to the initial end effector position.

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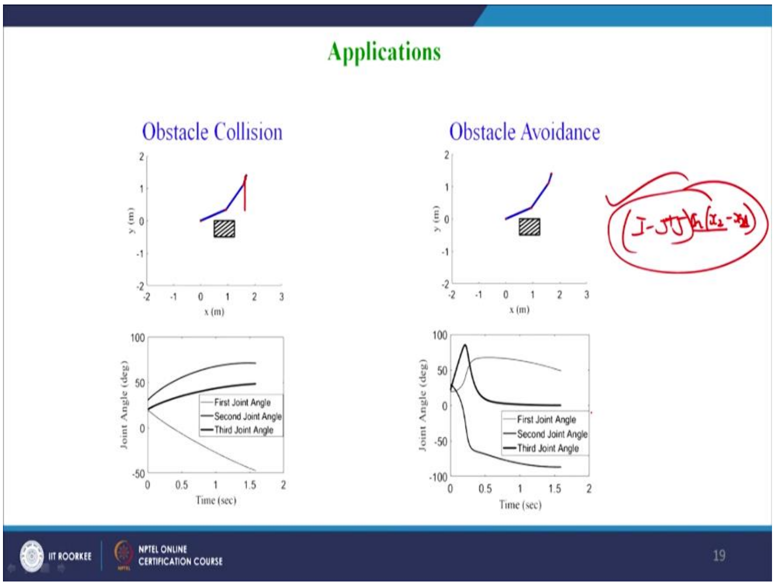

Now, coming to this simulation which has been done in MATLAB.

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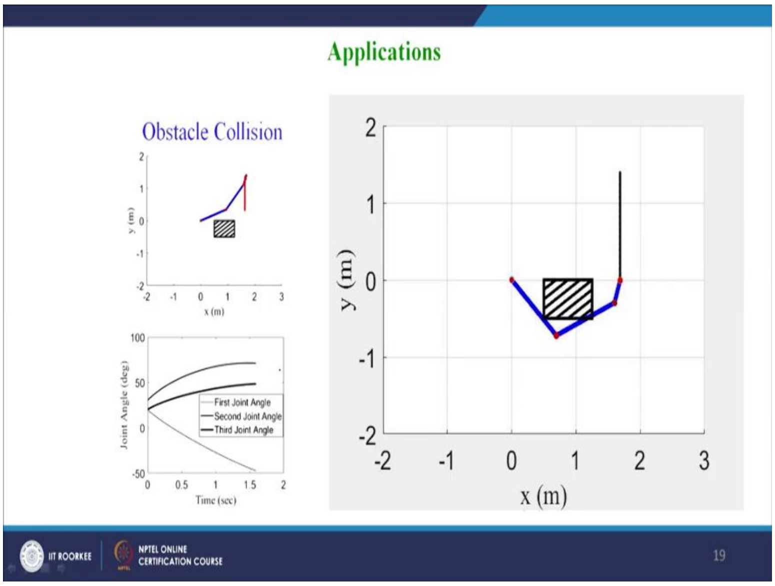

Where we have given as per the examples of Yoshikawa (Refer Time: 22:40) Yoshikawa, we have the obstacle which has to be avoided in traversing this straight line trajectory. So, traversing or matching this straightened trajectory is a 2 dimensional task, which as to be performed by this 3 degrees of freedom manipulator. And the corresponding after the simulation the joint angles are coming out to be the gradual increase in each of the joint variables.

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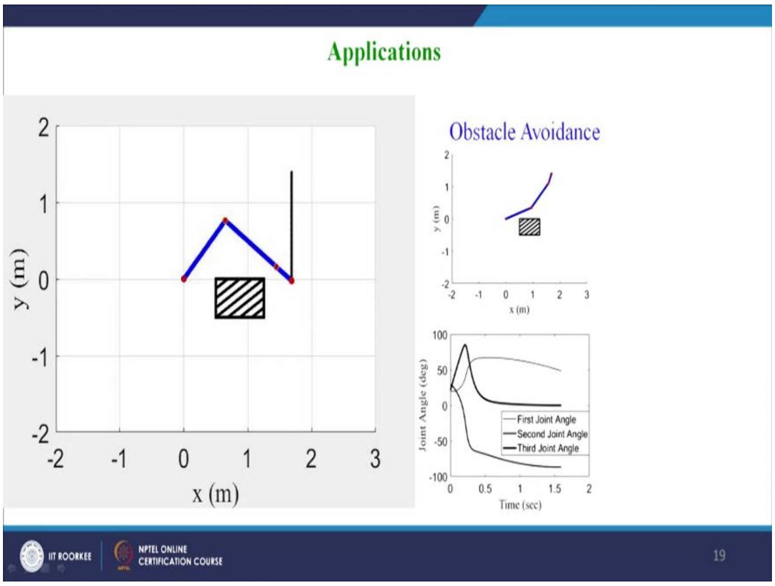

Similarly, when redundancy is considered, the obstacle is avoided. When redundancy is not considered the obstacle is not avoided it gets into the coalition with the obstacle. Whereas, when we consider the redundancy part that is the second term $(I - J^{+}J)$ into the redundancy part. Here the redundancy part is coming out to be the vector G into $x_2 = x_{2d}$. When you have this redundancy term part we will be avoiding this obstacle as shown by this joint angular configuration which are where the joint angle angles are varying differently in order to avoid this obstacle.

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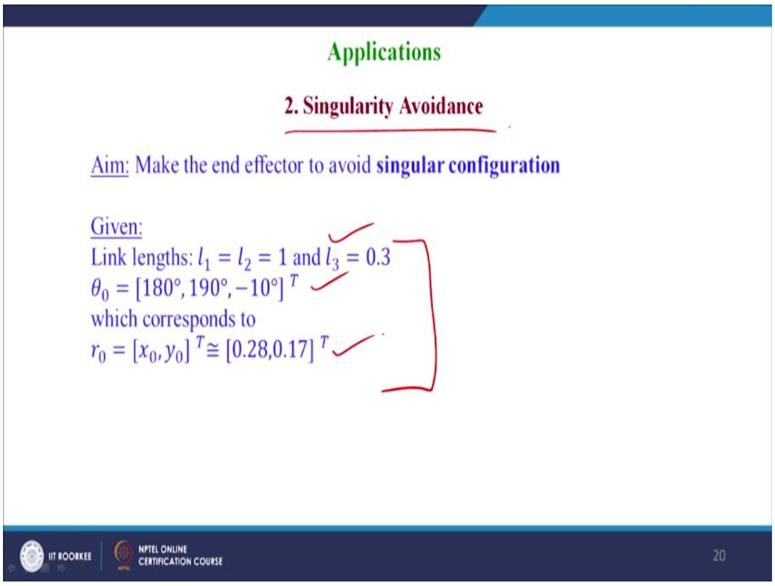

Now, coming to the second application which is the singularity avoidance that is to avoid the singular configuration in the joint space. So, similarly we have given the link parameters, the initial configuration of the manipulator and its end effector position.

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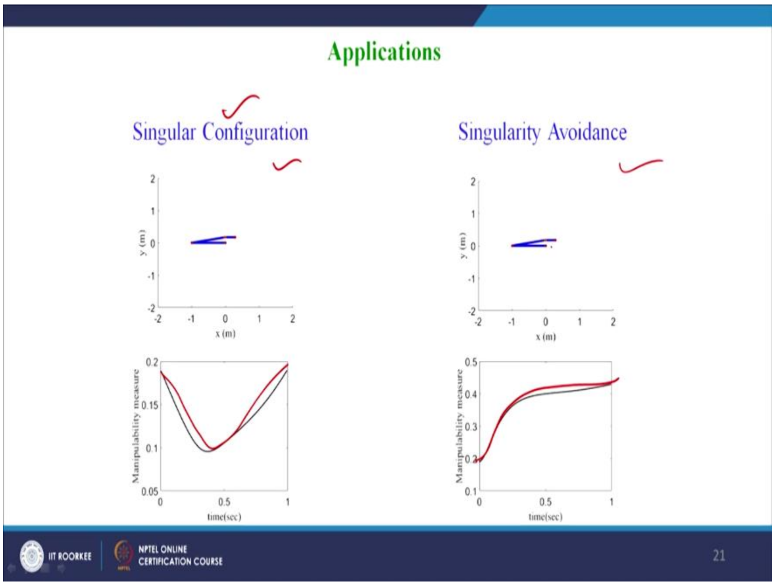

Now, coming to this mat lab simulation we have first case of the singular configuration 1. We are getting into the singular configuration, because we are not considering the redundancy part.

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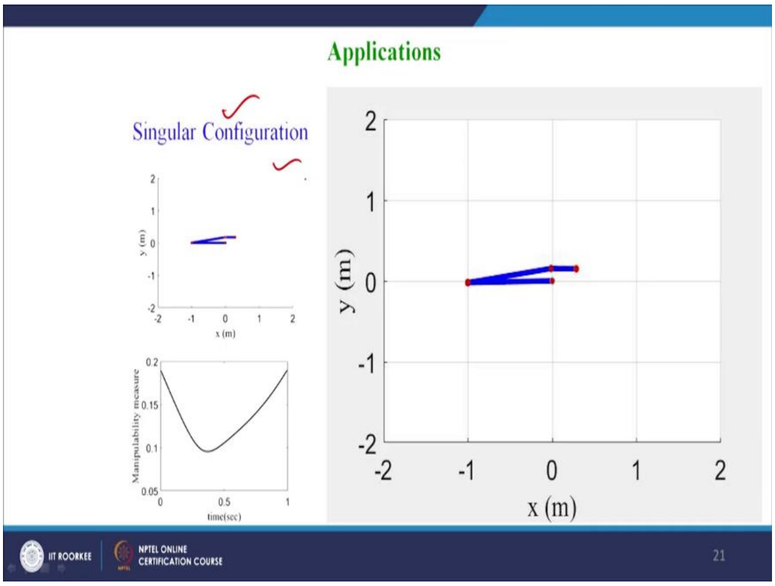

Next we consider the resonance part thereby we are avoiding the singular configuration.

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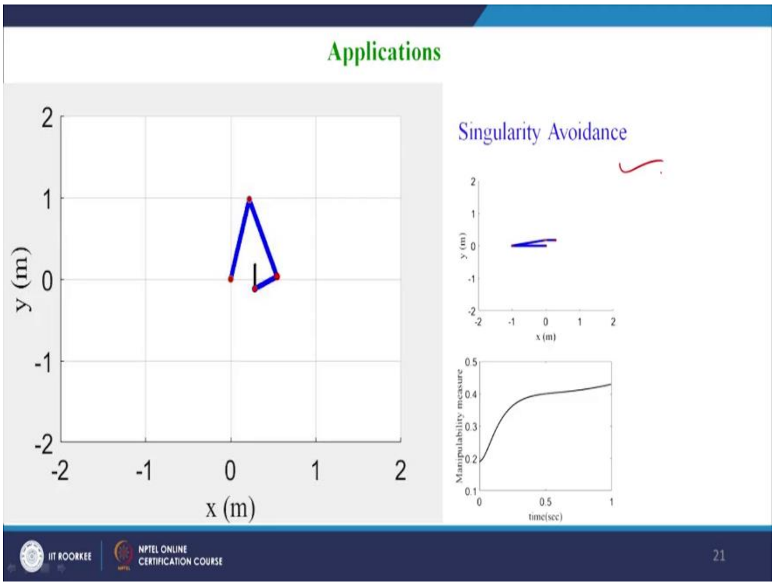

This singular configuration avoidance is quantified in terms of the manipulability measure, which is showing the manipulating capability of the manipulator in avoiding these singular configurations. Thus if we see the manipulability plot we have decrease in the manipulability value.

Whereas, in the case of acquiring or exploiting the singularity redundancy part of the manipulator, which is the extra 1 degrees of freedom available. When we explore the redundancy we are increasing the manipulability measure, such that the singular configuration of the manipulator in traversing the desired trajectory of the straight line is obtained.

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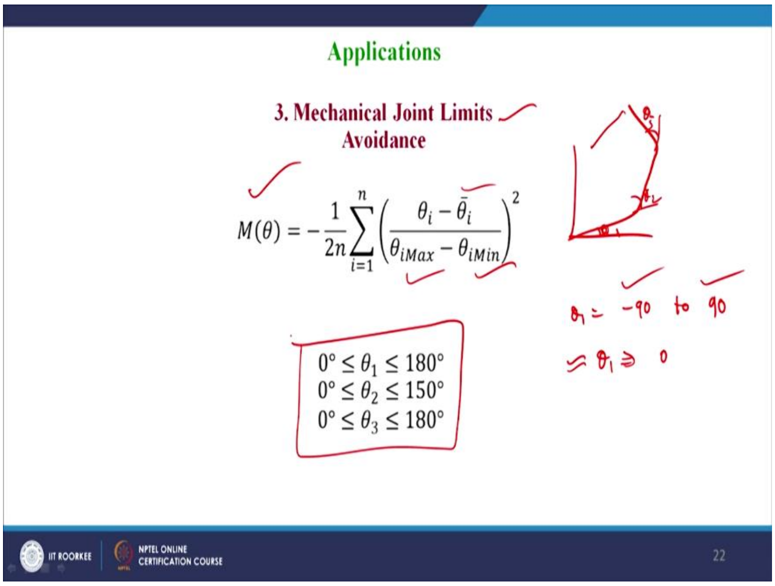

Now, coming to the final application which is the joint angles that is the mechanical joint limits is avoided in this last application. So, given the joint angles this a configuration, to obtain this trajectory we need to have the joint angles θ_1, θ_2 and θ_3 to be within the safe ranges of it is own joint angles. That is say θ_1 is varying from -90 degree to +90 degree.

Then we try to maintain that θ_1 joint angle remains in the center, almost close to the 0 degree angle. That is obtained by considering this objective criterion, which is given by

$$M(\theta) = -\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{\theta_i - \bar{\theta}_i}{\theta_{imax} - \theta_{imin}}\right)^2$$

Where $\bar{\theta}_i$ is the middle value of that joint range and θ_{imin} is the minimum value of θ_i and θ_{imax} is the maximum value of θ_i, and the ranges for this particular one example I have taken considering the previous conditions of the same manipulator with this ranges, θ_1 varies from 0 to 180 and θ_2 is varies from 0 to 150 and θ_3 is from 0 to 180.

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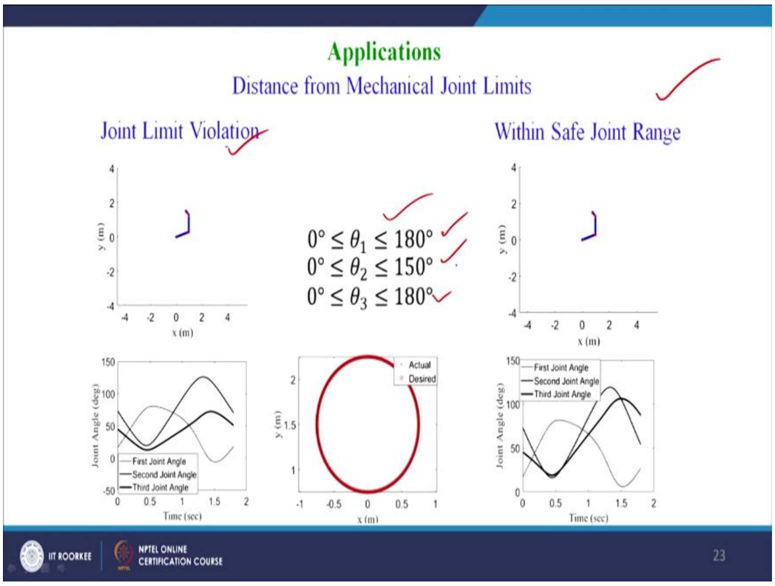

Now, coming to this simulation we have seen that the joint limit violation happens when we are not considering the redundancy.

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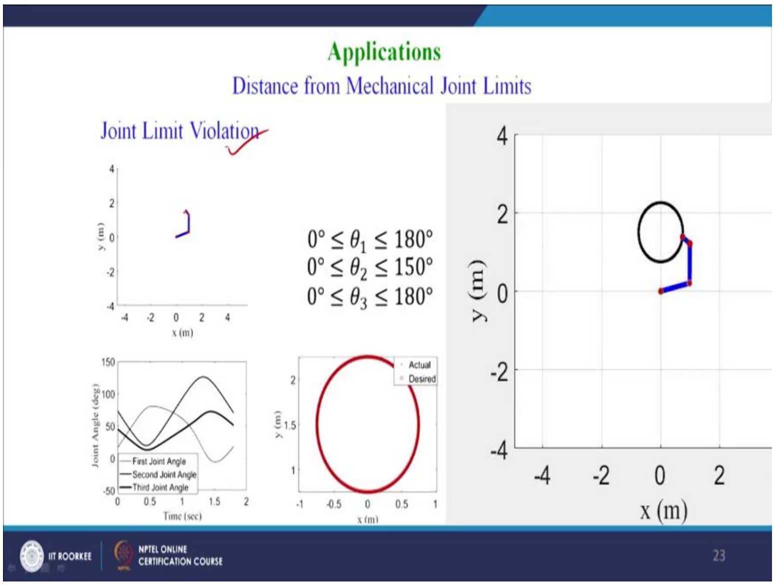

Thus the first joint angle is given by the thinner line as shown in this joint profile joint angular profile. We can see that this first joint angle range is to be within 0 to 180, but you can see that it is going to the negative region that is crossing this limit. When we do not consider the redundancy in traversing or following a desired a given circular trajectory of the end effector; when we consider the redundancy we are within this safe joint angular range.

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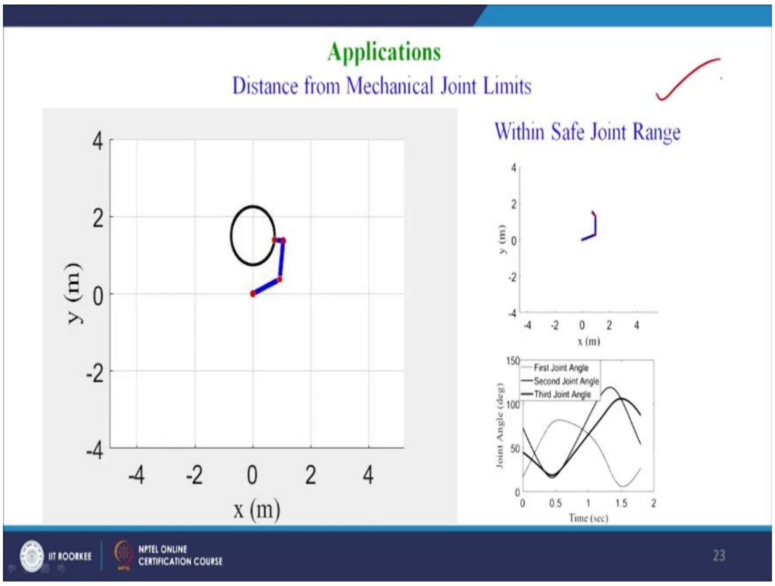

Where we can see that all the joint angles given by this thinner medium thick and the thicker most profiles or the joint angles they are lying within the joint angular ranges, given here 0 to 180, 0 to 150 and 0 to 180 for the 3 joints respectively.

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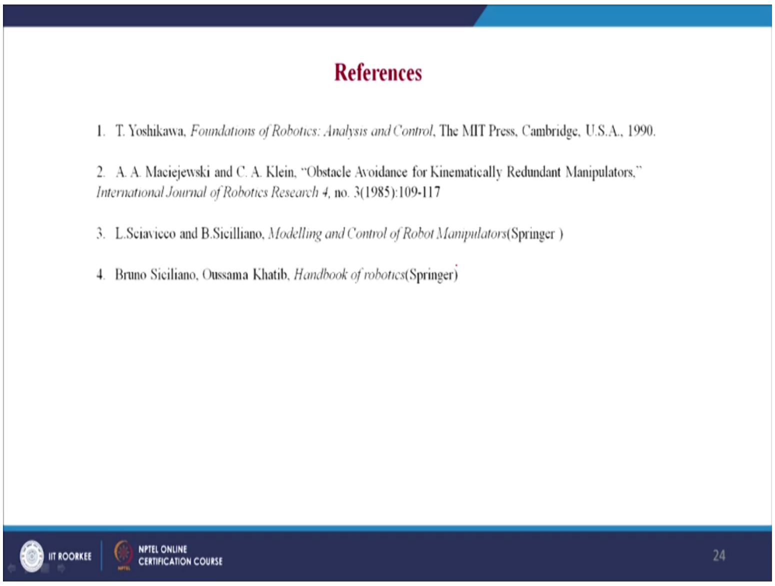

Now, coming to the final slides which is the references for this lecture, I have followed these references, one is the foundations of robotics analysis and control by Sony

Yoshikawa. Then the obstacle avoidance for kinematically redundant manipulator by A A Maciejewski and Klein, then modeling and control of robot manipulators by Bruno Siciliano and sciavicco and then finally the handbook of robotics by Springer by the authors Bruno Siciliano and Oussama.

Thank you very much.