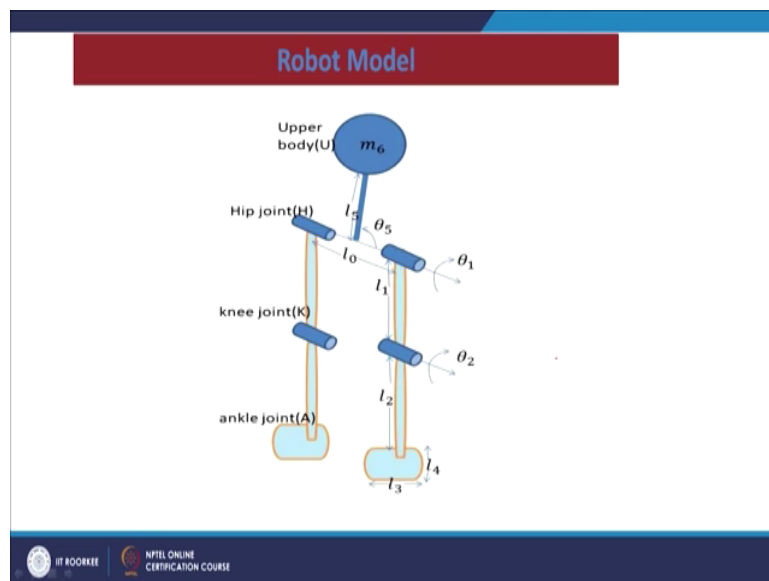


Robotics and Control: Theory and Practice
Prof. N. Sukavanam
Department of Mathematics
Indian Institute of Technology, Roorkee

Lecture –18
Biped Robot
Flat Foot and Toe Foot Model

(Refer Slide Time: 00:41)





So, this is the continuation of my last lecture on Biped Robot with Flat Foot Model. So, in the last lecture, we have seen the flat foot model as given in this picture. We have seen that the two legs of the biped are like two arm manipulators with 2 degrees of freedom each and the upper body is placed the centre of mass of the upper body is denoted by U here.

(Refer Slide Time: 00:57)

Parameters				
Link	Length	Value	Mass	Value
HK	l_1	14 inches	m_1	4kg
KA	l_2	14 inches	m_2	4kg
HU	l_5	10 inches	m_6	50kg
HH	l_0	8 inches	m_5	4kg

Table 3.1: Parameters

 IIT ROORKEE  NPTEL ONLINE CERTIFICATION COURSE

And, the details of the parameters are given in this table. The length of each leg is from ankle to from hip to knee is l_1 and ankle to knee is l_2 and the hip to the upper body is l_3 and the width of the hip joints is l_0 . And, the masses are given as m_1 , m_2 , m_6 and m_5 as given in this table.

(Refer Slide Time: 01:35)

- In Figure, each leg of biped robot have 2 degrees of freedom (DOF) with flat foot.
- All the joints are revolute which are called hip joint (H), knee joint (K) and ankle joint (A).
- Centre of mass of upper body is denoted by (U).
- Robot's walk can be considered as a repetition of one-step motion.
- The walking sequence can be determined by computing the trajectory of the hip, ankle and upper body joints.
- For hip trajectory, stable ankle joint is considered as a base and hip as the end effector.
- For biped robot walking on a plane, motion of the stable leg is assumed to be like an inverted pendulum considering it's ankle joint as base and hip as end effector.
- While walking, humans do not fold their stable leg as the whole body weight lies on it.
- Flat foot is attached at the ankle joint of each leg.
- Let the robot walk in sagittal plane (xz-plane).



(Refer Slide Time: 01:50)

Swing leg's trajectories:

Boundary Conditions of Ankle Trajectory

$$x_A(t_0) = x_i; x_A(t_f) = x_i + x_f; \dot{x}_A(t_0) = 0; \dot{x}_A(t_f) = 0.$$
$$z_A(x_0) = 0; z_A(x_f) = 0; z_A(x_m) = h_1; \dot{z}_A(x_m) = 0.$$

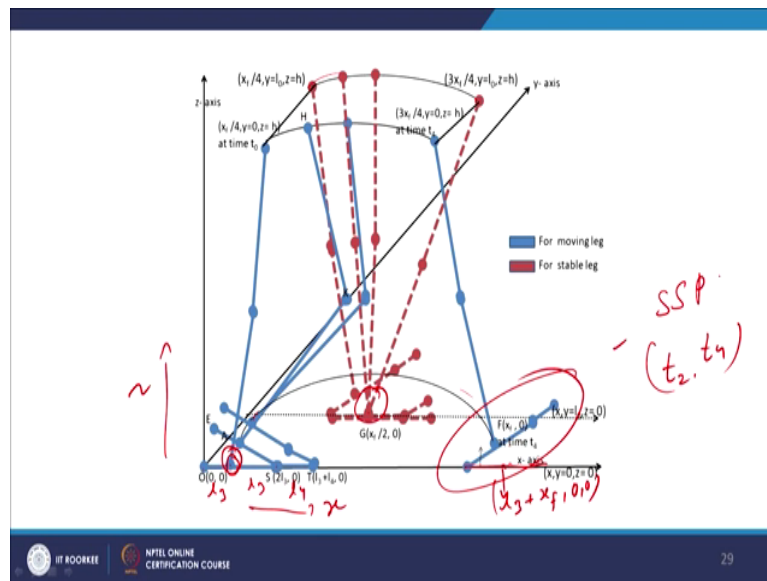
Ankle Trajectory

$$x_A(t) = x_i + \left(\frac{3x_f}{t_f^2}\right)t^2 - \left(\frac{2x_f}{t_f^3}\right)t^3;$$
$$z_A(t) = \frac{h(-(x_f + x_i)^2 x_i)}{(x_m - x_i)(x_m - x_f - x_i)^2} + \frac{h(x_f + x_i)(x_f + 3x_i)x_A(t)}{(x_m - x_i)(x_m - x_f - x_i)^2} - \frac{h(2x_f + 3x_i)x_A(t)^2 + hx_A(t)^3}{(x_m - x_i)(x_m - x_f - x_i)^2}$$

NPTEL ONLINE CERTIFICATION COURSE

So, in the previous lecture we have seen how to derive the trajectories of the ankle, hip of each leg the swing leg and the stable leg and how to solve the inverse kinematics for those trajectories.

(Refer Slide Time: 01:55)



So, here the swing leg is given in the blue colour and the stable leg is in the red colour. The trajectory for this stable leg is the stable foot is the centre and l_1 plus l_2 the length of the total length of the leg is the radius and the hip is moving in the circular path. So, that is given as the trajectory for the stable leg.

And, for the swing leg the ankle moves in this trajectory and how to derive the trajectory as a function of time is given by these parameters, initial position of the ankle and the final position of the ankle and initial velocity, final velocity and the height in the middle of the trajectory.

So, all this are taken into account and yeah polynomial trajectories derived for the ankle for the swing leg. The X-axis X coordinate of the swing leg ankle X A of t is given by the

polynomial of degree 3 because there are four constraints as given here; initial position, final position, initial velocity and final velocity are given.

Similarly, the Z coordinate of the ankle is given as a function of X co-ordinate Z of X. So, initial and final and the middle position of the Z axis Z co-ordinate and the velocity at the middle position is 0. So, these are the constraints so that we can derive the trajectory this particular trajectory this swing leg trajectory. So, this we have seen in detail the previous lecture.

(Refer Slide Time: 03:57)

Stable leg's trajectories:

Boundary Conditions of Hip Trajectory

$$x_H(t_0) = x_i + x_f/4; x_H(t_f) = x_i + 3x_f/4; \dot{x}_H(t_0) = v_{si}; \dot{x}_H(t_f) = v_{se}.$$



$$z_H(t_0) = h; z_H(t_f) = h; \dot{z}_H(t_0) = v_{zs}; \dot{z}_H(t_f) = v_{ze}.$$

Hip Trajectory

$$x_H(t) = \frac{x_f}{4} + v_s t + \left(\frac{(v_e - v_s)}{2t_f} - r_4 \frac{3t_f}{2} \right) t^2 - 2 \left(\frac{x_f}{2t_f^3} - \frac{(v_s + v_e)}{2t_f^2} \right) t^3,$$

$$z_H(t) = \sqrt{(h + h_2)^2 - (x_H(t) - (x_i + x_f/2))^2}.$$

where $r_4 = -2 \left(\frac{x_f}{2t_f^3} - \frac{(v_s + v_e)}{2t_f^2} \right)$.



8

And, similarly for the stable leg it is the circular path. The hip is moving in a circular path and the ankle is this centre of the circle.

(Refer Slide Time: 04:10)

Forward Kinematics

For Swing leg

$$x_A(t) - x_H(t) = l_1 \cos \theta_1(t) + l_2 \cos(\theta_1(t) + \theta_2(t));$$
$$z_A(t) - z_H(t) = l_1 \sin \theta_1(t) + l_2 \sin(\theta_1(t) + \theta_2(t));$$

where $(x_A(t), z_A(t))$ and $(x_H(t), z_H(t))$ are defined as earlier.

For stable leg

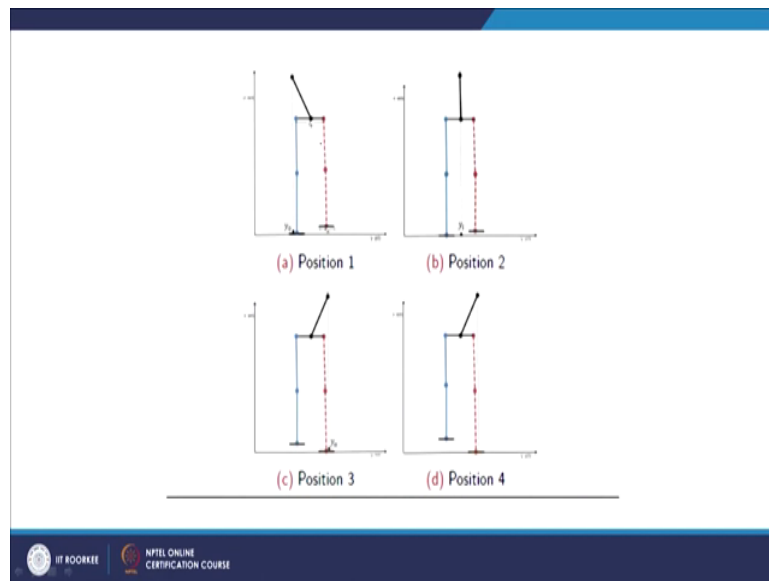
$$x_H(t) - \left(x_i + \frac{x_f}{2}\right) = (l_1 + l_2) \cos \theta_5(t);$$
$$z_H(t) = (l_1 + l_2) \sin \theta_5(t);$$

where $(x_i + x_f/2, l_0, 0)$ is the position of the stable leg's ankle joint which lies on the line $y=l_0$.

So, because we are considering both the legs as 2 degree of freedom manipulator, we can write the kinematic equation of the ankle trajectory X_A of t in terms of θ_1 and θ_2 . And, the stable legs hip it is a circular path therefore, it is in terms of \cos of this angle θ_5 and \sin angle θ_5 . So, these are the standard equations for a circular path.

So, up to this we have seen in detail in the last lecture. So, in this lecture, we will see that for achieving this trajectory, the robot should move in a stable manner that that it means it should not fall down.

(Refer Slide Time: 05:11)



So, for that the upper body's role is very important while the ankle is tracing a given trajectory, the upper body should move in the left and right direction in a suitable manner so that the centre of mass of the entire body as well as the zero moment point should lie in the stable region.

So, that, we will see how this upper body should move in the proper manner so that the entire body walks in a stable manner without falling down.

(Refer Slide Time: 05:45)

Trunk Motion

Trunk motion play an important role in ZMP stability.

Case-1

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1 .
- Stay there during time t_1 to t_3 .
- Again start moving towards middle of legs between t_3 to t_f time where $t_1 = t_f/4$ and $t_3 = 3t_f/4$.

The moving mass trajectory in y-direction is given below: $y_M(t) =$

$$\begin{cases}
 y_1 + y_v t + \left(\frac{3(y_2 - y_1)}{t_1^2} - \frac{2y_v}{t_1} \right) t^2 + \left(\frac{-2(y_2 - y_1)}{t_1^3} + \frac{2y_v}{t_1^2} \right) t^3, & t_0 \leq t \leq t_1 \\
 y_2, & t_1 \leq t \leq t_3 \\
 \left(y_2 + \frac{(-3t_1^2 + t_1^2)(y_1 - y_2)}{(t_3 - t_1)^2} + \frac{t_1 y_v}{(t_3 - t_1)} \right) \\
 + \left(\frac{6t_1(y_1 - y_2)}{(t_3 - t_1)^2} - \frac{(y_1^2 + 2t_1 y_v)}{(t_3 - t_1)} \right) t + \left(\frac{-3(y_1 - y_2)(y_1 + t_1)}{(t_3 - t_1)^3} \right. \\
 \left. + \frac{y_v(4t_1 + 2t_1^2)}{2(t_3 - t_1)^2} \right) t^2 + \left(\frac{2(y_1 - y_2)}{(t_3 - t_1)^3} - \frac{y_v}{(t_3 - t_1)^2} \right) t^3, & t_3 \leq t \leq t_f
 \end{cases}$$

$t_0 \quad t_1 \quad t_3 \quad t_f$
 $-y_v$

IIT KHARAGPUR
NPTEL ONLINE
CERTIFICATION COURSE

So, we consider different types of upper body motion. So, for achieving a stable walk, we will consider three types of upper body motion so, as given here. So, the total time for one step is divided into four parts. So, initial time is t_0 final time is t_f and we consider one-fourth is t_1 and half is t_2 and three-fourth time is at t_3 .

And, so, the case 1 of the upper body motion is the upper body moves from time t_0 to t_1 from the middle of the hip, the two hip joints and then it moves towards the stable leg and it stays there during the time interval t_1 to t_3 . It stays at the same position about the stable legs hip. Then, during the time interval t_3 to t_4 , the last one fourth of the time again it comes back to the middle of the two hips.

So, for performing this motion we can derive the trajectory using the initial and final condition. Initial condition is it is at the middle point and it starts with a velocity given by y_v

suffix v the initial velocity of the upper body motion and then it goes to the extreme. The extreme position is given by y suffix a ; the y suffix a is the extreme length of motion from the middle to the end of the hip.

So, using this constraints we can derive a polynomial of degree 3 again during the time interval t_0 to t_1 . And, during the time t_1 to t_3 it stays at the point the y coordinate of that point is y suffix a as given here and during the time interval t_3 to t_4 that is t_f it comes back from the extreme point y a to back to the middle of the two hips. So, that is derived using a polynomial of degree 3 and we can observe that the hip the upper body it moves with a velocity y suffix v from the middle, when it comes back again it should be with the same velocity in the opposite direction.

So, the velocity at t_f the final position is it is minus y suffix v as it has started with y suffix v at time t_0 it comes back with the same velocity minus y v at t_f . So, using this we can derive the polynomial of degree 3.

(Refer Slide Time: 08:57)

Case-2

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to $t_f/8$.
- Stay there during time $t_f/8$ to $7t_f/8$.
- Again start moving towards middle of legs between $7t_f/8$ to t_f time.

Then the trajectory can be calculated by case-1 equation.

Case-3

Similarly upper body start to move from middle position to stable foot from time t_0 to t_2 , then return back to initial condition. So the moving mass trajectory in y-direction is given below:

$$y_M(t) = \begin{cases} y_f + y_v t + \left(\frac{3(y_f - y_v)}{t_2} - \frac{2y_v}{t_2} \right) t^2 + \left(\frac{-2(y_f - y_v)}{t_2} + \frac{y_v}{t_2} \right) t^3 & t_0 \leq t \leq t_2 \\ \left(y_a + \frac{(-3t_2^2 + t_2^2)(y_f - y_a)}{(t_2 - t_1)^3} + \frac{t_2 y_v}{(t_2 - t_1)^2} \right) \\ + \left(\frac{6t_2(y_f - y_a)}{(t_2 - t_1)^3} - \frac{(t_2^2 + 2t_2 t_1)y_v}{(t_2 - t_1)^2} \right) t + \left(\frac{-3(y_f - y_a)(t_2 + t_1)}{(t_2 - t_1)^3} \right. \\ \left. + \frac{y_v(4t_2 + 2t_1)}{2(t_2 - t_1)^2} \right) t^2 + \left(\frac{2(y_f - y_a)}{(t_2 - t_1)^2} - \frac{y_v}{(t_2 - t_1)} \right) t^3 & t_2 \leq t \leq t_f \end{cases}$$

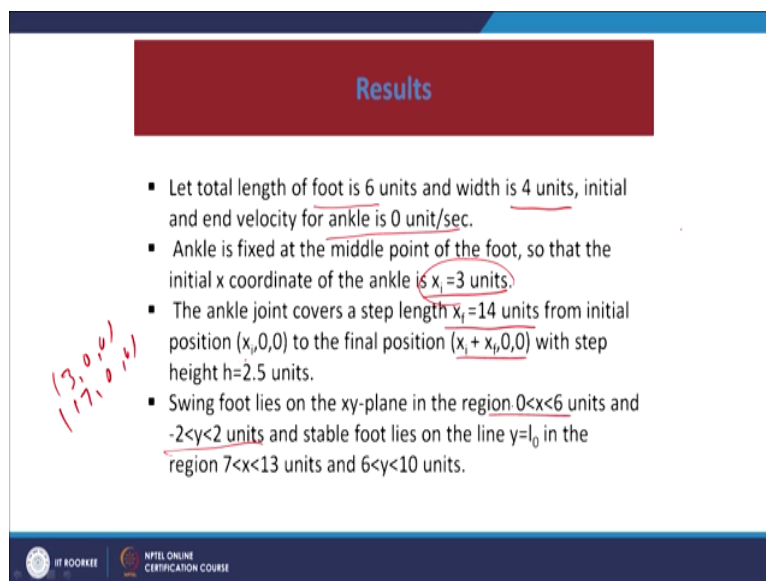
Case 2 of the upper body motion is it is dividing the first one-eighth of the time; t_0 to t_f by 8 during this time it starts from the middle of their hips and then it goes to the side of the stable leg. And, the remaining time and it stays there up to the entire time from $t_f/8$ to $7t_f/8$ that is all the rest of the time. So, the remaining one-eighth of the time it comes back to the middle of the legs. So, it is exactly similar to the previous case only the time is different the time of travel and then time at which stays there is different. So, a similar trajectory can be planned as given here except some coefficient may be different.

The third case is the upper body starts to move from the middle position of the to the stable foot side again in the same manner during the time t_0 to t_2 . So, for it moves bit slowly half of the time it travels from the middle to one side that is stable leg side and immediately return back to the middle of the all hips hip position. So, it never stays at a single point for any

duration of time. Therefore, it is having only two types of trajectory. During the time interval t_0 to t_2 and t_2 to t_4 2 different trajectories are there.

So, this is a type of trial and error type of constructing trajectories. There can be several types of trajectories one can construct and then experiment whether the robot walks in a stable manner or not, we are considering only three types of such trajectories.

(Refer Slide Time: 11:13)



Results

- Let total length of foot is 6 units and width is 4 units, initial and end velocity for ankle is 0 unit/sec.
- Ankle is fixed at the middle point of the foot, so that the initial x coordinate of the ankle is $x_i = 3$ units.
- The ankle joint covers a step length $x_f = 14$ units from initial position $(x_i, 0, 0)$ to the final position $(x_i + x_f, 0, 0)$ with step height $h = 2.5$ units.
- Swing foot lies on the xy-plane in the region $0 < x < 6$ units and $-2 < y < 2$ units and stable foot lies on the line $y = l_0$ in the region $7 < x < 13$ units and $6 < y < 10$ units.

(Handwritten red notes next to the third bullet point: $(13, 0, 0)$ and $(17, 0, 0)$)

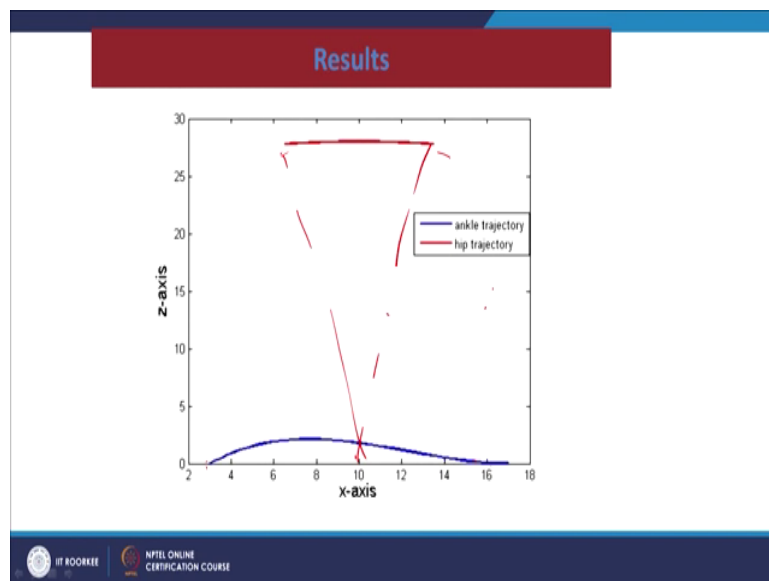
IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE

So, for simulation purpose we consider apart from the parameters which was given in the previous first table about the legs length, mass etcetera. We consider the length of the foot is 6 unit and its width is 4 units of length. And, the initial and end velocity of the ankle is taken to be 0. It starts with 0 velocity and then stops at the final time.

The ankle is fixed at the middle of the foot; that means, at 3 units the initial position of the ankle is there. So, the step length one step length is taken as 14 units from the initial position $x_1, 0, 0$ that is x_1 is 3 here because initial position of the ankle is 3 unit. So, this x_1 is 3; it is 3, 0, 0 is a initial point and final position is here 17, 0, 0; x the length of one foot is 14. So, it starts from here and end to this position with initial velocity 0 final velocity 0. So, we can derive the angle trajectory according to these values.

Similarly, according to the length and width of the foot they are placed in this particular region the rectangles in which they are placed are given by these measurements.

(Refer Slide Time: 12:52)

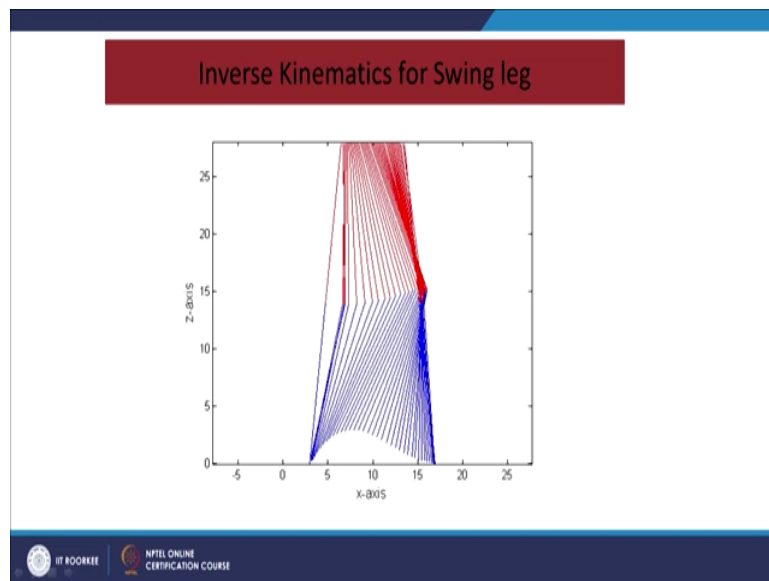


So, this is a simulation for the given data. The hip is moving in this particular manner it is a circular trajectory with centre at this point and radius is given by this total length of the

legs l_1 plus l_2 . And, the ankle moves from here to the point that is from 3 to 17 it is moving. During the same time the hip is moving from this position to this position in a circular path.

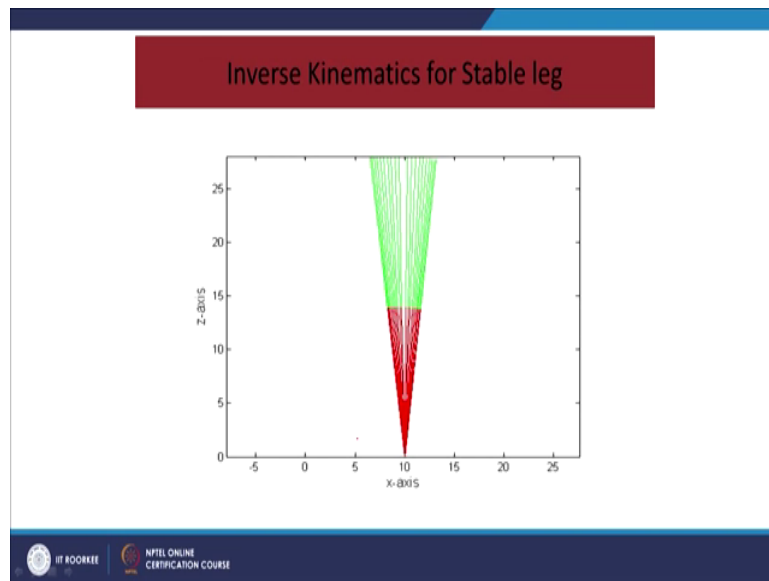
So, the simulation result is shown here.

(Refer Slide Time: 13:31)



And, how the swing leg the angles θ_1 and θ_2 are calculated using the inverse kinematics problems that is given for various trajectories of the ankle.

(Refer Slide Time: 13:40)



Similarly, the stable legs position; the hip is moving in a circular path and accordingly the angles are calculated at each instant of time.

(Refer Slide Time: 13:58)

Zero Moment Point

- Zero Moment Point (ZMP) may be defined as that point on the surface of the ground about which resultant sum of moments of all forces which are active is zero.
- ZMP can be calculated by following equations:

$$x_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) x_i - \sum_{i=1}^n m_i \ddot{x}_i z_i - \sum_{i=1}^n l_{iy} \ddot{\Omega}_{iy}}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) y_i - \sum_{i=1}^n m_i \ddot{y}_i z_i - \sum_{i=1}^n l_{ix} \ddot{\Omega}_{ix}}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

where m_i denotes mass of link i , respective inertial components are denoted by l_{ix} and l_{iy} , absolute angular velocities are denoted by $\ddot{\Omega}_{ix}$ and $\ddot{\Omega}_{iy}$, g denoted the acceleration due to gravity, $(x_{ZMP}, y_{ZMP}, 0)$ denotes coordinates for zero moment point and (x_i, y_i, z_i) denotes coordinates for center of mass of link i .

In link (x, y, z)

acceleration

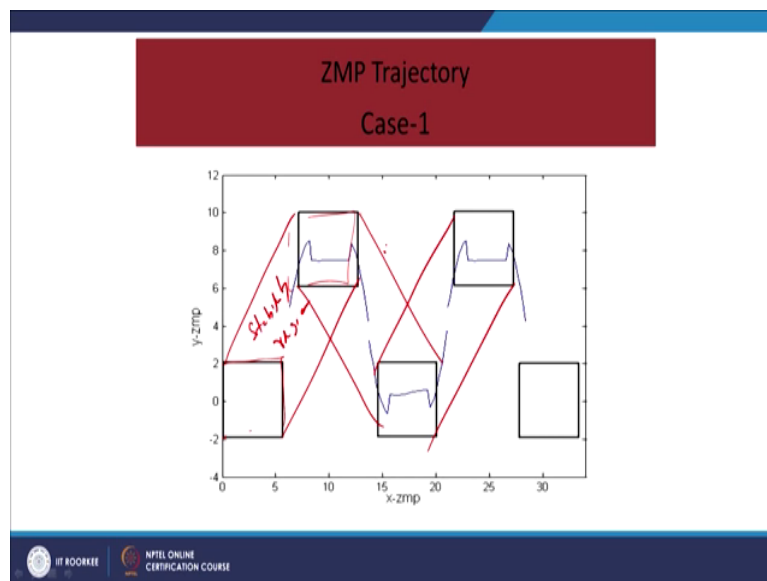
IIT Kharagpur NPTEL ONLINE CERTIFICATION COURSE 17

So, now for the stable walk the ZMP zero moment point is very important and the calculation of the zero moment point is given by this formula the x coordinate and the y coordinate of the ZMP is given by this one which was also seen in the last lecture.

So, here it shows that the omega iy it means that the angular sorry, this is acceleration. The angular acceleration about the axis y the omega y double dot and omega x double dot is angular acceleration about the x axis. \ddot{x}_i is the acceleration in the x direction, similarly in all other directions. So, the i denotes the particular link the i -th link and x_i, y_i, z_i these are the coordinates of the center of mass of a particular link that is a i -th link and if you take the $\dot{x}_i, \dot{y}_i, \dot{z}_i$ that is the velocity of the centre of mass of the i -th link and double derivative gives the acceleration.

So, these values at each instant of time we can substitute and then calculate the x and y coordinate of the zero moment point at each instant of time. So, it is a time because mass is fixed, but this mass gravity these are the fixed values, and \ddot{x}_i \ddot{y}_i etcetera these are the variables as the robot moves they are changing. So, we can calculate this x and y coordinates of the zero moment point as the function of time.

(Refer Slide Time: 16:11)

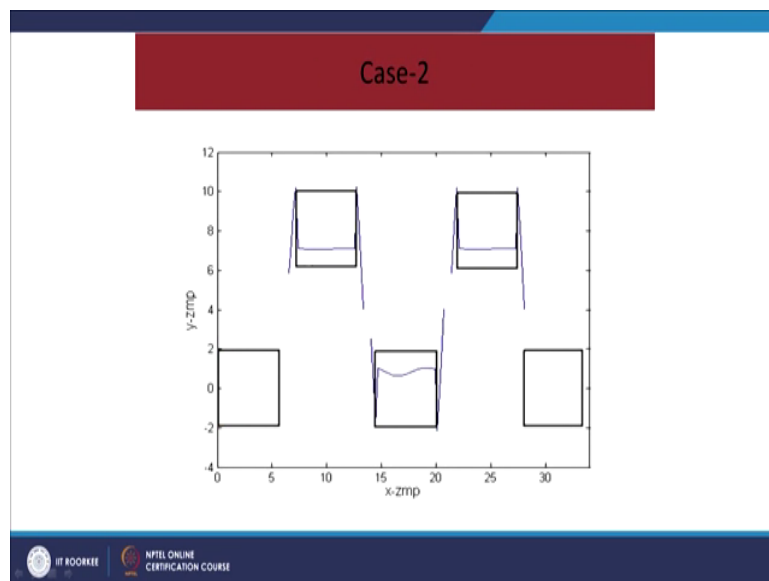


Now, this zero moment point should lie in this stable region. So, we can see that this is the foot of the swing leg and this is the foot of the stable leg and this region is the stable region stability region. So, if the zero moment point lies anywhere in this region then it is guaranteed that the robot will not fall down. So, during the walking the zero moment point always should lie within this region bounded by these red lines and if it goes out of this region then the robot will fall down that is the theoretical result.

So, now we have the calculation of the zero moment point using all the trajectories and the parameters etcetera and we can easily check whether it lies in the stability margin or not. So, for the case 1 when we draw the graph of the zero moment point it starts from the middle of the two hips and it goes to the stable leg above this stable leg and then it comes back again to the middle of this thing during the time interval t_0 to t_4 .

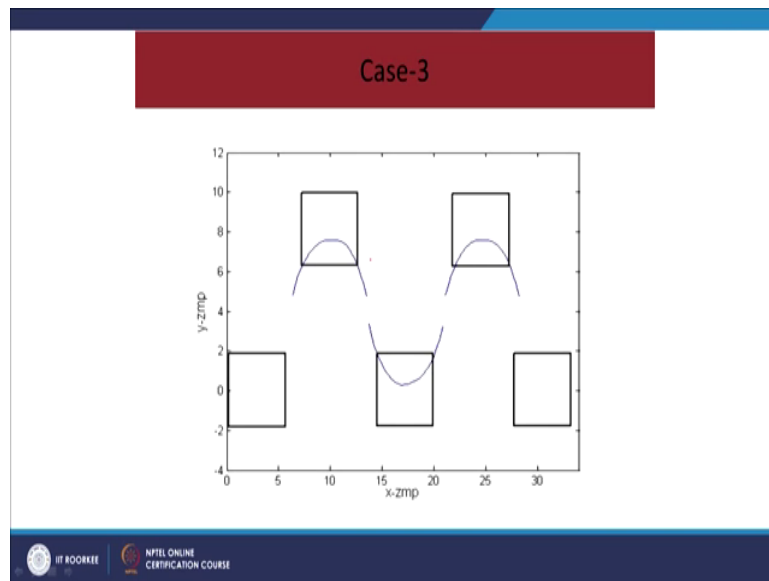
So, the graph of the motion is given by this particular picture and it shows that this is going out of the stable region that is above the foot it should be lying during that particular time interval, but at some position it is going outside the this thing.

(Refer Slide Time: 18:02)



Here in case 2 also we can see that in some position it is in the middle of this stable region just above the just inside the foot region and it comes out at some moment of time. So, during this moments of time it this robot maybe unstable.

(Refer Slide Time: 18:26)



And, for this particular case it is always within this stability region. In the case 3, we calculated for one particular data and it shows that it always lies within the stability region.

(Refer Slide Time: 18:45)

Hip		Upper Body		ZMP
velocity	Time	Trajectory	initial velocity	stability
$v_s = 2.3 \text{ in/s}$	3s	case-1	$y_v = 10 \text{ in/s}$	stable ✓
$v_s = 3.5 \text{ in/s}$	2s	case-1	$y_v = 15 \text{ in/s}$	stable but small margin ✓
$v_s = 4.7 \text{ in/s}$	1.5s	case-1	$y_v = 20 \text{ in/s}$	unstable
$v_s = 2.4 \text{ in/s}$	3s	case-2	$y_v = 16 \text{ in/s}$	unstable
$v_s = 3.5 \text{ in/s}$	2s	case-2	$y_v = 20 \text{ in/s}$	unstable
$v_s = 4.7 \text{ in/s}$	1.5s	case-2	$y_v = 22 \text{ in/s}$	unstable
$v_s = 2.3 \text{ in/s}$	3s ✓	case-3	$y_v = 7.3 \text{ in/s}$	stable
$v_s = 3.5 \text{ in/s}$	2s ✓	case-3	$y_v = 10.3 \text{ in/s}$	stable
$v_s = 4.7 \text{ in/s}$	1.5s ✓	case-3	$y_v = 11 \text{ in/s}$	stable

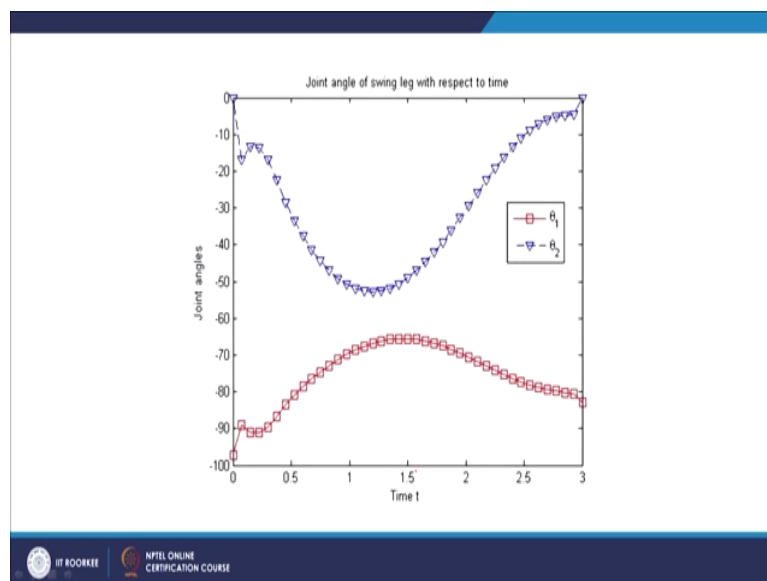
So, this is simulation result the v_s denote the velocity of the hip. Initial velocity of the hip is given by v_s for different types of velocity the simulation is done. The time means in 3 seconds for one step starting from initial position to final position the ankle moves in 3 second for one particular step and the upper body moves as in the case 1 with velocity y_v given by this much of that is 10 units per second is taken as a data and it shows that it is stable that the walking is stable; that means, the z m p lies in the stability region during all the 3 seconds.

But, if we reduce the time that is moves little bit faster within 2 second one step is there then using case 1 this becomes unstable at particular instant of time. So, it will fall down and if you reduce further all the time it is unstable, it will fall down. Similarly, for the other cases.

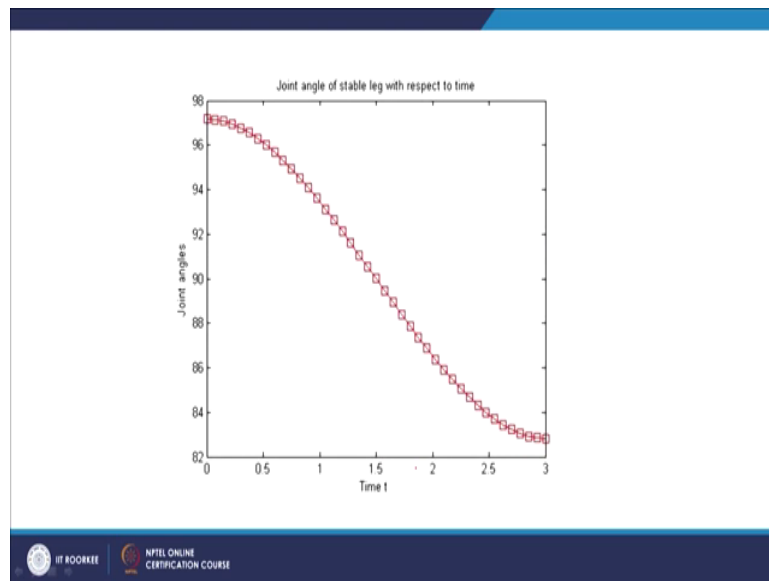
So, we observe that case 3 is a suitable trajectory for the upper body motion if you keep the upper body as given in this third case the upper body moves in this particular manner during the time interval t_0 to t_4 , then for the given trajectories of the ankle hip etcetera then it is showing always a stable motion whether it is 3 second, 2 second or 1.5 seconds of time, but here also we can see that if you reduce further the time then this case also becomes unstable.

So, it is a little bit of trial and error method type and because it is a introductory lecture we can see that we can get a idea about how to derive trajectories and how to check this stability of a walking robot and there are several other sophisticated methods which one can study from the literature.

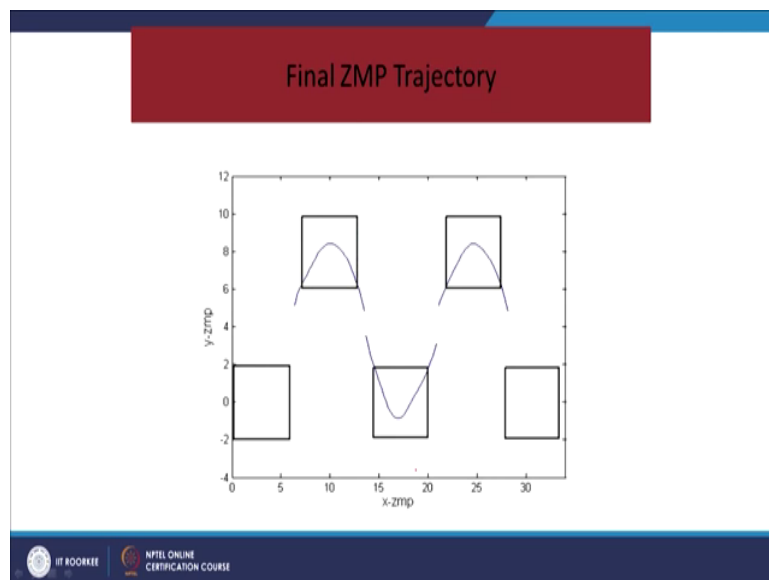
(Refer Slide Time: 21:26)



(Refer Slide Time: 21:27)



(Refer Slide Time: 21:30)



So, this shows that for the particular thing for this time 1.5 second that is the least in the simulation and the velocity is given by this much we have the simulation result that the ZMP is always within this stability margin and it is stable. And, the walking simulation is shown in this particular graph here.

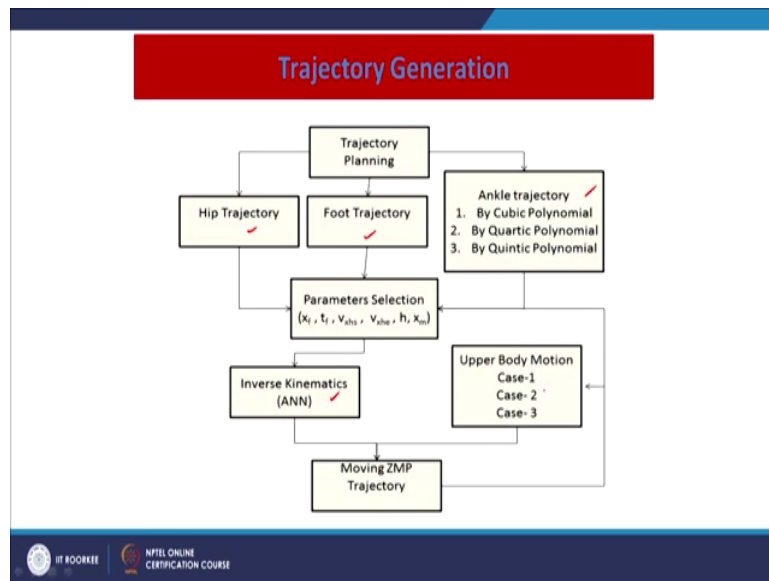
Now, we can see the toe footed model. So, the previous one it was a flat foot. There is no bending of the foot; it is like a rectangle only. Now, we can consider the foot itself is a two link manipulator because there are 2 degrees of freedom ah. So, each leg has 4 degrees of freedom here θ_1 and θ_2 and then θ_3 θ_4 there are 4 degrees of freedom and totally two legs have 8 degrees of freedom and there is a bending of the upper body from one side to the other. So, there are totally 9 degrees of freedom for the robot manipulator for the walking purpose.

And, these are the parameters. The link of the legs and the masses corresponding mass are given here and the units l_1 is 14 units that is from hip to knee is 14 units and knee to ankle is 14 units and the here the ES. So, this heel is denoted by E and the sole joint is denoted by S, toe is denoted by T here. So, in this table ES is 21 3 and ST is 1 4, HU is 1 5 etcetera. So, the data is given this particular table. So, we observed here that the heel to the sole joint it is 21 3 and the ankle is fixed in the middle of ES.

So, l_3 this is 21 3 from heel to the ankle it is l_3 and ankle to the sole is again l_3 and the sole to the toe is l_4 . So, the initial position of the ankle is given by $l_3 \ 0, 0$; this is x direction and y direction is this and z direction is the vertical direction. So, $l_3 \ 0, 0$ is a initial position of the ankle and the final position when it comes flat on the ground it will be l_3 plus the total this thing will be $x \ f \ 0, 0$ that is $x \ 1$ plus $x \ f \ 0, 0$ will be the final position of the ankle.

Other things are as in the case of the flat foot only. The motion of the hip and the trajectory of the ankle and similarly the motion of the hip of the stable leg circular motion with centre at here and radius as l_1 plus l_2 . So, everything is similar to the flat foot model and the procedure is also similar except that we have in addition two link manipulator for the foot because it has it itself has 2 degrees of freedom. So, we have more flexibility for walking. So, this will give some advantage in the stability of the walking.

(Refer Slide Time: 25:24)



So, we have to derive hip trajectory, foot trajectory and ankle trajectory and different types of upper body motion then we can find the inverse kinematics for finding the angles at each joints, different cases of upper body motion for stability and then we can analyze the stability using ZMP ok.

(Refer Slide Time: 26:23)

Swing leg's trajectories:

Ankle Trajectory during DSP

$$x_A(t) = 2l_3 + l_4 + l_4 \cos \theta_{4t}(t) + l_3 \cos(\theta_{3t}(t) + \theta_{4t}(t));$$

$$z_A(t) = l_4 \sin \theta_{4t}(t) + l_3 \sin(\theta_{3t}(t) + \theta_{4t}(t))$$

$$\theta_{3t}(t) = \begin{cases} (\theta_a) \left(\frac{3t^2}{T_s^2} - 2\frac{t^3}{T_s^3} \right) & 0 \leq t \leq t_1 \\ (\theta_a) \left(-4 + 12\frac{t}{T_s} - \frac{9t^2}{T_s^2} + 2\frac{t^3}{T_s^3} \right) & t_1 \leq t \leq t_2 \end{cases}$$

$$\theta_{4t}(t) = \begin{cases} \pi & 0 \leq t \leq t_1 \\ \pi + (\theta_b) \left(-5 + 12\frac{t}{T_s} - 9\frac{t^2}{T_s^2} + 2\frac{t^3}{T_s^3} \right) & t_1 \leq t \leq t_2 \end{cases}$$

Ankle Trajectory during SSP

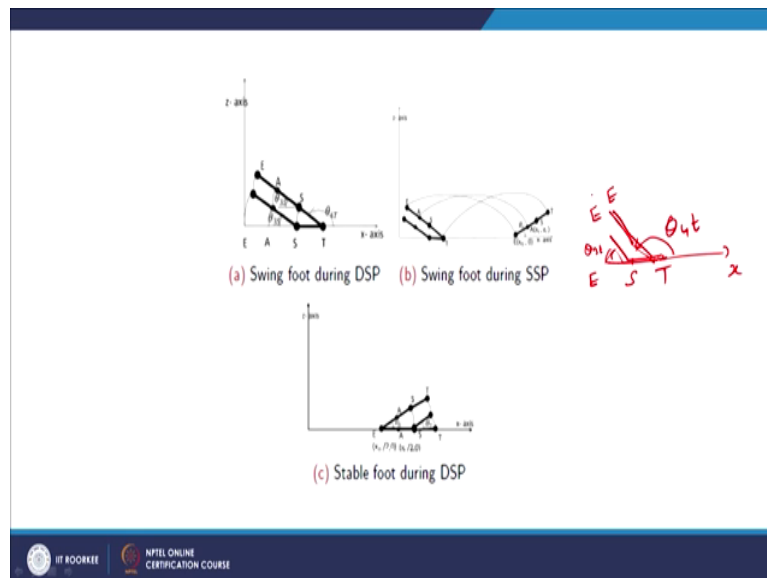
During SSP (t_2, t_f), boundary conditions are

$$x_A(t_2) = x_e; x_A(t_f) = x_f; \dot{x}_A(t_2) = \dot{x}_v; \dot{x}_A(t_f) = 0$$

$$z_A(t_2) = z_e; z_A(t_f) = 0; z_A(x_m) = h_1; \dot{z}_A(t_f) = 0$$

So, these are the various trajectories using the initial condition of the ankle final condition initial velocity etcetera and we can write it as a two arm manipulator kinematics equation and the solution of this inverse kinematics is finding the angles θ_{3t} . So, here this can be explained little bit this θ_{3t} is given by this particular thing.

(Refer Slide Time: 26:56)



The foot initially the foot is on the ground it coinciding with the ground this is E, S and tau. Then, after sometime it rises from the sole to the heel is rising and then toe to sole is also rising. So, we will get the foot like this in exactly the similar manner and this is the x direction, the forward motion. The angles are measured from the x direction to this one. So, this angle is called this one θ_{4t} , this is θ_{4t} and the angle between the extension and this one this is called θ_{3t} at each instant of time. For example, here the θ_{3t} is 0 the TS and SE are coinciding. So, the angle is 0 at this instant of time; at this position it is TS and SE, they are having the angle measured in this direction.

So, using this 2 degree of freedom model we can write the kinematics equation and then we can solve we can write the motion θ_{3t} as a polynomial because we are dividing the 0 to t 2 this is the double support phase that is both the legs are on the ground and half of the time both the legs are on the ground and from t 2 onwards. So, during double support phase it is t 0

to t_2 and this single support phase is when the one of the foot is rising and then moving in the air and then coming to the ground during the time t_2 to t_4 this is single support phase this. So, here we have.

So, this trajectory is for the double support phase, both the legs are on the ground. And, for the single support phase the ankle trajectory it has the initial and final conditions as given.

(Refer Slide Time: 29:47)

Swing leg's ankle joint trajectory

Ankle Trajectory during SSP

By substituting the boundary conditions, we get

$$n_0 + n_1 t_2 + n_2 t_2^2 + n_3 t_2^3 = x_e; \quad (3)$$

$$n_0 + n_1 t_4 + n_2 t_4^2 + n_3 t_4^3 = x_f; \quad (4)$$

$$n_1 + 2n_2 t_2 + 3n_3 t_2^2 = x_v; \quad (5)$$



$$n_1 + 2n_2 t_4 + 3n_3 t_4^2 = 0. \quad (6)$$

The matrix representation for these equations (3-6) is

$$M_{4 \times 1} = A_{4 \times 4} \cdot N_{4 \times 1}$$

Then, the coefficients of the polynomial can be calculated by

$$N_{4 \times 1} = A_{4 \times 4}^{-1} \cdot M_{4 \times 1}$$

 IIT ROORKEE
 NPTEL ONLINE
CERTIFICATION COURSE
33

In this expression, we can derive a polynomial trajectory for the ankle as given here because there are four conditions initial and final position initial and final velocity, we can derive a polynomial of degree 3 has given here.


(Refer Slide Time: 30:06)

Stable leg's trajectories:

Sole trajectory
 $x_S = \frac{x_f}{2} + l_3 \cos \theta_6(t), \quad z_S = l_3 \sin \theta_6(t)$

Toe trajectory
 $x_T = \frac{x_f}{2} + l_3 \cos \theta_6(t) + (l_3 + l_4) \cos(\theta_6(t) + \theta_7(t))$
 $z_T = l_3 \sin \theta_6(t) + (l_3 + l_4) \sin(\theta_6(t) + \theta_7(t))$

Hip Trajectory
 $x_H(t_0) = \frac{x_f}{4}, \quad x_H(t_f) = \frac{3x_f}{4}, \quad \dot{x}_H(t_0) = v_{xhs}, \quad \dot{x}_H(t_f) = v_{xhs},$
 $x_H(t) = \frac{x_f}{4} + \frac{3(x_f)}{2t_f^2} t^2 - \frac{(x_f)}{t_f^3} t^3$
 $z_H(t) = \sqrt{(l_1 + l_2)^2 - (x_H(t) - (x_f)/2)^2}$



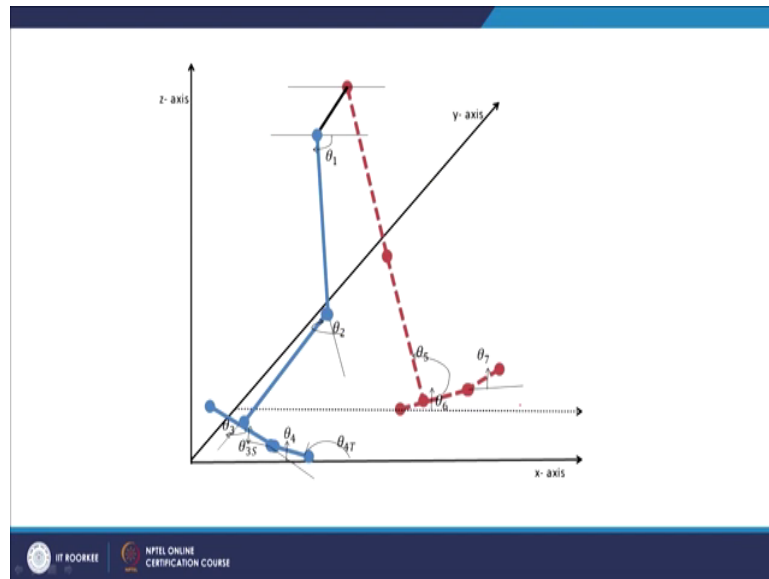
NPTEL ONLINE CERTIFICATION COURSE

34

And, now this stable leg trajectory, the stable leg trajectory as we have already seen it is a circular pass the hip is moving in a circle and the angles for the leg can be calculated according to the speed of the hip etcetera. So, this equations they give the dynamic the kinematics equation of the various position, the sole trajectory and the toe trajectory.

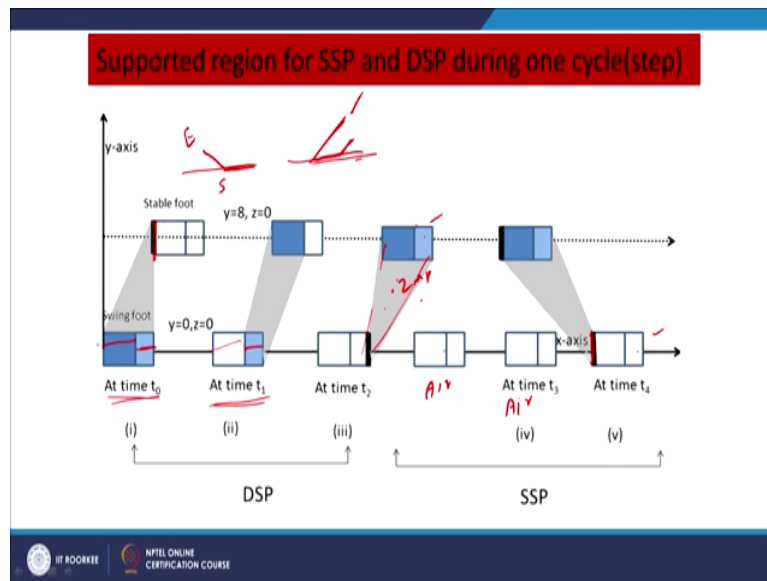
Sole trajectory means the motion of the portion ES; toe trajectory means the motion of the portion TS. The toe to the sole portion that link is moving in this particular manner; the hip trajectory is the motion of the hip this is for the stable leg. So, it is a circular motion has given this particular case.

(Refer Slide Time: 31:04)



So, we see here the stable legs hip will move in a circular path this parallelly this swing legs hip also move in a circular path and the various positions of the foot is given during the motion of the ankle.

(Refer Slide Time: 31:28)



And, the calculation of these ZMP can be done in the same manner as we have seen earlier. And, this picture shows that the foot at various time instance are given. At time t_0 the swing leg is completely on the ground. They are coinciding with the ground and this portion is the toe portion and this is the sole to the heel end of the foot. At the same time the stable foot is also on the this is about to step on the ground.

So, only the heel portion is on the ground and remaining portion is on the air. So, it is touching the ground and it is in this position the white portion means they are all on the ground. Then at time instant t_1 a little bit portion only the toe portion is coinciding with the ground and the remaining portion is lifted above the this portion is on the ground and the S to E portion is lifted here. At the same time the leg the stable leg the most of the portion has

come to the ground and only the toe portion is lifted on the ground and the next instant the entire foot is on the ground that is the stable foot.

At the same time the swing foot is moving in the air here also it is in the air and then it steps down on the ground; here a little bit it is touching the ground remaining portion is it is exactly in the same way the swing leg has become the stable leg from this point onwards. So, the total one particular step is shown here, similarly it will be repeated. And, this shadow region shows the stability region where the ZMP should lie. ZMP if it lies within this stability region then the robot will not fall otherwise if it goes outside the region, then it will fall down at that particular instant of time.

(Refer Slide Time: 34:03)

ZMP Stability Analysis and Trunk Motion

ZMP

$$x_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{x}_i (z_i + g) - \dot{x}_i \dot{z}_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{y}_i (z_i + g) - \dot{y}_i \dot{z}_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

Upper body motion: Case-1

$$y_U(t) = \begin{cases} y_0 + y_v t + \frac{-2y_v t_2 + 3(y_v + y_m/2 - y_0)}{t_2^2} t^2 + \frac{-2(y_v + y_m/2 - y_0) + y_v t_2}{t_2^3} t^3 & t_0 \leq t \leq t_2 \\ y_a & t_2 \leq t \leq t_4 \end{cases}$$

So, this are the calculation of ZMP here we can also add the previous formula we had one more term that is summation m_i summation I the inertia multiplied by ω $\ddot{\theta}$.

But, here we need not have this because we are considering the entire motion as x, y, z linear motion and the rotational motion is not considered because the trajectory itself is giving the position x, y, z at each instant of time as a linear motion.

So, the torque portion is not considered. So, we can omit this particular portion for calculating the ZMP. Even in the previous model we need not consider this one because that is a most general formula where the inertia and the torque etcetera are also taken into account.

(Refer Slide Time: 35:14)

ZMP


$$x_{ZMP} = \frac{\sum_{i=1}^n m_i (x_i (\ddot{z}_i + g) - \ddot{x}_i z_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$


$$y_{ZMP} = \frac{\sum_{i=1}^n m_i (y_i (\ddot{z}_i + g) - \ddot{y}_i z_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

Upper body motion: Case-1

U starts to move from swing leg's hip to stable leg's hip during DSP (t_0, t_2) and stays there all the time during SSP (t_2, t_4).

$$y_U(t) = \begin{cases} y_0 + y_v t + \frac{-2y_v t_2 + 3(y_a + y_m/2 - y_0)}{t_2^2} t^2 + \frac{-2(y_a + y_m/2 - y_0) + y_v t_2}{t_2^3} t^3 & t_0 \leq t \leq t_2 \\ y_a & t_2 \leq t \leq t_4 \end{cases}$$



 NPTEL ONLINE
CERTIFICATION COURSE

(Refer Slide Time: 35:18)

Case-2

Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1 .
Stay there during time t_1 to t_3 .
Again start moving towards middle of legs between t_3 to t_f time where $t_1 = t_f/4$ and $t_3 = 3t_f/4$.

$$y_U(t) = \begin{cases} \frac{y_1}{2} + \frac{3(y_1+y_2)}{4(t_1)^3} t^2 - \frac{y_1+y_2}{2(t_1)^3} t^3 & t_0 \leq t \leq t_1 \\ y_a & t_1 \leq t \leq t_3 \\ \left(\frac{(16(t_3-t_1)^3 - 9(y_1+y_2)t_1^2 t_3 - (y_1+y_2)t_1^3)}{4(t_3-t_1)^3} + \right. \\ \left. \left(\frac{3(y_1+y_2)}{2(t_3-t_1)^3} t_4 t_3 \right) t + \frac{3(y_1+y_2)(t_1+t_3)}{4(t_3-t_1)^3} t^2 \right. \\ \left. - \frac{(y_1+y_2)}{2(t_3-t_1)^3} t^3 \right) & t_3 \leq t \leq t_4 \end{cases}$$

Case-3

Start to move from middle position to stable leg's hip in half time (t_0, t_2), then return back to initial condition in rest of the time (t_2, t_f).

$$y_U(t) = \begin{cases} \frac{y_1}{2} + \frac{3(y_1+y_2)}{4(t_2)^3} t^2 - \frac{y_1+y_2}{2(t_2)^3} t^3 & t_0 \leq t \leq t_2 \\ \left(\frac{(16(t_2-t_4)^3 - 9(y_1+y_2)t_4^2 t_2 - (y_1+y_2)t_4^3)}{4(t_2-t_4)^3} + \right. \\ \left. \left(\frac{3(y_1+y_2)}{2(t_2-t_4)^3} t_4 t_2 \right) t + \frac{3(y_1+y_2)(t_4+t_2)}{4(t_2-t_4)^3} t^2 - \frac{(y_1+y_2)}{2(t_2-t_4)^3} t^3 \right) & t_2 \leq t \leq t_4 \end{cases}$$



IIT Kharagpur



NPTEL ONLINE
CERTIFICATION COURSE

(Refer Slide Time: 35:22)

ZMP



$$x_{ZMP} = \frac{\sum_{i=1}^n m_i (x_i (\ddot{z}_i + g) - \ddot{x}_i z_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^n m_i (y_i (\ddot{z}_i + g) - \ddot{y}_i z_i)}{\sum_{i=1}^n m_i (\ddot{z}_i + g)}$$

Upper body motion: Case-1

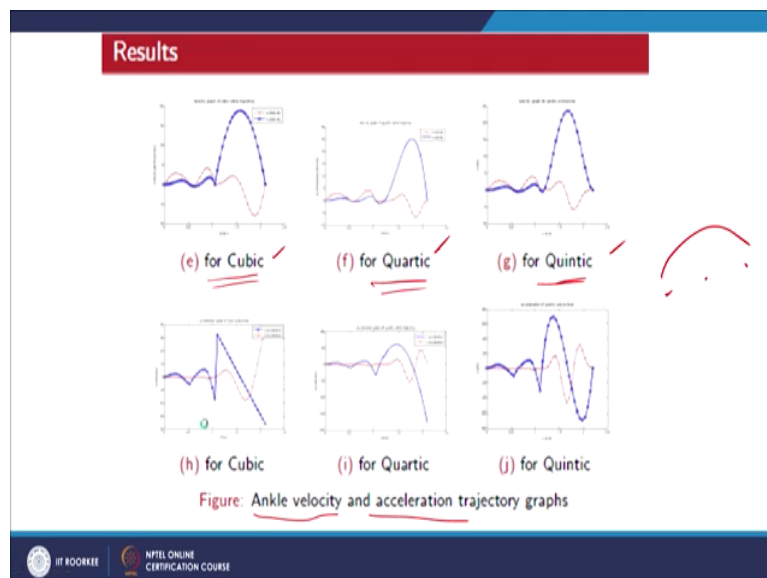
U starts to move from swing leg's hip to stable leg's hip during DSP (t_0, t_2) and stays there all the time during SSP (t_2, t_4).

$$y_U(t) = \begin{cases} y_0 + y_v t + \frac{-2y_v t_2 + 3(y_a + y_m/2 - y_0)}{t_2^2} t^2 + \frac{-2(y_a + y_m/2 - y_0) + y_v t_2}{t_2^3} t^3 & t_0 \leq t \leq t_2 \\ y_a & t_2 \leq t \leq t_4 \end{cases}$$


IIT KHARAGPUR

NPTEL ONLINE
CERTIFICATION COURSE

So, after calculating ZMP we can just consider the same type of cases 3 case that is how the upper body moves here. Here in the first case the upper body from t_0 to t_2 it starts from the swing leg to the stable leg side and it stays there all the time. During the time t_2 to t_4 it is always above the stable leg, only the motion is from t_0 to t_2 . And, similarly the case 2 is described in this case and the case 3 is described exactly similar to the ah flat foot model except that the way of motion of the upper body maybe slightly changed.

(Refer Slide Time: 36:08)



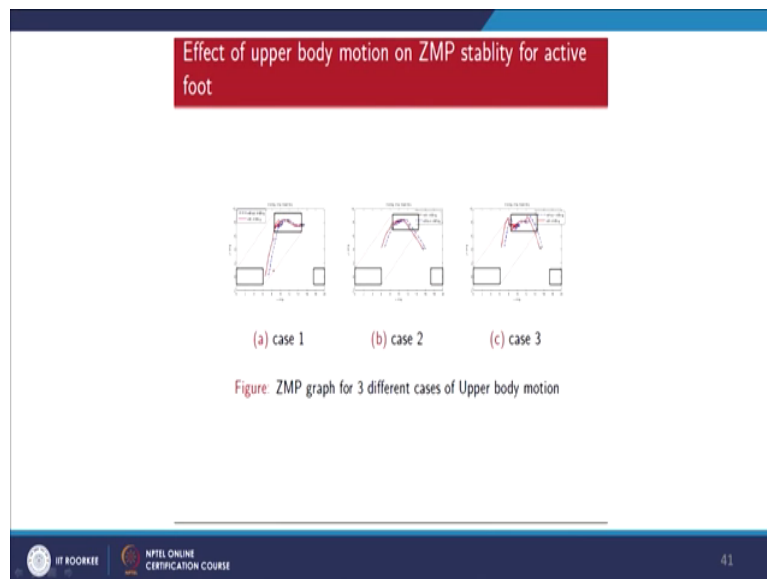
So, in all the 3 cases, we plot the ZMP for various types of trajectories because the trajectory of the ankle is also different for various cases that is if you have four conditions for the motion of the ankle, then we will have a cubic polynomial. If you have five cases five initial and boundary condition we have the fourth degree polynomial and for six conditions, we have fifth degree polynomial.

So, for a various initial and boundary conditions and the middle condition we have various types of a polynomial for the trajectory the ankle trajectory to move from initial to final. For different trajectories, we plot this graphs as shown in this picture. This is the ankle velocity and the ankle acceleration trajectories for various types of polynomial.

So, for example, the ankle trajectory is given here the ankle trajectories are given if you differentiate once with respect to t then it gives the velocity and once with respect to a second

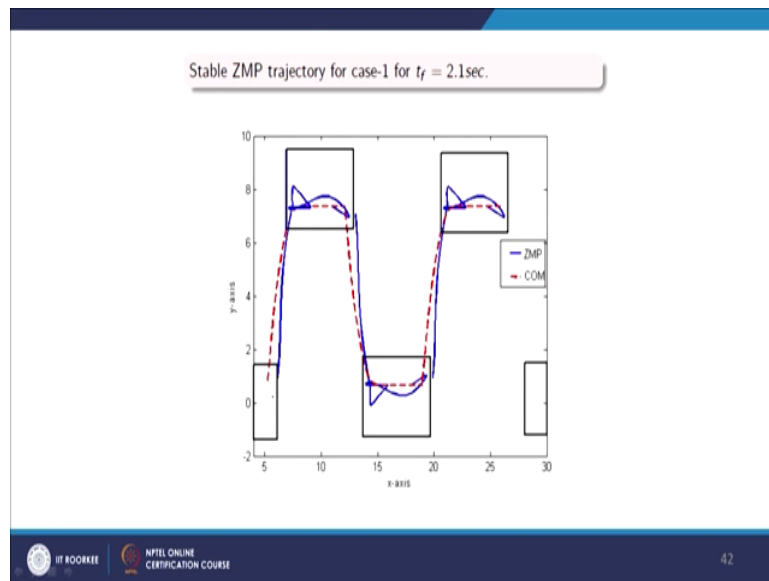
time with respect to t it gives the acceleration. So, these are plotted for various types of trajectories in this model, in this graph and it shows that the acceleration curve is discontinuous in some cases, but it is continuous in the case of the fifth degree polynomial. So, we will get more smoother curve using a fifth degree polynomial ah.

(Refer Slide Time: 38:11)



So, and this picture shows the ZMP of the entire motion using various types of polynomial. This is for case 1 of the upper body motion for case 2 and case 3 the ZMPs are plotted here and it shows the position of the ZMP for various cases.

(Refer Slide Time: 38:28)




This is the stable ZMP trajectory for case 1 for the time 2.1 second it is shown here. So, the blue colour represents ZMP and the red one represent the centre of mass of the object.

(Refer Slide Time: 38:55)

Results				
Upper body	Ankle trajectory	Step Time	Initial hip velocity	Conclusion
No upper body	cubic,quartic .quantic	4.5	$v_h=2.95$	x-ZMP is in the region for $t > 4.5s$ but y-ZMP at middle of hip
Fixed	cubic,quartic .quantic	3	$v_h=2.55$	x-ZMP is in the region for $t > 3s$ but y-ZMP at middle of hip
Moving				

Table: Simulations based observation



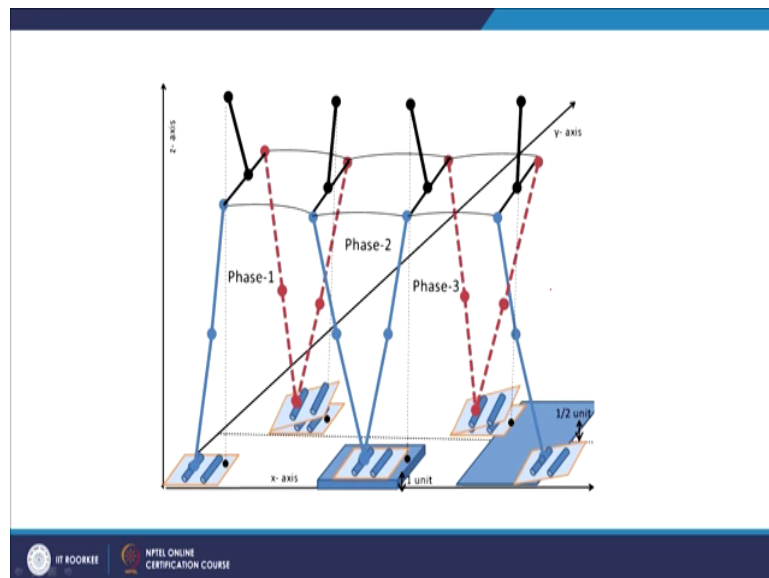
Now, there are three different types of upper body motion. There is no upper body that is only the two legs are considered without any upper body mass is considered to be 0. So, in this case mostly it is unstable. There is a fixed upper body, only it will not move the left and right direction it is always fixed. And, the ankle trajectory is taken to be let us say the cubic for various polynomials, it is again not very stable it shows the some unstable situation.

(Refer Slide Time: 39:35)

Effect of Upper body, ankle trajectory and other Parameters				
Upper body	Ankle trajectory	Step Time	Initial hip velocity	Conclusion
case-2	cubic,quartic,quantic	3	$v_h=2$ $y_g=8$	for last 4 time instant ZMP is out of region
case-3	cubic,quartic,quantic	3	$v_h=2$	unstable
case-1	cubic	2.5	$v_h=2.8$	stable
	cubic	2.2	$v_h=3.15$	unstable
	cubic	2.1	$v_h=3.55$	unstable
	quartic	2.5	$v_h=2.8$	stable
	quartic	2.2	$v_h=3.15$	stable
	quartic	2.1	$v_h=3.55$	unstable
	quantic	2.5	$v_h=2.8$	stable
	quantic	2.2	$v_h=3.15$	stable
	quantic	2.1	$v_h=3.55$	stable
	quantic	2	$v_h=3.65$	stable
	quantic	1.8	$v_h=3.65$	unstable

But, if you take moving upper body for case one for different types of angle trajectory cubic trajectory, fourth degree, fifth degree polynomials and for different time instances that is one step takes 2.5 seconds or 2.2. So, different types of time steps and the velocity the initial velocity of the hip is given by this numbers. So, for various combinations we try the stability of the robot.

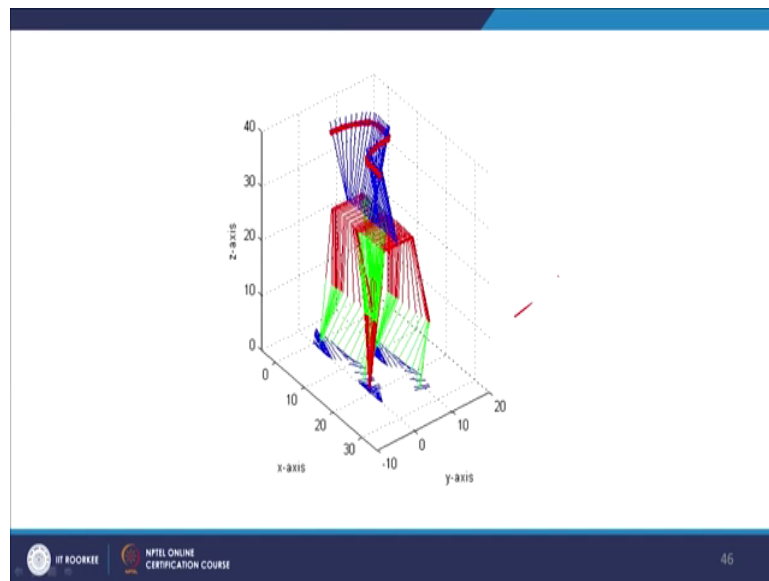
(Refer Slide Time: 40:12)



And, it shows that in most of the cases it is stable. For example, in one 2.1 second with velocity 3.5 it is stable the stability margin is more in this particular case. So, that is shown in this particular picture that it is always within this stability region and the robot is moving in a stable manner.

In other cases also it is stable ah. So, the case 1 is considered to be the best possible trajectory for the upper body motion so that the robot does not fall.

(Refer Slide Time: 40:55)



So, so this picture shows how the robot moves for the 2.1 for one step it takes 2.1 second, yeah. So, it is a slow walking pattern and with this particular model we are able to show some simulation for the flat foot and toe foot models.

So, the next lecture we will consider some neural network based the derivation of trajectories and the kinematics, inverse kinematics etcetera for biped robot as well as the robot manipulators.

Thank you.