Robotics and Control: Theory and Practice
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Lecture – 11
Trajectory Planning

In this lecture we will see how to plan a trajectory for a robot manipulator. So, path planning and Trajectory Planning are very important aspects of a robot manipulator, when it is performing a work like pick and place or spot welding painting or various other tracking problems. So, a let us see first what is the meaning of a trajectory or the difference between a path and trajectory.

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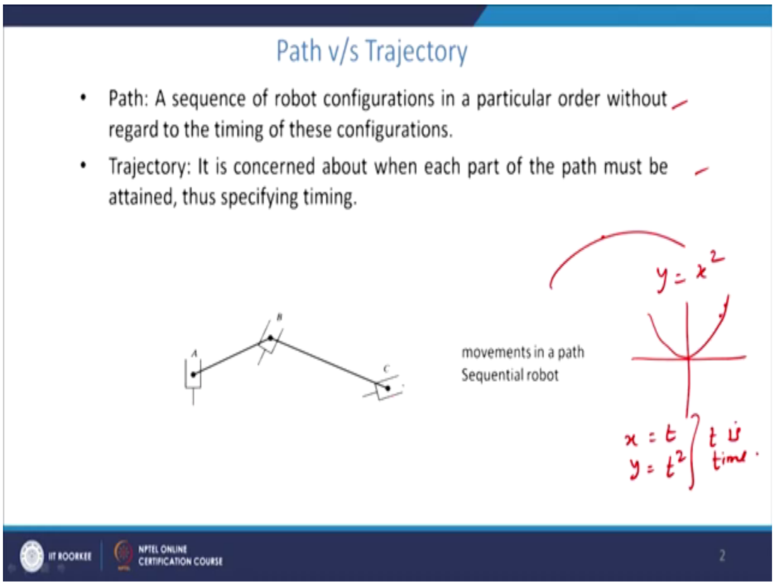

So, a path is a simple curve in the space, where there is no association of the point on the curve and the time. In other words it is simply a relation between the coordinates of the point.

So for example, if I write y is equal to x square in the two dimensional space. So, then it represent a path only, there is no association of each and every point and the time. But the same curve if we can give x is equal to t and y is equal to t square, where t is time. Then it says that the point at time t equal to 0 the point is at 0 0 and when t is equal to 1 it is at 1 comma 1 etcetera. So, each and every point on the path is associated with a time.

So, a trajectory is a path to which the time is associated and without association of time when it when a curve is given then it is called a path here. So, in robotics the trajectory planning is a very important aspect. So, the end effector in this picture it is shown that at three different time t 1, t 2, t 3 the orientation of the end effector should be as shown here as well as the position is shown as in the picture.

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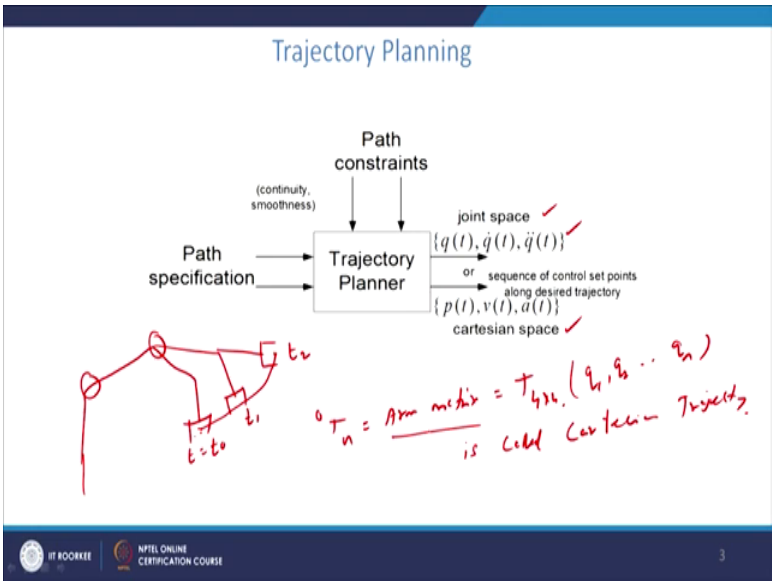

So, the trajectory planning is of two types for a robot manipulator that is Joint space trajectory as well as the Cartesian space trajectory. So, the meaning of this is if we have a robot manipulator and if you are interested in moving the manipulator to different configuration like this. So, at t equal to t 0 here is at t 1 at t 2 at three different instant of time the robot is having this configuration. So, here we are mainly concerned about the end effector position and orientation at a different instant of time, so that is called a Cartesian trajectory.

It should travel along a path associated with this time and the specification is about the end effector only here. So, end effector with respect to the base of the manipulator as we have seen it is 0 T n is the arm matrix and it is a 4 by 4 matrix and it is a function of q 1 q 2 q n, where q 1 q 2 q n are the joint variables. So, the end effector with respect to the base is given by the arm matrix as shown here.

So, this configuration is given for various instant of time, then it is called the Cartesian trajectory. And for performing this Cartesian trajectory if you are giving the angles that is q 1 q 2 q n the variables the joint variables are given as functions of time, then it is called the joint space trajectory. So, one can plan to move a robot manipulator either using the end effector trajectory or using the joint space trajectory, but if one is given another is can be obtained by the inverse kinematics solution.

If the end effector trajectory is given if the arm matrix is given we know that we can solve inverse kinematics and obtain the values of q 1, q 2, q n. So, if the Cartesian trajectories given we can obtain the joint space trajectory by solving the inverse kinematics. Similarly the joint trajectory is given directly by substituting in the kinematics equation we can get to the end effector traje trajectory. So, both of them are related concepts.

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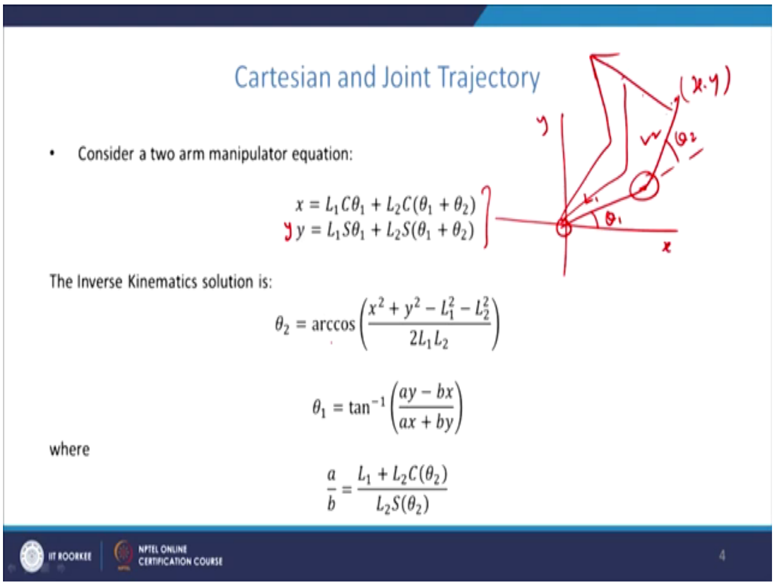

So, here for example for the two arm manipulator x y axis you have an manipulate like this and the actuators at this joints the angle is measured at the x axis theta 1 and this is theta 2. The end effector position is x y. So, x is given as L1 cos theta 1 plus L2cos theta 1 plus theta 2, where L1 is the link length and L2 is the second link length.

So, this is a very simple geometrical concept we can easily obtain the. So, from here we can solve theta 1 and theta 2 by using simple algebraic manipulations and this a and b are given by this expression as a equal to L1 plus L2 cos theta 2 and b is L2 sine theta 2 and by substituting you will get it the values of this thing. So, given the equation for x and y we can obtain the inverse kinematics solution theta 1 and theta 2 in this manner.

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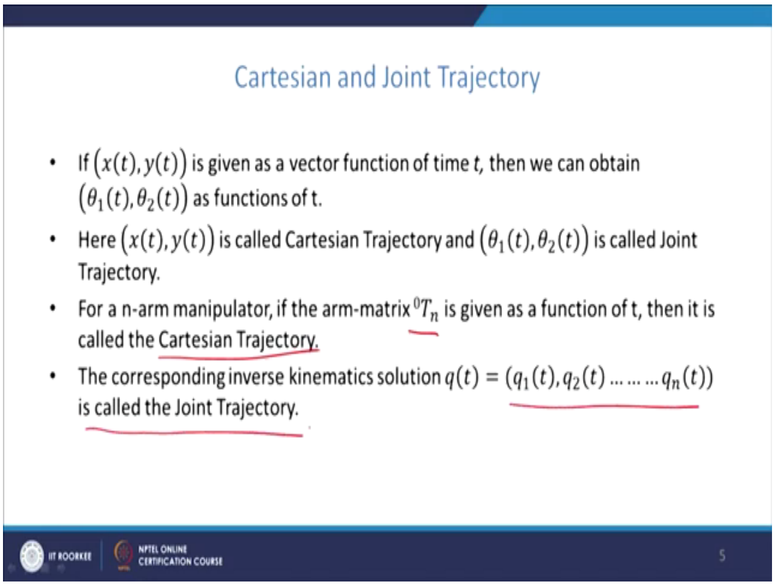

So, now if you plan here a trajectory, if x of t y of t is given as a function of time then a we call it as the Cartesian trajectory. That is the if you want that the end effector should travel along a straight line should travel along some a straight line in this manner. Then equation of the straight line can be given as a function of time at each and every instant what should be the position of the end effector. So, that can be calculated. So, for each and every end effector position we can solve the Inverse Kinematic.

So, if the end effector position is given as a function of time that is the Cartesian trajectory and if you solve the inverse kinematic and obtain the theta 1 and theta 2 that is the joint trajectory for this thing the two arm robot manipulator. So, in general for the n arm manipulator if the 0 t n is given as function of time it is called the Cartesian trajectory and the angles are given as function of time it is called the joint trajectory.

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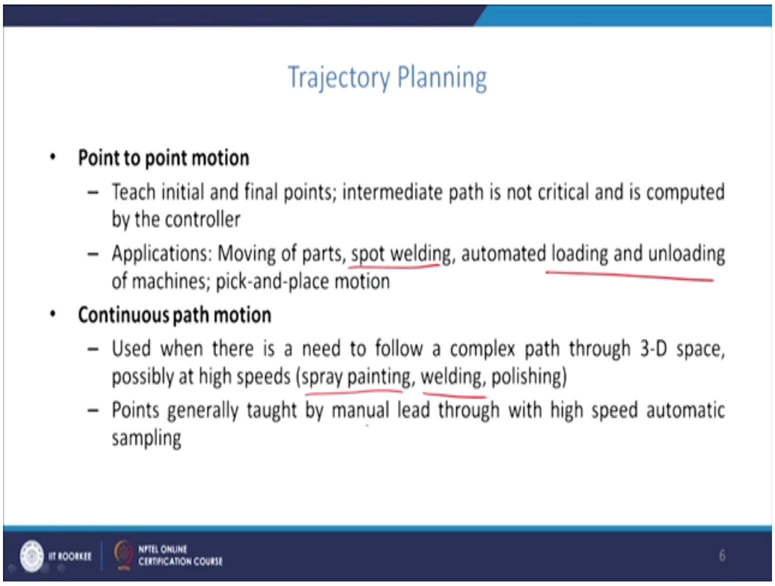

Now the trajectory planning is of two types Point to point motion as shown here. So, at different instant of time the positions are given that is point to point motion, we are not concerned about the intermediate position that can be calculated by using some simple formula. Because we are not specific about the intermediate position of the end effector, only aim is to reach these three specific points at these instant of time which can be planned. So, that is point to point motion like.

For example the loading and unloading work or the spot welding type of things can be done using the point to point motion, because taking an object and putting it only two positions and orientation are important and the initial and final time are important, intermediate positions one can calculate by various types of form. But continuous path motion is we are not only interested in the initial and final, but all the intermediate positions are also important they are also to be planned in a suitable manner.

For example, the spray painting welding continuous welding of metals etcetera. So, they need a continuous travel of the end effector of a robot manipulator. So, in both the cases we can follow a the procedure of finding the trajectory using polynomials.

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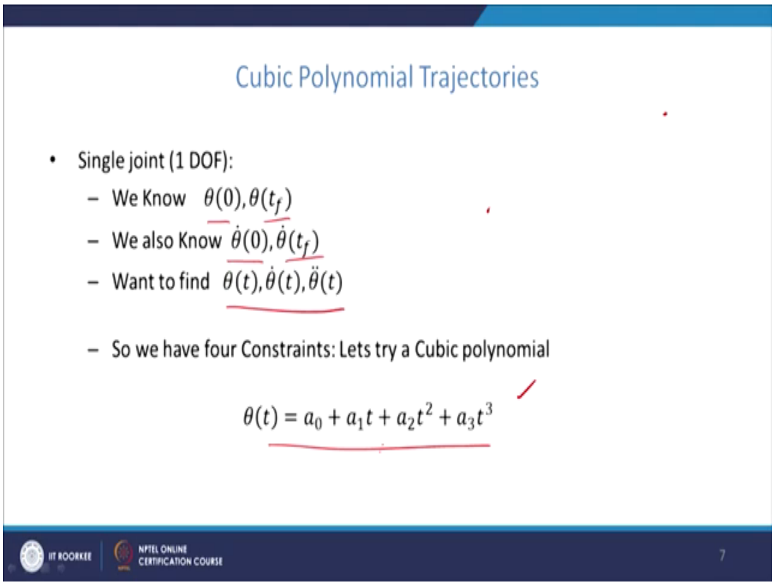

So, generally initial position and final positions are given or more conditions may be given. So, if we know the at the 0th initial time t equal to 0 and final time t f. So, the position 0 theta of 0 and theta of t f is known and the velocity at initial time and final time the derivative theta dot is known. Then we can find the intermediate values of theta of t for various values of t. So, how to find that can be easily fit, because there are four conditions given theta 0 theta t f and derivatives at the two points. So, we can consider a polynomial of degree three then by substituting the values we can obtain the coefficients a 0, a 1, a 2, a 3.

So, it is a very standard way of fitting a polynomial for given constraints. So, if there are n conditions given like this then we can fit a polynomial of degree n minus 1. So, that there are n minus n coefficients and which can be obtained uniquely by solving.

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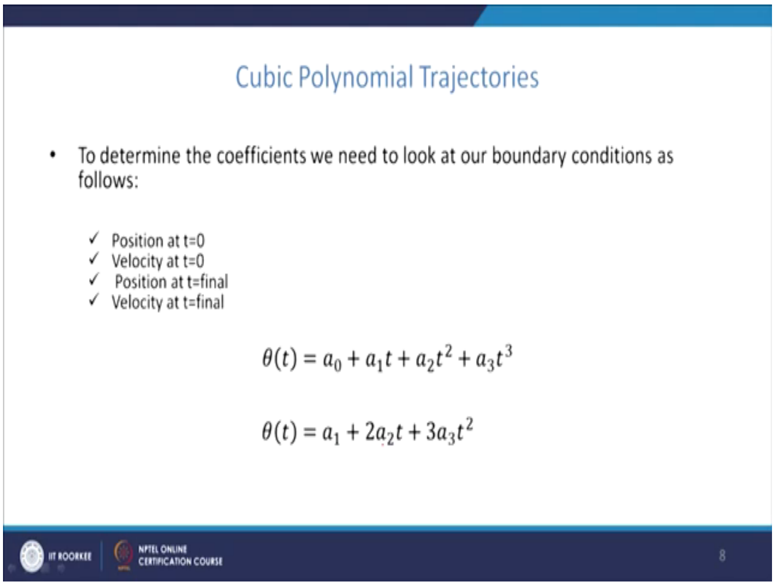

So, we can easily see that if theta of t is this one it is derivative theta dot of t is given by this directly differentiating and then substituting t equal to 0 we will get theta of 0 is a 0. And the derivative if we put t equal to 0 we get a one a one is theta dot of 0, so these two are directly obtained.

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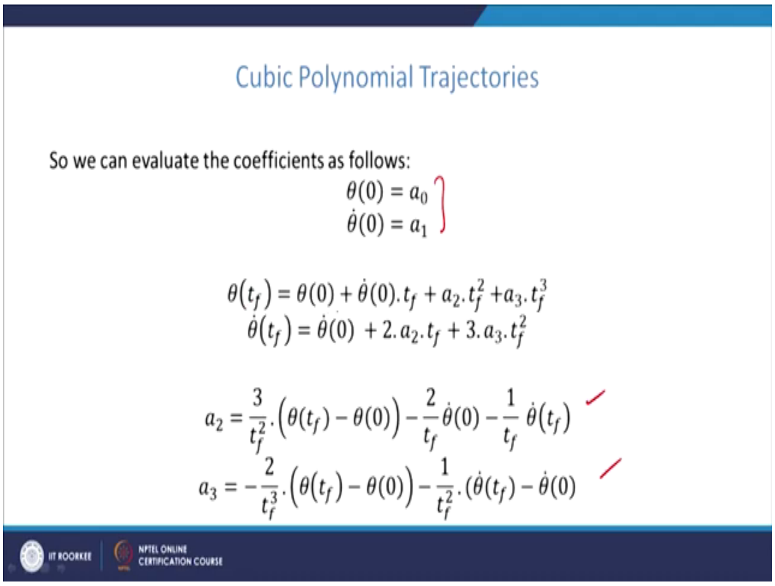

Now, by substituting the final time t f in this expression, t f in this expression we get already we have obtained a 0 and a 1. So, a 2 and a 3 are obtained by this simple procedure by solving the two equations. So, by this procedure we are able to get given the initial and final condition and initial and final velocity, we can fit a curve theta of t for all the intermediate positions. So, these procedure can be adopted for any number of such conditions.

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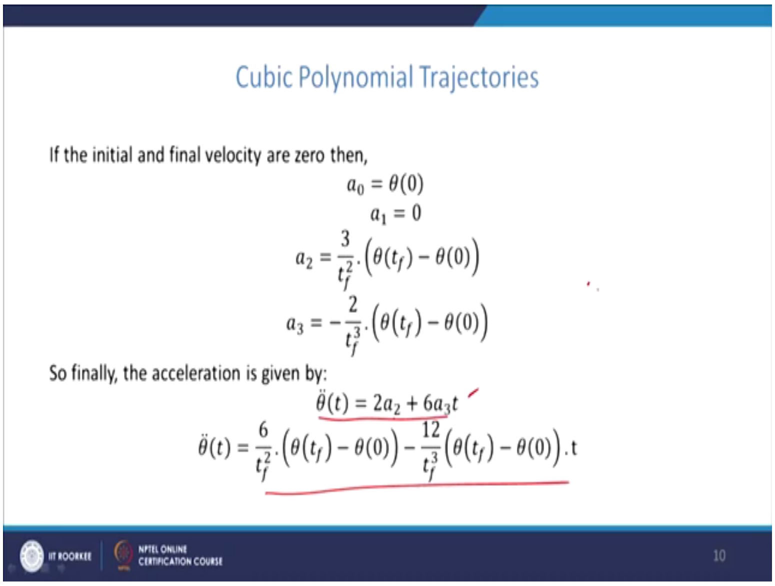

Now, if you consider the second derivative of the theta this polynomial. First derivative is this further if you differentiate we will get to a 2 plus 6 a 2 t this expression and now if we substitute a 2 and a 3 value we get this acceleration the second derivative is called the acceleration is given by this expression.

Now, we can observe that when we put equal t equal to 0, this acceleration is constant some value is obtained. So, it is a little bit of disadvantage, when a robot is moving at t equal to 0 the acceleration is some non 0 value. It means there will be a sudden discontinuity in the acceleration at the initial time. So, here for a smooth motion of a robot manipulator if there is a discontinuity at the in the acceleration it is not desirable.

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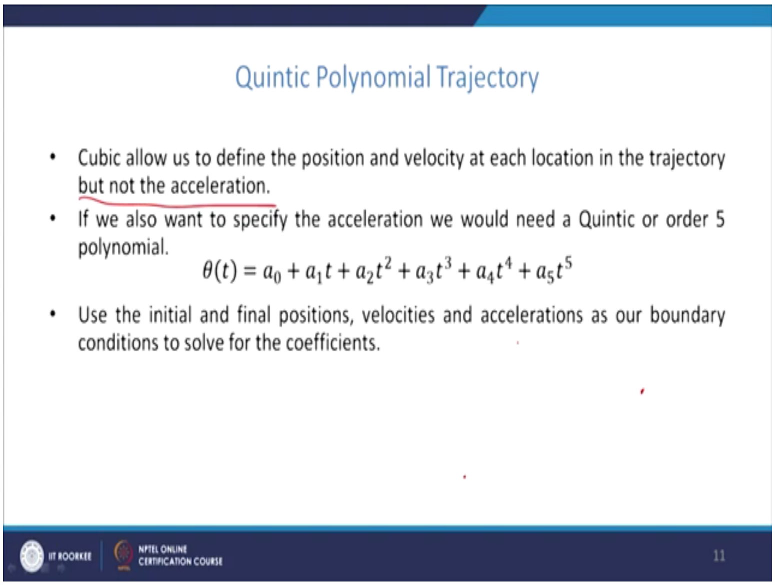

So, here we see that we cannot specify the acceleration on our own, because automatically it takes some value at t equal to 0. So, to avoid that we can increase the number of polynomial the degree of polynomial.

So, if you have initial position velocity and acceleration is specified and similarly final position velocity and acceleration is specified. So, there are six conditions using six condition we can fit a polynomial of degree 5 like this and then using six condition we can find the constants a 0 a 1 up to a 5 in a very simple manner. Directly substituting we get a system of equation.

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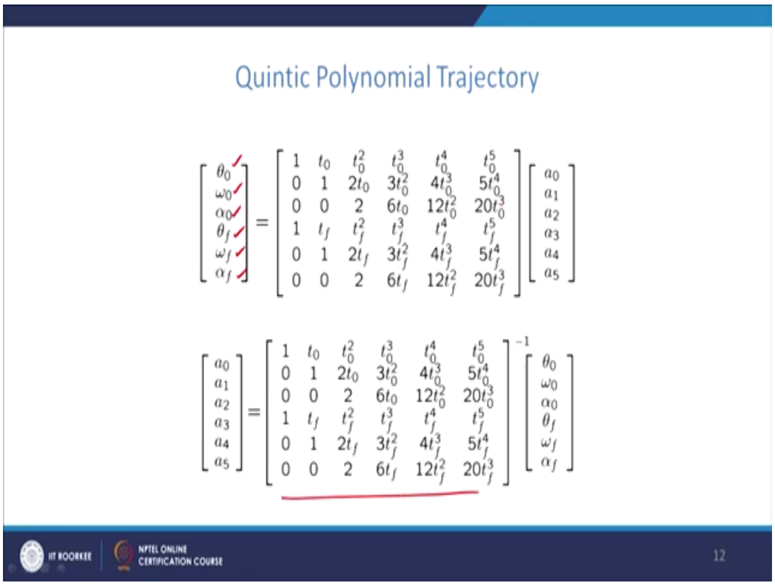

And the co efficience can be obtained like this a 0 a 1 a 2 up to a 5 is obtained by the inverse of this matrix multiplied by the known values, this is the it is given that initial position initial velocity initial acceleration this is final position final velocity final acceleration. So, if this six values are given now we can substitute and then multiplied by the inverse of this matrix, we obtained the coefficient directly like this. So, it gives the fitting of a polynomial if some constraints are given for any type of problem.

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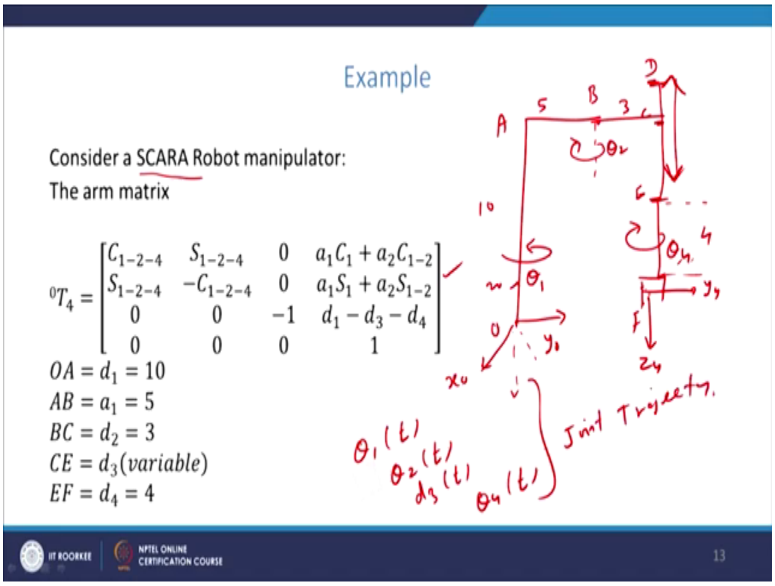

So, this can be applied for the robot manipulator problem like this. So, let us consider a the SCARA manipulator the standard manipulator of this form. Let us take this is O A and B and here there is revolute joint and this is prismatic joint. So, the first joint is a revolute one, this seconds this also revalued, the third one is a prismatic moving above and below and the fourth one is a revolute joint.

So, we call it as theta 1 this rotation theta 2 and this is theta four and O A B C D E F. So, if you take O A to B 10 this length and A B is 5 here and B C is 3 C E it is a variable this one from, because it is going up and down. This C E is a variable and E F is 4; E F the value is 4 here. So, the using the DH algorithm and then fixing all the co ordinate frames, then writing the parameters of the manipulator we can easily calculate the arm matrix 0 T 4 to be like this.

The relation between the base frame x 0, y 0, z 0, the end effector frame here is z 4 y 4 x 4. So, the relation between the end effector frame with respect to the base frame is given by the arm matrix like this for the given parameters.

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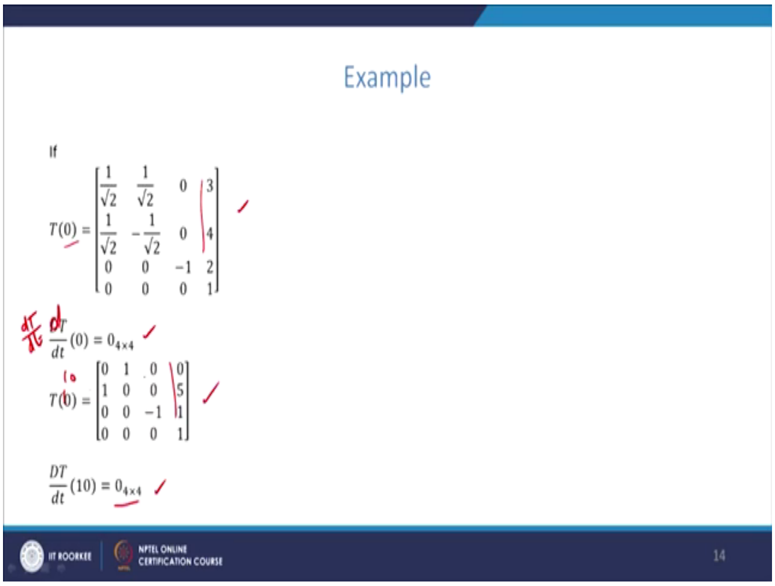

Now, if you want that the robot should travel between two points, that is two instant of time if you are giving at T 0. The time equal to 0 t of 0 is given by this expression and this is 10 T at time instant 10 is given by the homogenous transformation as given here. So, it means the x co ordinate of the end efector is parallel to the vector 1 by root 2 1 by root 2 0, in this the x co ordinate the end effector with respect to base.

In base we can say 1 by root 2; 1 by root 2 is the line with 45 degrees between x and y axis, the y coordinate is 1 by root 2 minus 1 by root 2 0 the z is 0 0 minus 1. So, if we observed that the z axis of the end effector is opposite to the z axis of the base so it is 0 0 minus 1; so as so as given here.

Similarly at time T equal to 10 units, the x axis will be 0 1 0 of the end effector 0 1 0 is the y axis here for the base. So, the x axis of the end effector will be parallel to the y axis of the base at T equal to 10. So, it should turn so much of angle, so that it adjust itself from time t equal to 0 to this one and the origin of the end effector is 0 3 0 sorry 3 4 2 at time T equal to 0 and it is 0 5 1 at time T equal to 10. So, it is shifting the origin should be shifting to the values as given here the fourth column of the this thing. So, for performing this position and orientation between these two time instant 0 and 10.

Similarly the velocity of the end effector is 0 at t equal to 0 and it should stop at the t equal to 10. So, what we have is we have four conditions at two instant of time. So, we can simply fit a third degree polynomial with four coefficients.

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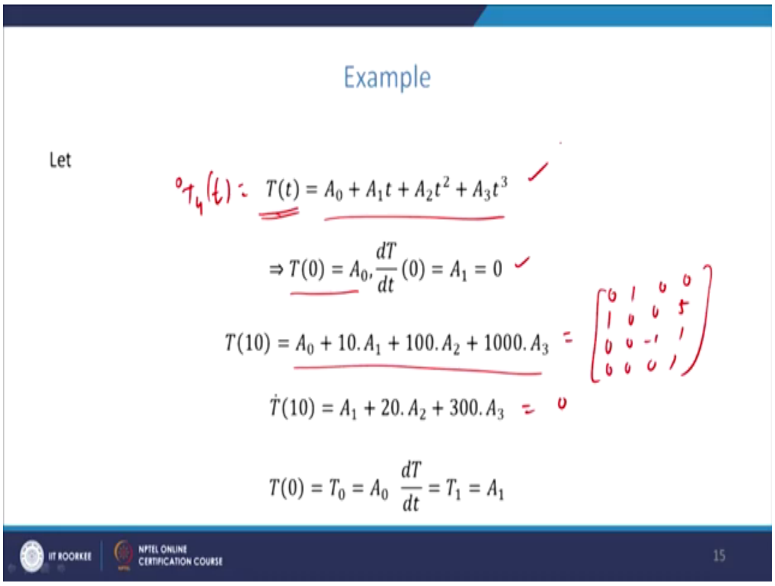

So, we will write that the end effector o T 4 as a function of time should follow this polynomial trajectory T equal to 0 it is A 0 can be calculated directly to be T of 0. So, A 0 A 0 is the matrix given by T of 0 as given here and when we differentiate it we will get A 1 is nothing but d by d t at 0 that is equal to 0 here, so A1 is the 0 matrix. Similarly we can find when we substitute at t equal to 10 small t equal to 10 in this expression, we get this value that is equal to as given by this 1 0 1 0 etcetera ok.

So, we get that t of 10 as given here and the derivative is 0. Now when we solve these two equation we will get the values of A0 A1 A2 A3 etcetera. So, once we obtain all this four coefficient matrices by substituting we get the entire trajectory. So, the trajectory planning is performed in this particular manner for the given SCARA Robot Manipulator.

So, the procedure is same for any type of manipulator, once the initial final conditions are given. Whether it there are two conditions we will fit a polynomial of degree one and if there are n conditions we will fit a polynomial of degree n minus 1 and find a trajectory. So, that one can find the end effector position for each instant of time. Whatever we have done here it is the Cartesian trajectory because it is giving the end effector position as a function of time.

Now, if you solve the inverse kinematics solution for each and every instant of time, we will get theta 1 and theta 2 as a function of time d 3 and theta 4 by solving inverse kinematics. So, this is called the joint trajectory. For any type of robot manipulator we can calculate the Cartesian trajectory or the joint trajectory by using the number of conditions. If there are n conditions given initial and final conditions, then we can get we can fit a polynomial of degree n minus 1 for the Cartesian trajectory. And then solving the inverse kinematics solution we will get the joint trajectory of the manipulator ok.

Thank you.