Robotics and Control: Theory and Practice
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Lecture – 16
Manipulator Control

Hello viewers, this lecture is on Manipulator Control. So, in this lecture, we will see how to compute the control of your robot manipulator for performing a assigned task. So, in the previous lectures, we have seen how to derive the dynamics equation of a robot manipulator.

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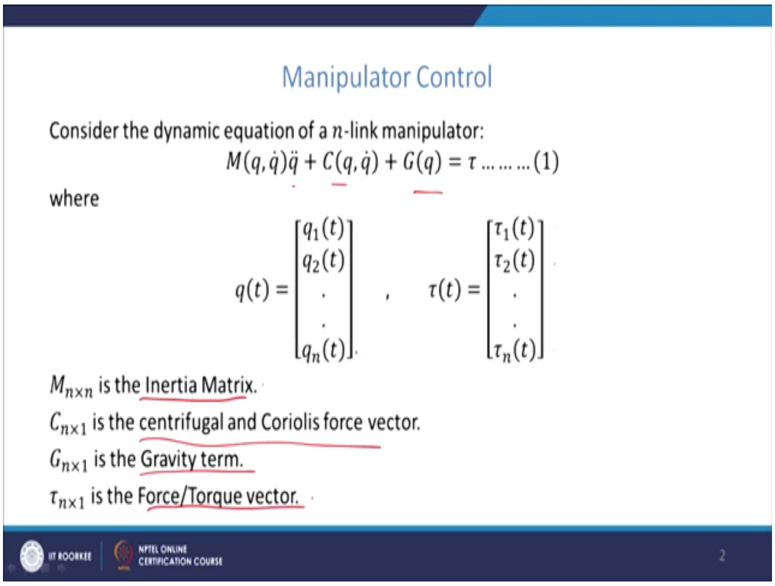

So, we have seen that it is written in the form of a system of second order differential equations, like M q double dot and C times C of q q dot plus G of q equal to tau, where q is the vector containing all the joint variables q 1 q 2 q n ah. So, some of them may be revolute joint some of them may be prismatic.

So, if it is revolute joint q i denotes the joint angle and if it is a prismatic joint then q denotes the linear motion of the joint. Similarly, tau denotes the torque at each joint tau 1, tau 2 up to tau n. ah. For revolute joint tau i denotes the torque and for prismatic joint tau tau i denotes the linear force applied at that joint.

So, here this M q q dot denotes the inertia matrix and C qq dot denotes the centrifugal and Coriolis force, G is the gravity term and tau denotes the force or torque vector applied at each joint. So, for each robot manipulator, we can derive such a system of differential equation representing, the equation of motion.

Now, the problem is to find the control, control is here the torque vector tau of t for a particular task for an assigned task how to find the control torque.

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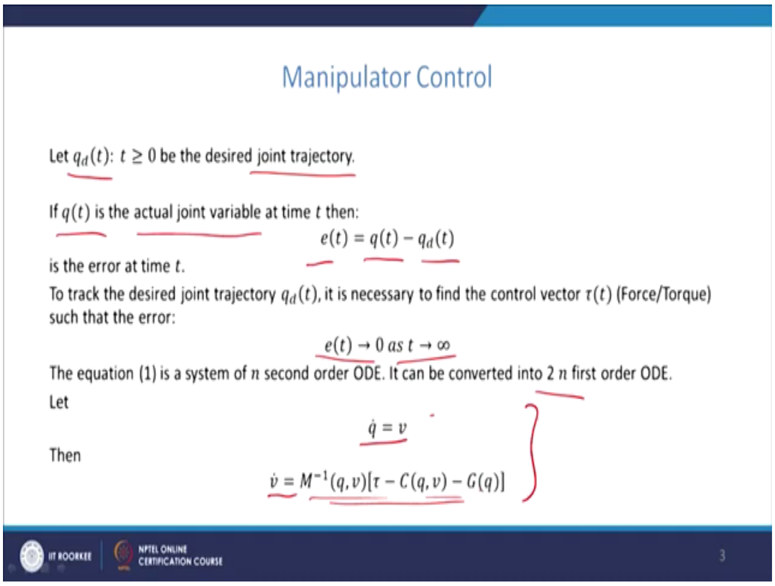

So, let us assume that q d denotes the desired joint trajectory. So, joint trajectory means the desired angles or the linear motion of each joint. So, the q of t the desired value is given by q t and q t denotes the actual joint variables. So, the current value of q is given by q t. So, the error between the current value and the desired value denotes the error at time t. So, it is a vector n dimensional vector. So, the aim is to find the control the torque vector, in such a way that the error converges to 0 as t tends to infinity.

So, in theoretically we say that error should tend to 0 as t tends to infinity, but in practice the error must tend to 0 as quickly as possible ah. So, that the desired trajectory is tracked. Now, we can convert the given system of second order equation into 2 n first order differential equation.

So, each second order equation can be converted into 2, first order equation by substituting. So, if you define q dot equal to v, then you can write v dot equal to q double dot and the q double dot from this equation 1. If, you keep the q double dot in 1 side and take all the remaining terms to the right hand side, we get v dot is q double dot which is equal to M inverse of tau minus C of q v minus G of q.

So, it is the same equation written in 2 first order differential equation form. And, each one is a n equation because q is a vector containing n elements. So, here there are n equations and the second one there are n equations, totally there are 2 n first order differential equations.

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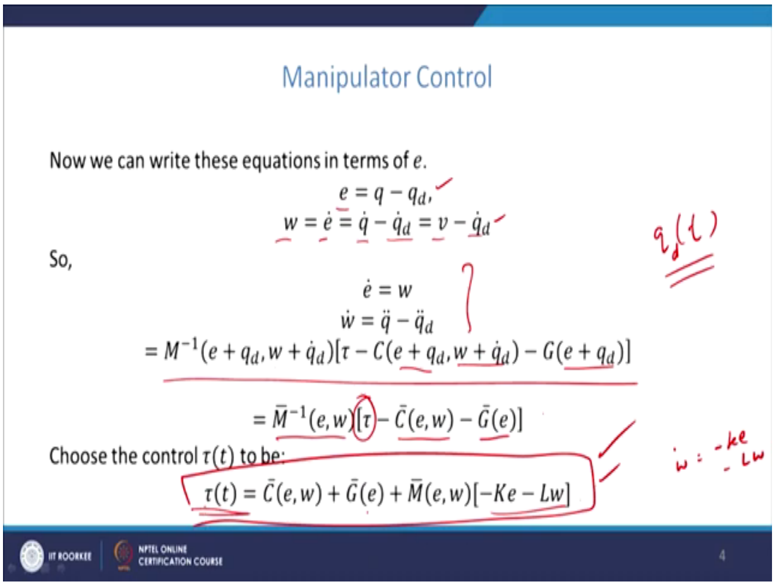

Now, according to the notation introduced earlier e is q minus q d and w, if you call is e dot we can write the same equation in the error form. So, e dot is equal to w we can give this notation and w dot is nothing, but e double dot. So, e double dot is from here it is q double dot minus q d double dot or in other words it is p dot minus q d double dot. So, we can get w dot is given by from the previous slide, we can write this one v dot is given by this last line.

So, ultimately w dot is given by this expression the M inverse e plus qd, because q means e plus q d from this equation and w plus q d dot from this equation. Similarly, C of this is q and this is q dot and G of q is substituted. Now, when we write the same equation only in terms of e and w we can give this particular notation, instead of writing M inverse of e plus qd etcetera we can write it in a shorter form as M bar inverse e w, a function of e and w. And, similarly C of e plus q d etcetera can be written in the shorter form as C bar e w and this can be written as G bar e.

So, the same dynamic equation is converted into a form in terms of the error vector e and it is derivative w in this way. Now, we have to compute the control, control tau for tracking their desired trajectory q d t is the desired joint trajectory we want to track.

So, we can define this torque or the control in this form, that is C bar ew plus G bar e plus M bar multiplied by minus K e minus L w. How it is selected it is just the observing the equation, here we have tau minus C bar minus G bar. So, if you take tau to be C bar plus G bar this 2 will get cancelled. So, the remaining term is simply M bar into minus K e minus L w.

So, what we will get is if we substitute this particular tau we will get the equation to be the simple 1, the equation the second equation will become after substitution this M bar inverse will get cancelled with M bar, and the remaining will be simply w dot will be minus k e minus L w. So, only this much we will get.

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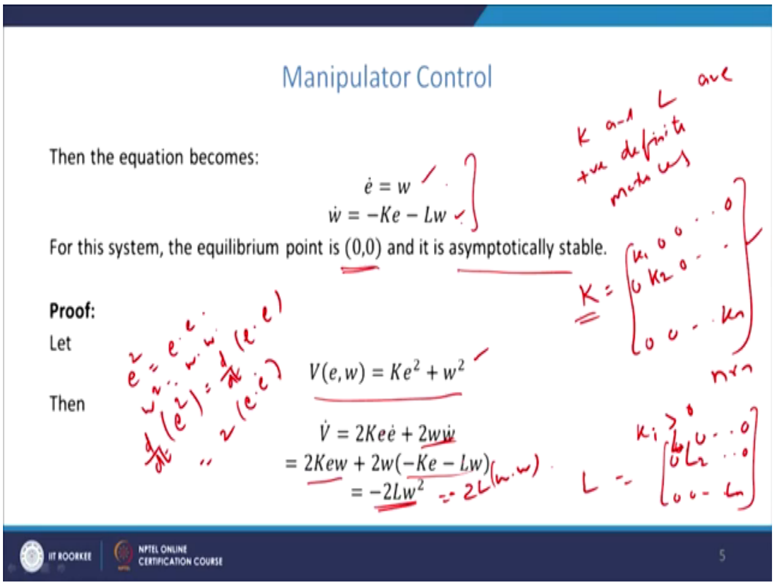

So, what we get the error system to be e dot is equal to w and w dot is minus K e minus L w, it is a very simple system of differential equation. And, we can easily verify that 0 0 is the equilibrium point. If, we equate the right hand side to 0, we will get w equal to 0 and putting it in the second equation, we get e equal to 0. Here, K and L are positive definite matrices.

So, for example, we can take K to be k 1 0 0 and 0 k 2 0 0 etcetera. Here n cross n diagonal matrix, where all the diagonal entries are positive numbers k i is greater than 0, strictly greater than 0, if we take then it is a positive definite matrix a example of a positive definite matrix. So, it this will be very useful in this type of control problems. A positive definite matrix can be it is defined as a matrix whose all eigenvalues are positive.

So, it need not be always a diagonal matrix, but here we can select this k to be like this similarly L to be here, diagonal matrix with all positive diagonal entries. So, so by selecting like this we can easily see that 0 0 is the equilibrium point for this system. And, we can also show that it becomes asymptotically stable by selecting this k and L suitably so, that we can see through the Lyapunov theory, which was done in the previous lecture.

So, let us assume that the Lyapunov function is taken to be like this, in terms of e and w v of ew is K e square plus w square. So, first thing is K is positive definite as we have selected and therefore, this function the right hand side is a positive definite function, because it is 0 only at e and w both are 0 and it is positive for all other values of e and w the matrix.

So, when we say because e is a vector here e square it means it is not the scalar square, it is the vector e dot e the norm of e squares, in similarly w square means w dot w the scalar product of these vectors. So, when we take the derivative v dot it is 2 k times e dot, when e is a vector e dot is also a vector, this is the dot product of this vectors, because when we say differentiation d by dt of e square. The meaning is d by dt of e dot e.

So, that is nothing, but 2 times e dot e dot ok. The derivative of e and e dot so, that is written in this way, similarly w square when we differentiate we will get 2 w dot it means the dot product of w vector and w dot vector. So, now we can take this expression by substituting the values of e dot and w dot from this 2 equation, we get 2 K e e dot is w that is the first term and 2 w and w dot is given by this.

So, it is easy to see that the ultimate value is 2 L w dot w this is nothing, but minus 2 L w dot w vector. So, this is negative definite function, because it is always negative value, this minus sign is there and all the things are positive.

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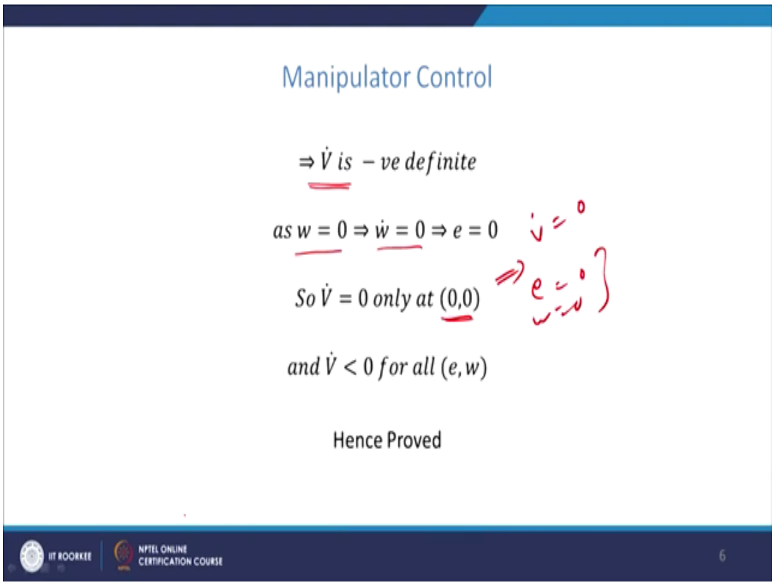

So, it is a negative definite value, even because when w is 0 automatically it will imply that e is also equal to 0, because when w equal to 0 w dot will be 0 and then both w and w dot or 0 from the equation given here, you can say that e is also equal to 0. Because, when w is 0 w dot is 0 K e is equal to 0 and K is positive definite therefore, the vector e is 0 vector.

So, the v dot the scalar is always negative and it is 0 only at the e and e is 0 and w equal to 0 ok, v dot equal to 0 that implies e is 0 and w 0. So, it is negative definite. So, this implies that, the system is asymptotically stable, when we select the control in this way. We get the asymptotic stability of this system; we have shown that the system is asymptotically stable in this way.

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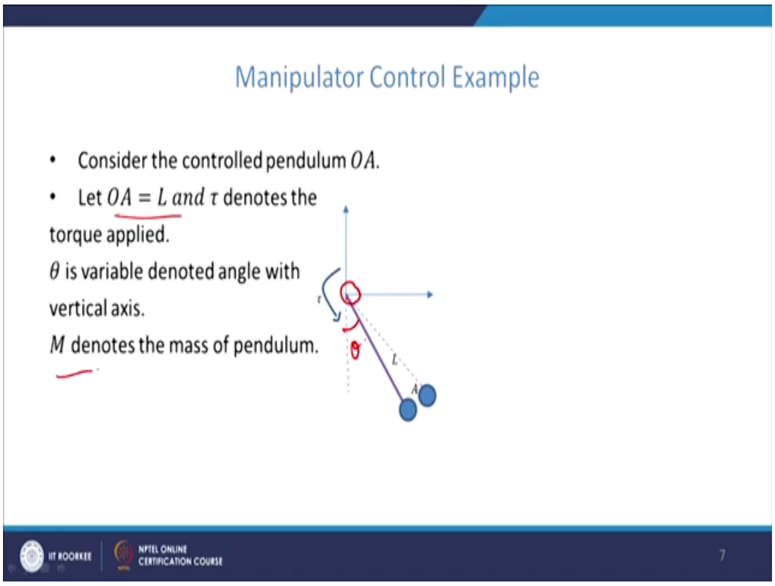

Now, we take an example of computing the control of a simple 1 arm manipulator ok, single arm manipulate or we can say that a pendulum with control torque, that is applied at the joint here.

So, if you assume that the simple the pendulum single arm manipulator, the length is L and theta is measured from the vertical downward vertical, then and M is the mass of the pendulum.

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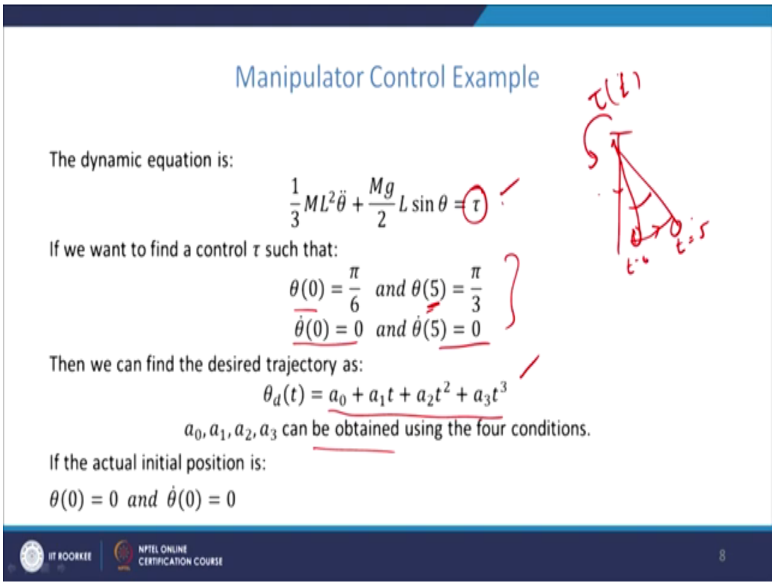

Then, we know that the equation is written in this particular form. We have already seen the dynamic equation of the single arm robot manipulator is written like this, where the torque access the control for this one. Now, we want to find the control of this pendulum, in this particular manner. That is a time T equal to 0 the angle is pi by 6, 30 degrees. And, a time t equal to 3 5 unit the angle is 60 degrees.

So, the pendulum is at 30 degree when t equal to 0, when t equal to 5 it is at 60 degrees and the derivative; that means, the velocity of the pendulum is 0 at time t equal to 0 and 5 both.

So, it should start from here and then stop at this point, that is the meaning here. For this how to find a torque the torque is applied at the joint as a function of time.

So, let us assume that the desired position of the pendulum theta desired is given by a polynomial of degree 3, because we have 4 conditions here. The trajectory is planned like this, we have seen in the trajectory planning lecture how to plan a polynomial trajectory. So, here we have 4 conditions. So, we can write a polynomial of degree 3 and a naught a 1, a 2, a 3 are the coefficients, which we have to find using this 4 conditions, that can be very easily obtained.

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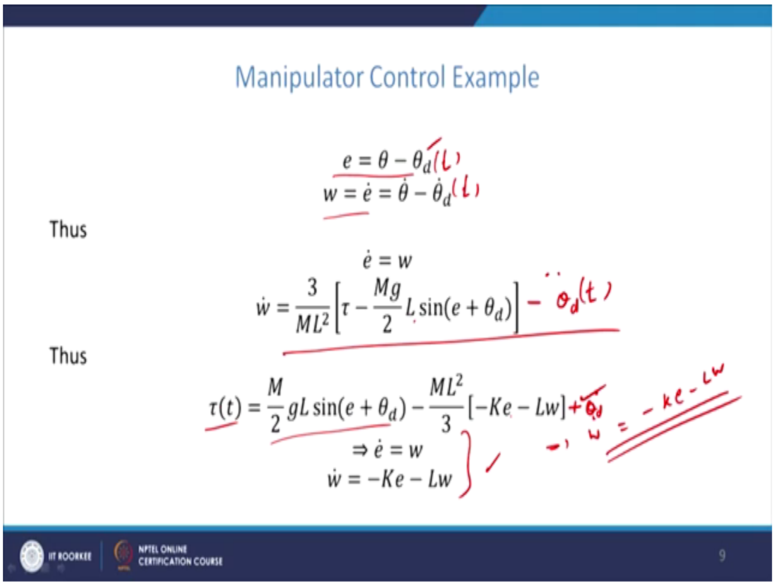

So, now after obtaining the desired trajectory as a function of time, we define the error actual angle is theta and desired angle is theta d as a function of time. These are all functions of time and w is e dot the derivative of e and that is nothing, but theta dot and theta desired dot the derivative. Now, we define the system in the form of error and it is derivative.

So, let us say e dot is w as given here and w dot is nothing, but theta double dot minus theta d double dot. So, if you substitute that from here we will get this expression w dot is nothing, but theta double dot, theta double dot is from this equation we can get it is tau minus of M g L sin theta by 2 etcetera. So, we can easily calculate from that expression this one.

So, w dot is given by this expression 3 by M L square tau minus M g by 2 L sin of e plus theta d, that is coming from this equation, this is for theta double dot. I think we have to add this term also because w is given by this minus theta d double dot of t. So, w dot is actually this term, then we can select this tau to be given by this equation.

So, tau if you take the first term this will cancel the term M g by 2 L sin theta of this thing. So, this term is getting cancelled with this term and then we can write M L square by 3 into minus K e minus L w.

So, if you substitute in the place of tau this whole expression, we will simply get the expression to be equal to this will imply w dot a is equal to we get simply minus K e minus L w. So, here one term is missing that is plus theta d double dot.

So, we have to include this theta d double dot in the selection of tau. So, that finally, we will get only this term. So, by selecting this control, what we get is in the in the form of error e dot equal to w, w dot is minus K e minus L w.

So, as we did in the previous description, exactly similar equation we are getting. So, by taking a Lyapunov function of this form K e square plus w square the same thing can be done here, we can show that this system is asymptotically stable. In other words it says that the tau which we have selected here will make the system the pendulum to travel along this particular desired trajectory. As t progresses the error of the desired trajectory and the actual trajectory will tend to 0. So, that is the meaning of this expression.

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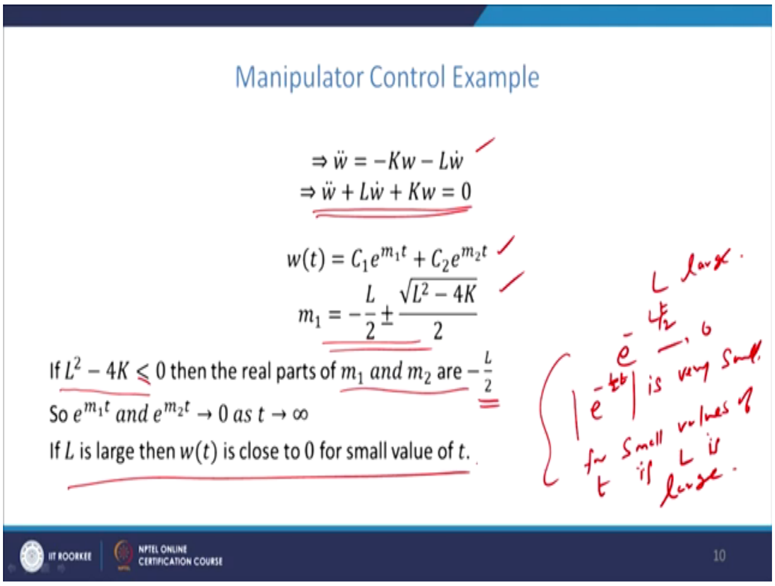

Now, we can see here how quickly we can make the error to be 0 by selecting suitably L and K. So, here w double dot is given by this expression minus K w minus L w dot, because from the previous equation, we have w dot equal to minus K e minus L w, if we take double dot of this we get this expression.

So, this is nothing, but a second order ordinary differential equation and this expression it gives the roots of this auxiliary equation to be m 1 and m 2. So, w of t is nothing, but C 1 e to the power m 1 t plus e 2 e power m 2 t, where the roots of the auxillary equation is given by this expression. If, you select L square minus 4 K less than or equal to 0, we can see that the real parts of the roots m 1 and m 2 are simply minus L by 2.

So, if we take L very large, then e to the power minus L by 2 tends to 0, L by 2 into t it converges to 0 or it converges to close to 0, in a shorter span of time. So, e power minus L by

2 t this modulus is very small for small value of t, if L is large. So, by selecting a large value of L we can make the error tending to 0 in a shorter period of time. So, in that case we can say that the trajectory will be tracked by using the control given by this formula.

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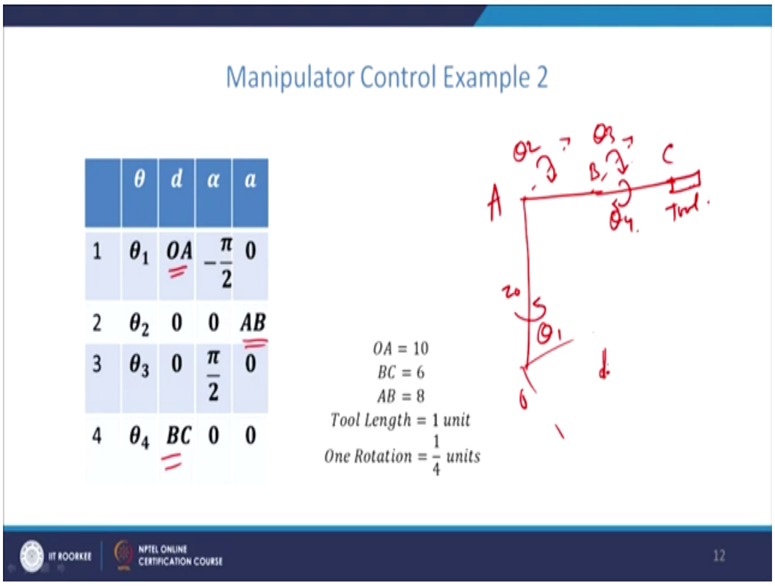

So, we have the robot manipulator, where OA is this and AB is the second link and BC is the third link with the tool in the as the end effector.

So, here the AB is a revolute joint, first joint is revolute, the second is revolute joint with rotation of this type, the third is revolute joint and the fourth is also a revolute joint. So, the 4 joints all of them are revolute and by using the d h algorithm, we have already seen we can write the coordinate frames and the joint and link parameters in this particular form.

So, here OA is given by this expression this is a joint distance. And, similarly BC is joint distance AB is the link length. According to the parameters theta 1, theta 2, theta 3, theta 4 these are the joint angles, theta 3 and theta 4.

So, with this adjustment of these 4 angles one can perform a desired task using the robot manipulator. So, here in this example it is starting from a initial position and then reaching a position on the ground and inserting the tool at a particular point.

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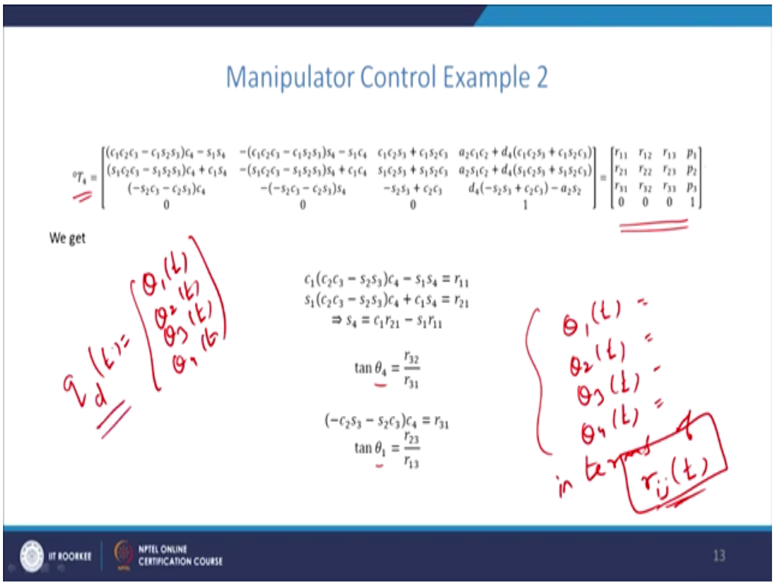

So, for performing this task we can design the problem in the following manner. So, firstly, the arm matrix 0 T 4 as we have seen in the inverse kinematics lecture, the 0 T 4 for this particular manipulator is given by this expression. So, if we know the 0 T 4 for each instant of time as a function of time, then we can solve the inverse problem for all the angles theta 1 as a function of time theta 2, theta 3, and theta 4.

So, all the angles can be obtained by solving the inverse kinematics problem, in terms of this r ij, these are all functions of time in terms of r ij as a function of time.

So, the main problem here is to give r i j as a function of time and then solving the inverse problem theta i i equal to 1 to 4 as a function of time. So, once this is done this is our Q desired, Q desired as a function of time is given by theta 1 t theta 2 t theta 4 t.

So, this is calculated by this one. So, main problem here is now to give this rij as a function of time. So, once it is given we can easily obtain what is Q desired, once Q desired is given we can get the torque as demonstrated in the previous examples.

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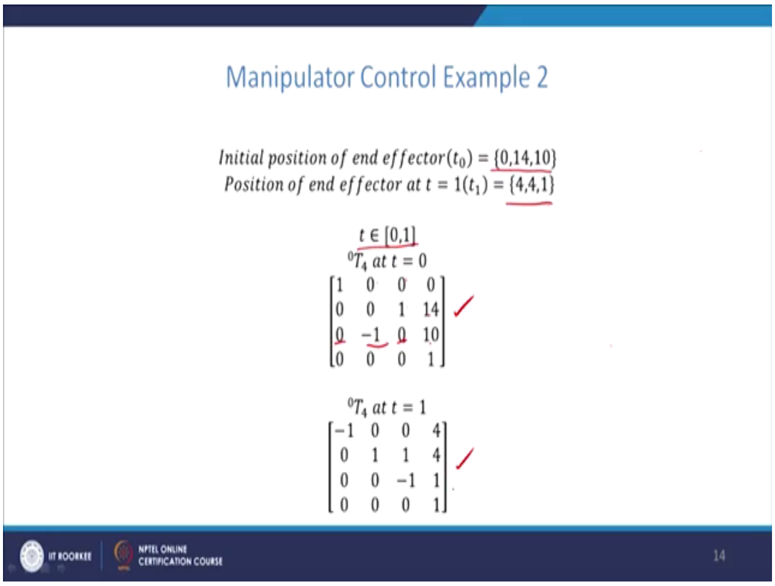

So, let us assume that the robot performs a desired task in 2 unit of time.

So, from the interval in the interval 0 to 1, time interval 0 to 1, it performs the job of starting from the initial position and reaching the final position. And, then in the interval z 1 to 2 it is performing the job of rotating the screw for 1 full rotation. So, these are the two segments of the task assigned to the robot.

So, let us assume that at time t equal to 0 the end effector is at this position 0, 14, 10. And, at the final time t equal to 1, it is a deposition 4, 4, 1, that is from the ground the z value is 1 unit above, that is the final position of the end effector. So, initial position is given final position, the origin is at the 0.014, 10 and the x axis of the end effector is 1 0 0 0 0 minus 1 and 0 1 0.

So, it this means that the end effector x axis is parallel to x axis of the base. And, it is the negative z axis of the base, the y axis is 0 0 minus 1, it means it is pointing downwards and the z axis is 0 1 0 it means the z axis of the end effector is parallel to the y axis of the base. Same way, we can define the required position and orientation of the end effector at t equal 1 is given by this expression.

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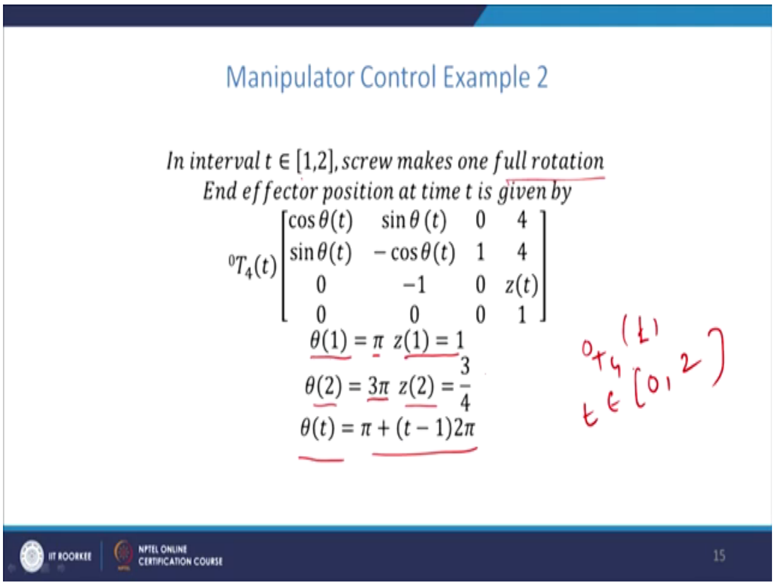

So, there are two conditions therefore, we can decide a trajectory as a polynomial of degree 2 degree 1. So, now, during the time 1 to 2, this crew makes 1 full rotation, that is angle 0 2 angle 2 pi it will make a rotation.

So, at time t equal to 1 the angle is pi and at the time t equal to 2 it is 3 pi. And, the height of the tool is at 1 unit at time t equal to 1, after 1 full rotation the screw has gone inside 1 by fourth unit. Therefore, from the ground the height will be 3 by 4 at time t equal to 2.

So, from this information we can easily calculate theta of t should be like this. If, you substitute t equal to 1 we will get pi at t equal to 2 we will get 3 pi.

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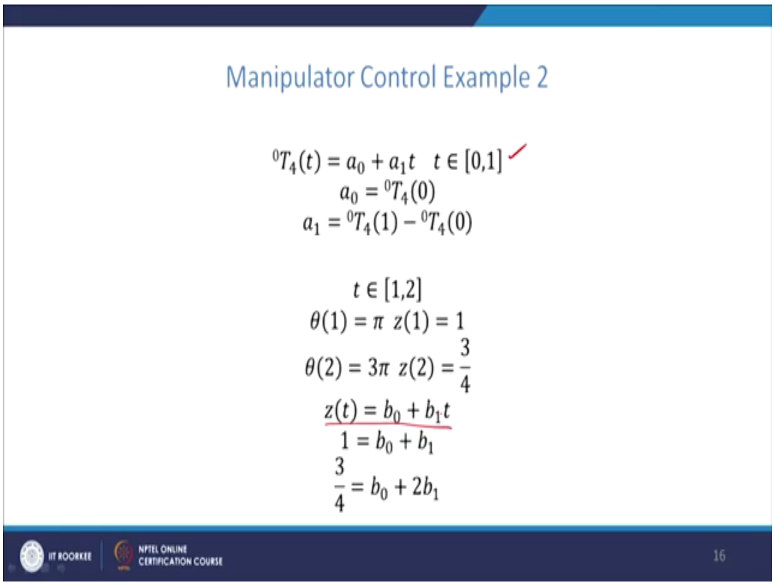

Similarly, we can calculate z of t as a function of time, and 0 T 4 as a function of time during the interval 0 to 1.

Because, there are 2 conditions given here, these are the 2 conditions at t equal to 0 and 1; we get a polynomial of degree 1. There are 2 coefficients a 0 a 1 which can be easily calculated from this expression. And, z of t is also calculated by using a polynomial with the 2 coefficients. So, it is done using the values z of 1 is 1 and z of 2 is 3 by 4. So, once we get all this information we get 0 t 4 as a function of time t belongs to 0 to 2, the total interval of time.

So, for each instant of time we solve the inverse kinematics problem using this formula, various formula is given.. Here theta 4 and theta 1 is solved. Similarly, we can solve for theta 2 and theta 3 using all these values. So, we can obtain q desired as a function of time during the entire interval. And, then this gives the control using this procedure. As we have seen the procedure for the general robot manipulator we have seen how to select the control. So, this procedure will give the control of the robot manipulator using the designed q value.

So, as we have seen for the general robot manipulator how to calculate the control using the formula given in the last line that is tau equal to C bar plus G bar plus M bar into minus K e minus L w.

And, where K and L are selected suitably to make the system asymptotically stable so, using this procedure we can compute the control for the example 2 and we get the desired torque. So, that the assigned task is performed in a robust manner ok.

Thank you.