Robotics and Control: Theory and Practice

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Lecture – 13
Dynamics of Manipulator (cont.)

This lecture is about the Dynamics Equation of the two arm manipulators. In the previous lecture we have seen, how to write the dynamic equation of one arm manipulator and the two arm manipulator with pointed mass. So, in this example we see two degree of freedom manipulator with uniformly distributed mass.

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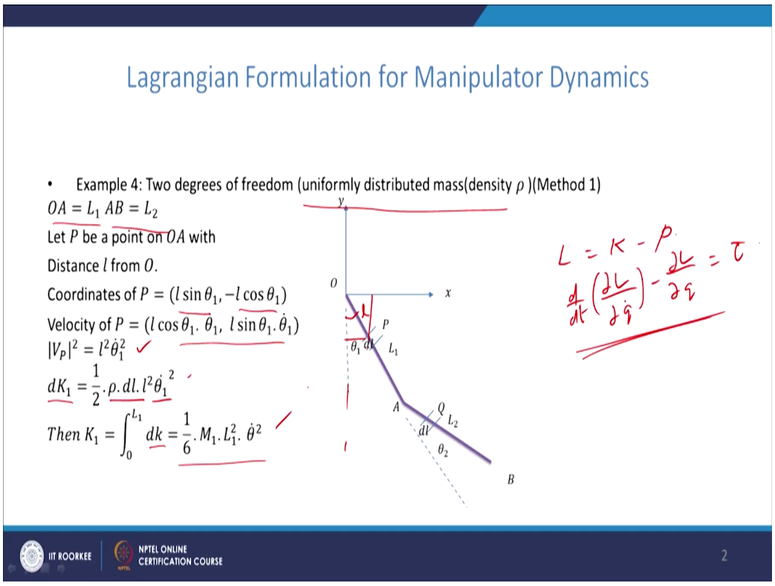

So, the density is constant throughout the manipulator. So, as we recall we have to first construct the Lagrangian, that is the kinetic energy minus potential energy, and then the equation can be formulated that is d by d t of del L by del q dot minus del L by del q equal to torque is the general dynamic equation of any number of arms.

So, now we consider this robot manipulator O A is the first link and AB is the second link and the mass is distributed uniformly. So, the length O A is L 1 and length AB is L 2 let P be any point on O A at a distance of L from the point O. Then, the coordinate of the point P is given by this thing the x coordinate and y coordinate.

So L sin theta is the x coordinate and minus L cos theta is the cos theta 1 is the y coordinate, where theta 1 is measured from the vertical line downwards. Then, the velocity square of that point P, the modulus of the velocity square is small l square theta 1 dot square, directly by finding the modulus square of this vector.

So, the kinetic energy of yeah element of length d l, small length d l is taken and so, it is given by half into mass is density into the length d l into the velocity square. Velocity is l square theta 1 dot square at that point. Now, if we integrate from the 0 to the total length L 1 of this kinetic energy of the element, we get the total kinetic energy of the first link.

So, we can easily see that the integration d k is given by this expression.

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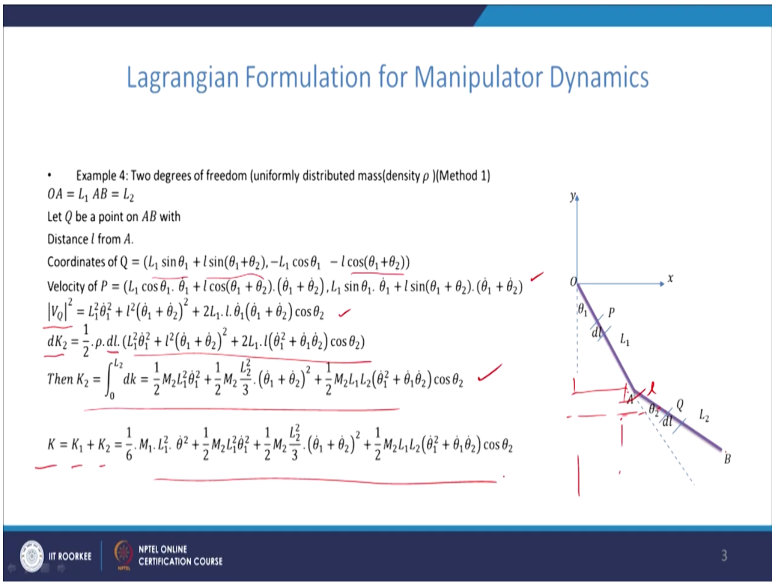

Now, we take the second link for the second link the let Q B any point on this link AB and it is at a distance small L from the point A. So, the coordinate of the point Q is the x coordinate. For example, it is it is L 1 sin theta 1 is this length plus the extra length that is L 1 sin theta 1 plus small l sin of theta 1 plus theta 2, the angle theta 1 plus theta 2. So, that is the x coordinate. Similarly, y coordinate is the negative y axis.

So, we get minus L 1 cos theta 1 minus small l cos of theta 1 plus theta 2. And, velocity is the time derivative of the position vector which is given by this expression, and the modulus of the velocity square can be directly calculated by taking the x component square plus y component square.

So, the kinetic energy of the small element of length l is half into mass in to the velocity square, it is this one and the total kinetic energy is the integration from 0 to L 2 the length of the second link that is given by this expression after directly integrating and substituting the limits we get.

So, the total kinetic energy is K 1 plus K 2 the kinetic energy of link 1 plus kinetic energy of link 2 that is given by this by adding the previous 2 equations.

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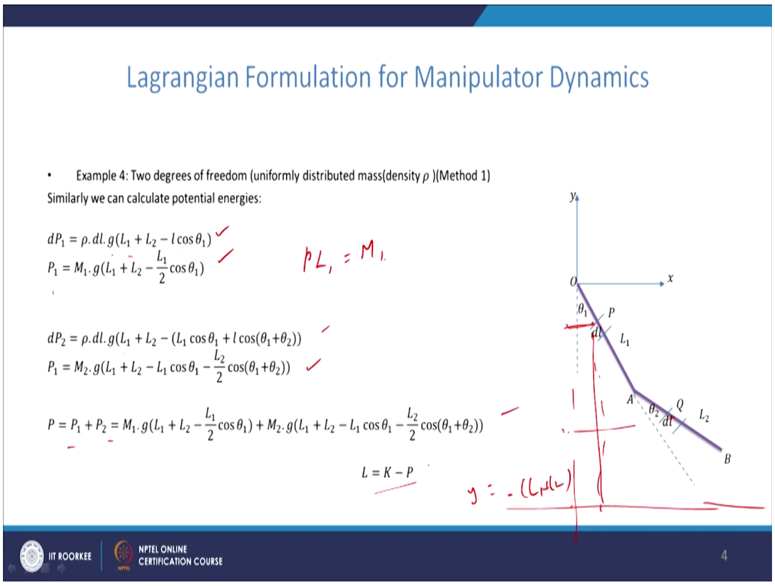

Now, the potential energy for the link 1 so, we take this line L 1 plus L 2 line minus of L 1 plus L 2, that is y. So, this line is the 0 potential energy line and so, far the point P the height is given by from this line is L 1 plus L 2 minus the y component, that is a L small l cos theta 1 is the y component distance. So, L 1 plus L 2 is this minus of this one is giving the height.

So, mass is rho into dl gravity and the height of the small element. So, the total potential energy is the integration from 0 to L 1.

So, that gives and because mass is the rho into the length is M 1 we get this formulation the total potential energy of link 1. Similarly, the potential energy of the small element at the point to Q is given by this expression, because the height is L 1 plus L 2 minus the y component, which is given by this expression.

So, the total potential energy is the integration from 0 to L 2, which can be easily calculated to be this. So, the potential energy P is P 1 plus P 2 is given by this and the Lagrangian is calculated as K minus P.

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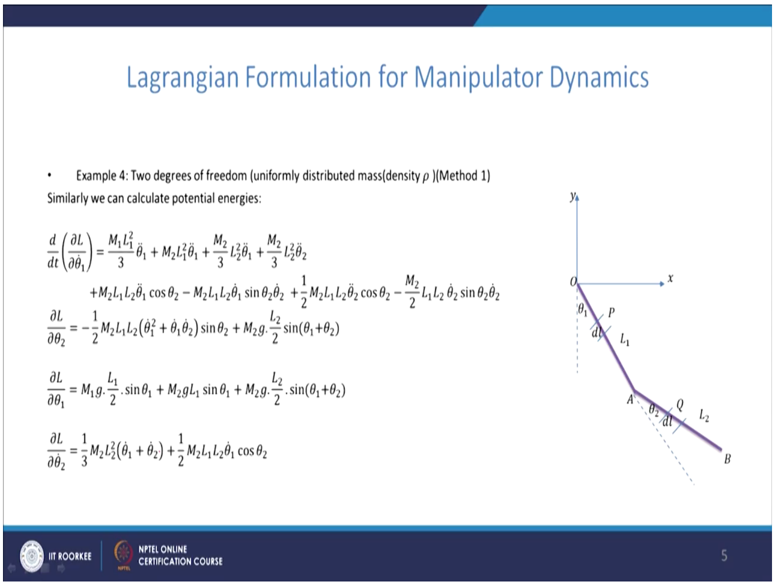

So, it is a the simple procedure, that we differentiate first with respect to theta 1 dot and then we differentiate with respect to time d by dt of this we get this expression. And, the derivative with respect to theta 2 dot is given by this, and derivative with respect to theta 1 and derivative of L with respect to theta 2.

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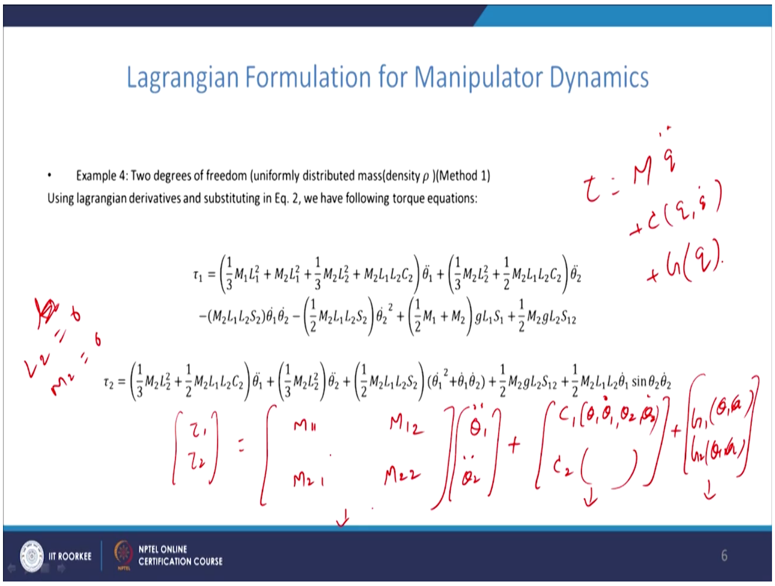

So, substituting in the dynamic equation we easily obtain the dynamic equation of the 2 arm robot manipulator a with uniform mass distribution. So, this equation can be written as in the matrix form tau 1 tau 2 is with theta 1 double dot theta 2 double dot. So, we get these components we call it as M 1 1 M 1 2, M 2 1, M 2 2 plus 2 components C 1 C 2 of theta 1, theta 1 dot, theta 2, theta 2 dot plus we get another term that is called the gravity term.

So, that is G 1 and G 2 of theta 1 theta 2, where the small g will appear in these terms. So, this is finally, written as tau is equal to M q double dot plus c q q dot plus G of q is the general dynamic equation of the robot manipulator, where the M matrix is a 2 by 2 matrix called the inertia matrix and a centrifugal and Coriolis term and the gravity term are given by this terms.

Now, we can easily verify that if you substitute L 2 the capital L 2 equal to 0 and the mass M 2 equal to 0, in this we will obtain the previous lecture we obtain, the dynamic equation of a single arm manipulator with uniform mass distribution that can be easily obtained here.

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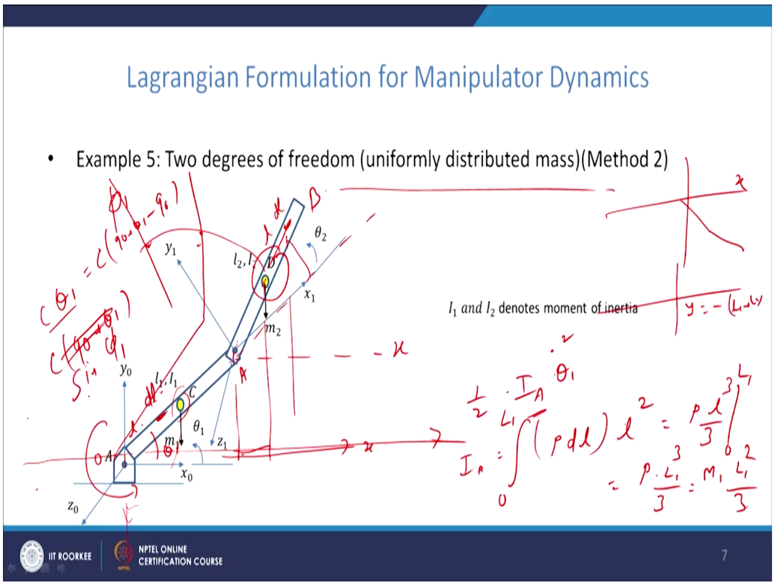

Now, we developed the dynamic equation of 2 arm manipulator with uniformly distributed mass the same problem using another procedure, instead of doing the integration for finding kinetic and potential energy. Another procedure is to find the kinetic and potential energy at the center of mass of the links.

So, if we consider the first link is given by O A this point and this is AB is the second link, the center of mass of the second link is situated at the point D, because the mass is uniformly distributed. The center of mass is at the centre of the points A and B. And, similarly the center of mass of the first link O A is at the center of this thing it is at the point C here.

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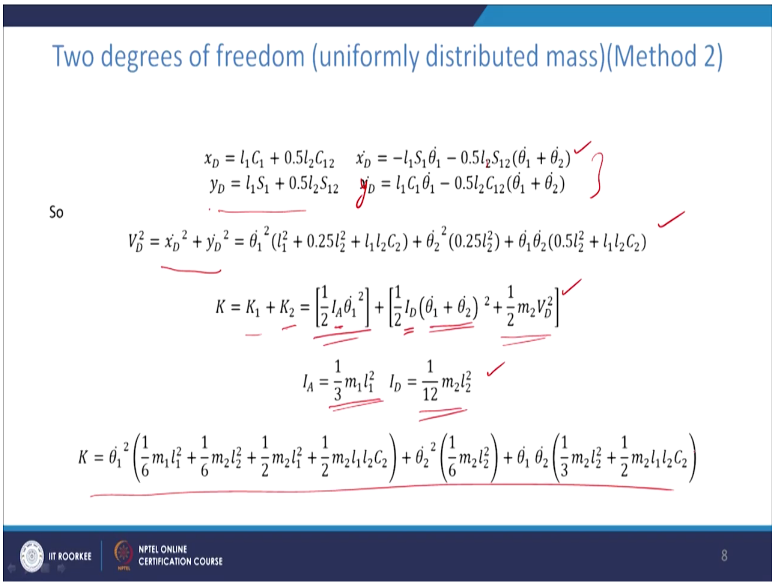

Now, we can calculate the kinetic energy and potential energy of the first and second link in the following way. So, if you take the D point the center of mass of the second link, it is a coordinate is given by the x coordinate is given by L 1 cos theta 1 plus half times L 2 cos of theta 1 plus theta 2. So, if you take this one if you measure the angle from the x axis here the theta 1 is from the x direction up to the first link, theta 2 is the extension of the first to link to the second link is theta 2.

So, the from the x axis to the second link the angle is theta 1 plus theta 2. So, it is slightly different from the previous example, where the theta is measured from the vertical line. From the negative y axis we are we were measuring the angle theta here we are measuring theta from the x axis theta 1. So, but we will arrive at the similar dynamic equation only irrespective of how we measure the angles with slight modification in the terms.

So, here the point x D and y D are given by this expression, from here we see that the x coordinate is L 1 cos theta 1 and this is L 1 by L 2 by 2 into sin cos of theta 1 plus theta 2. So, that is what is given in this expression. Similarly, the y component is given by this the derivative in the x direction is the derivative with respect to a time, D by D t of x D is given by this expression. And, y D is given and it is time derivative is given by the other expression.

So this is actually y y D dot derivative. Now, the velocity of the center of mass velocity square is given by x D square plus y D square derivative. So, directly it can be calculated by squaring and summing these 2 terms this one. So, the total kinetic energy is kinetic energy of the mass 1 and the length link 1 and kinetic energy of link 2 it is half into.

So, here we calculate the kinetic energy of the link 1 as it is a revolute joint. So, now, we calculate the kinetic energy of the first link using the formula half into inertia into the angular velocity square. So, that is the way of calculating for a revolute joint. So, here the inertia of this one, because the point at which the rotation is happening is the point O, the torque is given at the point O here and the joint is revolving.

So, we have to calculate the inertia of this it is mass into if you take a small mass of length dl. So, rho into dl is the small mass into the distance square the distance from the point at which it is revolving, the distance if you are assuming to be L here L square. So, mass into the distance square gives the inertia for the element of mass. So, we have to integrate it from 0 to L 1 the total length.

So, that gives rho is a constant l cube by 3 and the distance is 0 to L 1. So, we get rho into L 1 cube by 3 and rho into L 1 is the mass of the first link M 1 and L 1 square by 3. So, we get 1 by 3 M 1 into L 1 square as the inertia of the first link.

So, inertia I A we are denoting. So, I A into theta 1 dot square that gives the kinetic energy of the first link. Similarly, the kinetic energy of the second link it consist of 2 terms; one is half into inertia of the center of mass into the angular velocity theta 1 dot plus theta 2 dot whole square, because the motion of the center of mass.

So, the second link it is revolving about the center of mass, while it is moving the centre it is moving. So, we can see that when we observe only the second link we can see that it is revolving about the center of mass at various positions and apart from that it is also moving with a linear velocity. So, there are two types of motion happening to the second link, one is the linear motion.

So, it is given by the kinetic energy is given by half into mass into the linear velocity square. The second is happening because the link is rotating with respect to the center of mass. So, the angle angular velocity it is due to the 2 angles, because when the second link is moving theta 1 as well as theta 2 are making a different changes at different time.

So, theta 1 dot plus theta 2 dot whole square causes the rotation. And, the inertia ID the inertia of inertia with respect to the central of mass is calculated by this expression. So, here also it is the same type of calculation, we take any arbitrary point from the center of mass. Let us say L is the distance between the center of mass and at any arbitrary point and a small element of a length dl is taken.

So, a similar to this rho into dl into l square the only difference is the distance is a 0 to L 2 by 2, because from the center of mass the maximum distance from either side can be L 2 by 2. So, 0 2 L 2 by 2 if we integrate the quantity rho into dl into l square we will obtain, the value 1 by 12 times mass M 2 into the length square l 2 square.

So, this is obtained and when we substitute this inertia and as well as the linear the kinetic energy due to the linear velocity, we obtain the total kinetic energy to be like this. The K 1 plus K 2 is given by this particular expression.

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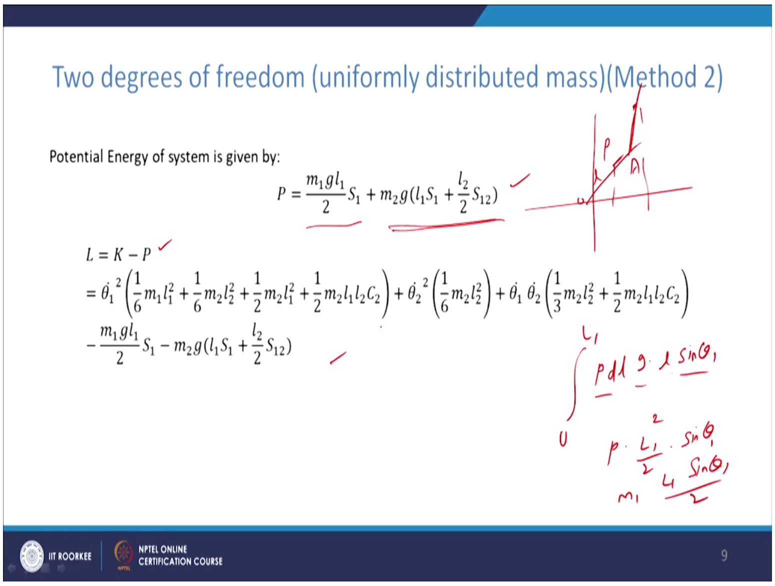

Similarly, the potential energy, the potential energy due to the link one is given by this expression the mass into gravity into the length the height is given by this.

So, here the point the line of the 0 potential energy is taken to be the x axis. So, we note that whichever line we take as the 0 potential energy, it is not going to affect the dynamic equation. The previous example, we have taken the line y is equal to minus l 1 plus l 2 this line.

So, this is the x axis and this equation is y equal to minus L 1 plus L 2 as the 0 potential energy line, instead we can take any horizontal line as the 0 potential and the result will not affect the dynamic equation yeah. So, here the height is to be taken from that point.

So, the height is if you take any arbitrary length L. So, we calculate from here for the link 1 mass into the gravity into if you take any arbitrary for the first link. If, you take any arbitrary point P the height is given by because the length is small l from the origin. So, the mass is rho into dl and gravity multiplied by the height. The height is nothing, but small l into sin theta mass in mg h is this one is given by.

Now, if you integrate this from 0 to the total length, we will get L square by 2 here and when we substitute the limit we will get L 1s. So, we will get rho in to L 1 square by 2 to sin theta 1. So, now, rho into L 1 is we will give the mass M 1. So, we get M 1 into L 1 sin theta 1 by 2 is the potential energy for the link 1.

Similarly, we can calculate the potential energy for the second link by taking any arbitrary point from the point A and calculating the height from the x axis. So, the Lagrangian is K minus P kinetic minus potential energy and the usual substitution of K and B will give L like this.

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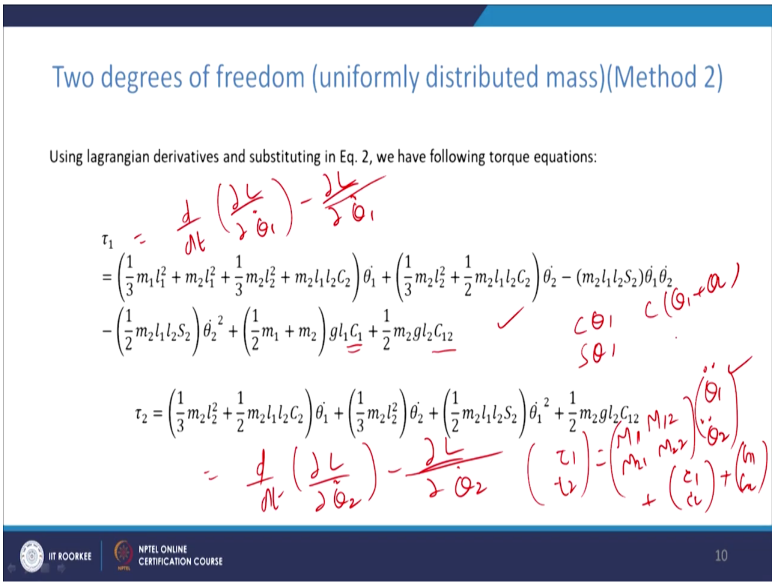

And, then the dynamic equation tau 1 is given by d by d t of del L by del theta 1 dot minus del L by del theta 1.

And, so, directly by calculating these derivatives we get tau 1 as given here and tau 2 is this is nothing, but d by dt of del L by del theta 2 dot minus del L by del theta 2 ok, this expression. So, this gives the dynamic equation of this one and again we can write it in the form of the matrix tau 1 tau 2 as we did in the previous example, we get M 1 1, M 1 2, M 2 1, M 2 2 theta 1 double dot theta 2 double dot plus C 1 C 2 the gravity the centrifugal term and G 1 G 2 the gravity terms.

So, gender dynamic question is obtained for it 2 link manipulator. So, here we note that the previous example we measured the angle from the downward vertical line. So, if you convert.

So, these 2 equations will be slightly different, because here we are measuring from the x axis and previously we were measuring from the downward y direction.

So if we convert this theta to the previous example, we can easily see that there is difference of because wherever here we are getting cos of theta 1. And, the previous one, we are requiring this; we have to get cos of 90 plus theta 1 is to be substituted for the previous one. If, you are converting this cos theta 1 into this particular angle that is nothing, but cos of 90 plus theta 1 minus 90 isn't it the if you measure from the bottom vertical line it is 90 plus theta 1, but we are having theta 1 only.

So, 90 plus theta 1 minus 90 is the cos theta 1 in this example. So, that is if you call this 90 plus theta 1 as some other angle phi 1. So, we get cos of phi 1 minus 90 that is nothing, but sin of phi 1 we get so, wherever cos theta 1 appears if you replace it with a sin phi 1 we get the previous formula.

Similarly, wherever sin theta 1 appears we can replace it with cos phi 1. So, we can observe that in this dynamic equation, wherever this theta 1 is appearing, because theta 2 is a no difference, because it is the same type of measurement in both the examples. In these examples we extend the first link and then measure the theta 2 from the extended line to the second link that is the same thing in the previous case also.

So, theta 2 is measured in this same manner, only theta 1 is measured in a different way. So, we can easily see that wherever theta 1 is appearing in the equation for example, here cos theta 1 is appearing. So, this should be replaced by sin of phi 1 which is the measured from the lower vertical. Here also cos of theta 1 plus theta 2, we can replace it by sin of phi 1 plus theta 2. And, similarly in the wherever the cos theta 1 and sin theta 1 appears or cos of theta 1 plus theta 2 appears, we can just replace it by a suitable angle measured from the lower vertical. We get the dynamic equation of the previous example.

So, we one can use in one can use the dynamic equation in any particular manner. So, these examples only show how to derive the dynamic equation of either a single arm manipulator or a 2 arm manipulator so far. So, in the first method we have seen how to calculate the kinetic and potential energy using the integration method.

We have taken a small element of mass and then calculate the kinetic and potential energy for that element of mass, and then integrating throughout the particular link. So, like that we calculate the energies for all the links, both the links, link 1 and link 2 and then adding the total kinetic and potential energy as shown in the formulae.

So, this same procedure we will adapt for a general robot manipulator. In the next lecture, where we will take a small element of mass at each link and then calculating it is the kinetic and potential energy with respect to the base frame and then a dynamic equation is obtained. Finally, in the similar manner as describe in the method one hare. So, that procedure we will see in the next lecture.

Thank you.