Robotics and Control: Theory and Practice
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Lecture – 10
Manipulator Jacobian Example

In this lecture we will see how to compute the Manipulator Jacobian using a example; example of a robot manipulator.

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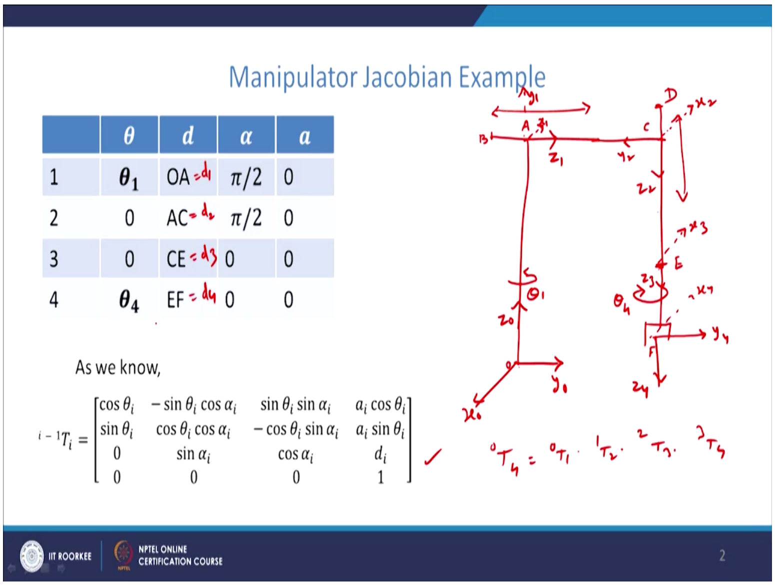

So, let us consider the robot manipulator shown in this figure; where the first joint is a revolute joint that is theta 1 and the second joint A is it is a prismatic joint, it slides in the forward and backward direction; the third joint C is also a prismatic joint moving up and the down; the fourth joint E is a revolute joint it is a rotational the angle is denoted by theta 4.

Now using the DH procedure, we can fix the coordinate frames at each joint and then write the robot parameters as shown in this table.

So, if the z 0 is the z axis of the base frame and the z 1 is the sliding direction; z 2 is the sliding direction of the third frame and z 3 is the revolute axis the axis of rotation; the z 4 this is z 3, z 4 is the approach direction at the end effector. Then we can fix the x, x 0 y 0 axis and then the axis are fixed as the perpendicular direction to z 0 and z 1 is the y direction x direction; x 1 and y 1 is the vertical direction. And similarly the x 2 axis is parallel to x 1 axis and the y 2 axis is the direction, this direction.

Similarly the x 3 axis is parallel to x 2 axis and the sliding direction is y 4; the perpendicular direction is x 4. So, this can be this we have seen elaborately in the previous lecture on DH procedure. So, after this we I can fix the joint angles; joint angles are theta 1 and theta 4 and the these are the two variables.

And theta 2 theta 3 they are 0 here; because the x 2 and x 1 x 2 axis and x 2 x 3 axis are parallel. So, the angles between them are 0 and the joint distance d O A is fixed here the and A C is the variable here; the sliding the prismatic joint A C is a variable we can call it as d 2, this is d 1. And similarly C E is a variable call it as d 3 and E F is d 4 here. So, d 1 and d 4 are constants; d 2 and d 3 are variables here. So, there are four variables theta 1, d 2, d 3 and theta 4.

And so, once the parameters are fixed in this using DH algorithm; then we can write the link coordinate frames that is the i th coordinate frame with respect to the i minus 1th coordinate frame is the standard expression for the homogeneous transformation is given by this one; cos theta i minus sin theta i into cos alpha i sin theta i sin alpha i alpha i into cos theta i etcetera.

So, this is the standard way of writing the coordinate frame i with respect to the co ordinate frame i minus 1 here the. So, using this we can write the arm matrix that is base the end effector frame with respect to base frame by multiplying all this 0 T 4 is the arm matrix which relates the (Refer Time: 06:00) end effector frame 4th frame with respect to the base 0th frame. So, that is 0 T 1, 1 T 2, 2 T 3, 3 T 4. So, this product will give expression where we have to substitute various values given in the parametric table.

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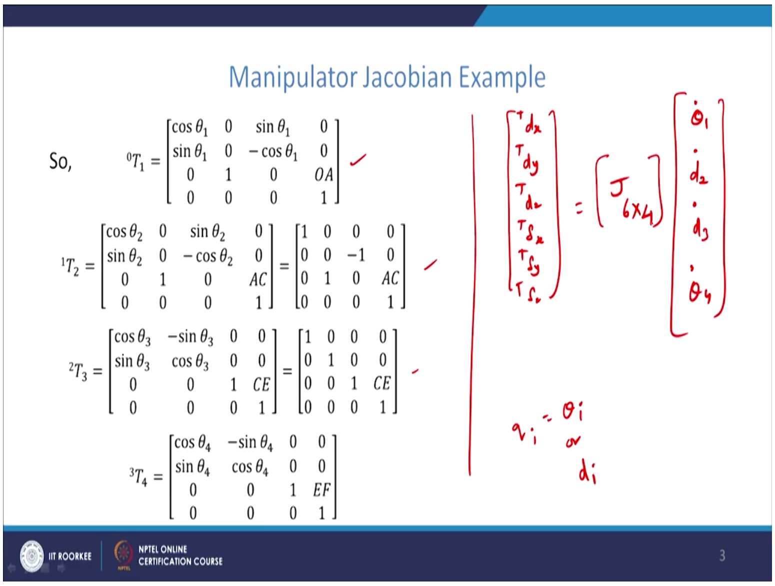

So, now by substituting the parameter values we can obtain 0 T 1 as given here 0 1 T 2; if you put i is equal to 2 we get 1 T 2 in the previous expression. And i equal to 3 will give 2 T 3 as given here etcetera. So, the four matrix; 0 T 1, 1 T 2, 2 T 3, 3 T 4 is obtained for the previous example in the by substituting the parameter values.

So, now how to compute the Manipulator Jacobian for this manipulator, this particular example? So, if you recall the manipulator Jacobian definition, it is the relation between the d x T, d y T, d z T, del x T, del y T, del z T; this is related with Jacobian, it is 6 cross n matrix, here n equal to 4, the number of link is 4 here. So, 6 cross 4 matrix multiplied by the joint velocities; that is theta 1 the first joint variable and d 2 dot, d 3 dot, theta 4 dot.

So, the general form is q 1 dot, q 2 dot up to q n dot. So, if joint is revolute joint q i that is equal to theta i and if the revolute joint is prismatic, we denote it by d i. So, the velocity of, the joint velocity at each joint is given by the derivative of this variable. So, theta i dot represent the angular velocity at the joint i, if it is a revolute joint and d i dot denotes the linear velocity, if it is a prismatic joint.

So, the relation between the joint velocities and the end effector velocity with respect to itself; here d x T, d y T, d z T represent the linear velocity with respect to the x y z axis of the end effector frame. And del x, del y, del z this T super fix T; it represent the rotational velocity of the end effector with respect to the end effector frame. So, this related. So, how to find the Jacobian matrix J 6 cross 4?

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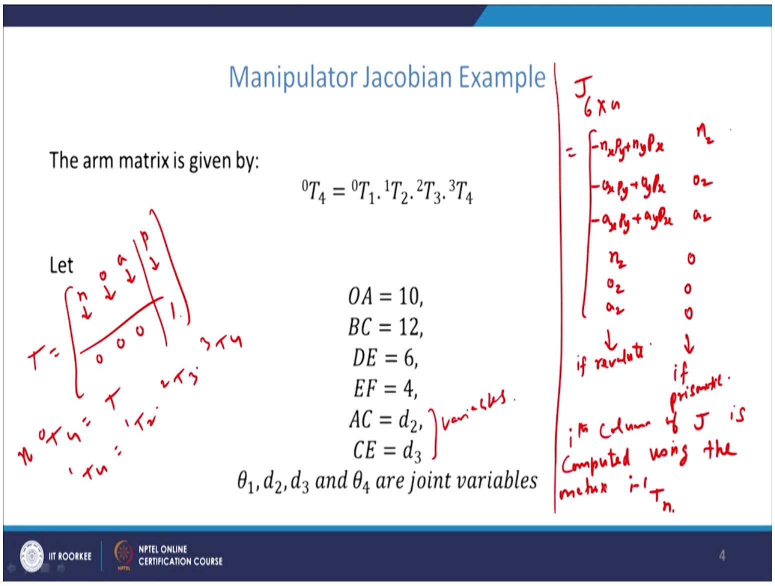

So, that we have seen in the previous lecture as the following. So, the J 6 cross n matrix in general; how it is calculated? It is calculated by the following formula; it is minus n x P y plus n y P x minus o x P y plus o y P x minus a x P y plus a y P x and n z del dot n, so n z, o z, a z. So, this is the first column. Similarly we can calculate the second column, if it is revolute joint. And if it is a prismatic joint we will get; n z, o z, a z, 0, 0, 0 if prismatic.

Now what is this n, o, a, p etcetera; this notation as we have seen the earlier lecture. If you have a homogeneous transformation T; the first column is denoted by n vector, second is o vector, third is a vector, forth is that vector p and 0 0 0 1. The i th column of the Jacobian matrix is computed using the matrix i minus 1 T n; where n is the number of link for the manipulator, in this example n equal to 4.

So, if you want to calculate the first column of the Jacobian matrix, we put i equal to 1; it means 0 T 4, 0 T 4. If we call it as T, then we have the first column as n, o, a, p etcetera that can be substituted in this formula; because the first joint is the revolute joint, this formula will hold good.

Now, if you want to calculate the second column of the Jacobin matrix, we put i equal to 2. So, we get 1 T 4. So, if you take 1 T 4 that as the T matrix; for 1 T 4 the how to calculate 1 T 4? It is 1 T 2, 2 T 3, 3 T 4. If you multiply this three matrix as given here that is 1 T 2 multiplied by 2 T 3 into 3 T 4, this product it will give the matrix 1 T 4.

So, from that we select the n o a p columns and then substitute in this second value; because this formula is for the prismatic joint that we have seen in the previous lecture. Similarly the third column it is a prismatic joint. So, that is 2 T 4 to be calculated, from that we make use of the similar expression for the third column. Then the fourth column is 3 T 4; 3 T 4 is already given here. So, the first column is n, second is o, and third is a and p.

So, substituting this in this formula; because the fourth is the revolute joint, the same formula as given in the first column is valid, only thing is the n o a p is calculated from 3 T 4. So, the procedure for calculating the Jacobian matrix is in this manner. So, we can actually calculate all of them.

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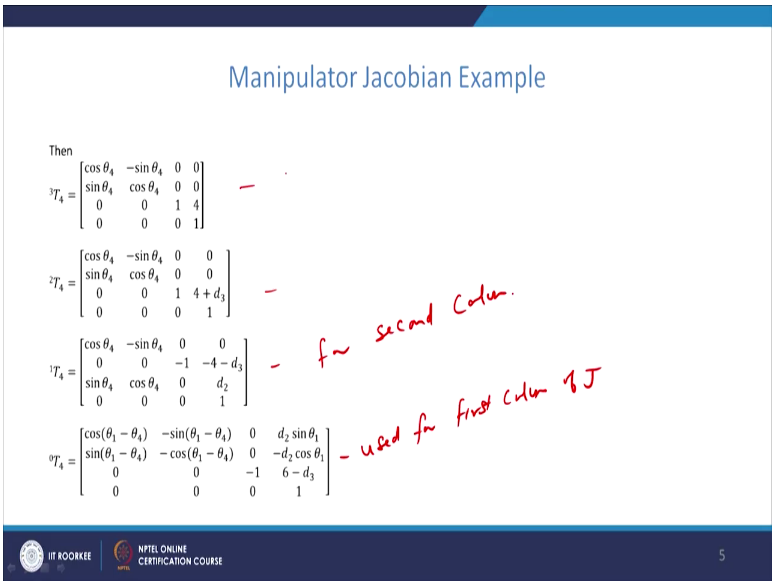

Now, 3 T 4 is given as it is for the parameter values. So, let us assume that the length is O A is 10, B C is 12, D E is 6, E F is 4 etcetera, so this values are given. And A C and C E these are the variables; because the joints are prismatic joints. So, substituting the values in this expression we get the values to be like this; the calculation of 3 T 4, 2 T 4, 1 T 4 and 0 T 4.

So, this is used for first column of the Jacobian and 1 T 4 is for the second column; this is for third column, and this one is for the fourth column of the matrix. So, using that, we can get the Jacobian matrix.

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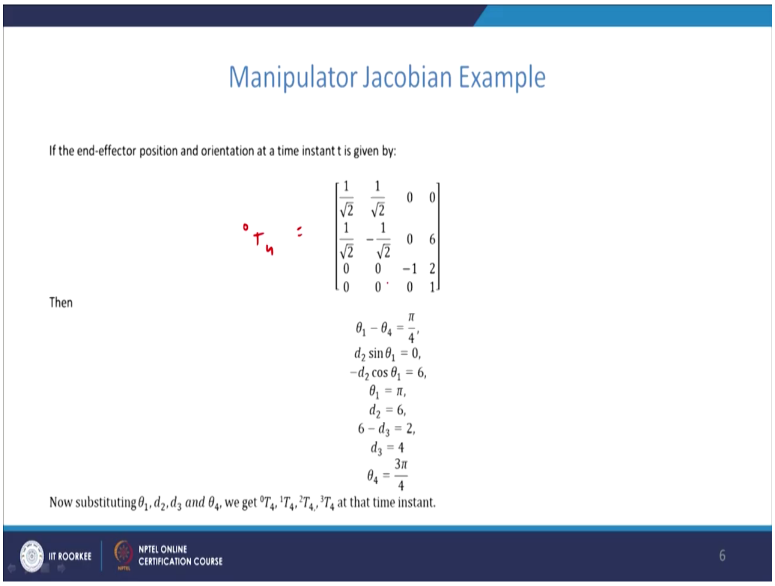

So, the Jacobian matrix is calculated at every instant of time; actually when the robot is moving to various position at a particular position what is the value of the Jacobian that can be calculated, because all this theta 1, d 2, d 3, theta 4 are varying. So, as they are changing what is the Jacobian matrix. So, at a particular instant of time; let us say the end effector is in this configuration.

The end effector coordinate frame with respect to base coordinates; this is 0 T 4 is given by this expression. Now 0 T 4 is calculated by the last expression 0 T 4 is given. So, if you compare each element 0 T 4 with the required configuration of the end effector and then we can easily solve the equations.

There are 12 such equation we can solve them, most of them are 0 here. So, it is very easy to solve just by comparing 0 T 4 expression with the constant matrix and then we will get this configuration; that is when theta 1 minus theta 4 is pi by 4 and we get the values d 2 to be equal to 6 and d 3 4, etcetera. So, these values are obtained directly by solving the inverse kinematics solution of the this particular configuration. So, at this value what is the manipulator Jacobian, so that is calculated by the following.

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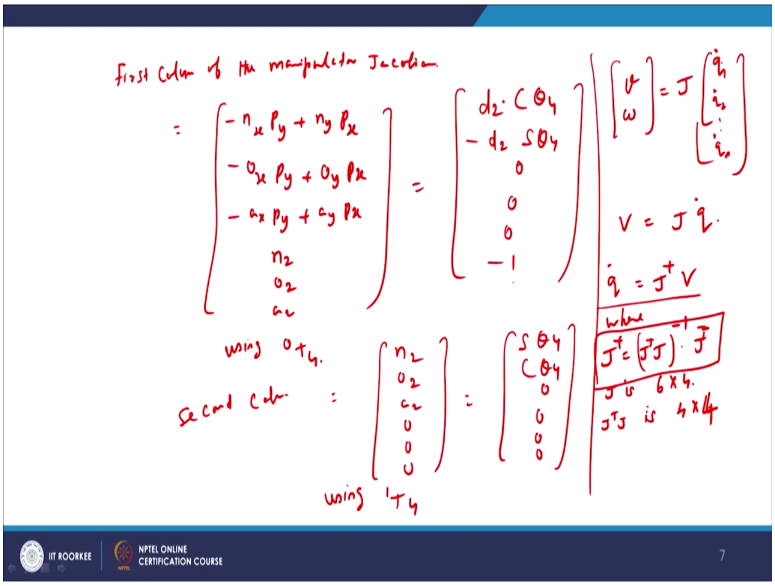

So, now let us calculate; so the first column of the manipulator Jacobian. So, it is actually given by the values minus n x P y plus n y P x minus o x P y plus o y P x and n z, o z, a z. So, this is using 0 T 4 columns at that particular values; so we can substitute here directly by seeing n x P y etcetera from the expression given here, 0 T 4. You can see that n x is cos of theta 1 minus theta 4 and P y is minus d 2 cos theta 1.

So, this plus n y P x; n y is sign of theta 1 minus theta 4 and P x is d 2 sin theta 4. If you multiply and add substituting in this formula, we will get the value to be d 2 cos of theta 4. Similarly we can see from the matrix 0 T 4 this values, we will get minus d 2 sin theta 4; then

0 0 0 and minus 1. That is n z from here we can see that, the third value n z is 0; o z is 0 and a z is minus 1.

So, this is substituted here 0 0 minus 1. So, the first, similarly we can calculate the second column of this thing; the second column is it is given by n z, o z, a z, 0, 0, 0. It is using 1 T 4; because it is a prismatic joint we have this expression. So, 1 T 4 if you see here, n z is sin theta 4 and o z is cos theta 4 and then 0. So, we can substitute those values here. So, we get sin theta 4, cos theta 4, 0, 0. So, this is the second column. So, similarly we can calculate the third, fourth column etcetera of the manipulator Jacobian.

Now, having computed the manipulator Jacobian; so what we get is, the left side we have the linear velocity and angular velocity. If we denote the linear velocity to be; so let us say the velocity linear velocity and the angular velocity d x T, d y T, d z T we call it as v and del x T, del y T, del z T call it as w angular velocity with respect to the end effector frame it is equal to Jacobian multiplied by this q 1 dot q 2 dot etcetera q n dot in the general case.

So, we can denote it by v is equal to J q; for example, in the simple notation v is the left side, J into q is the vector given as q 1 q 2 q n dot this is the expression. Now we can calculate q dot to be, if J is a square matrix then we can take the inverse of J; but normally in this example for example, here it is 6 cross 4 matrix. So, in general it is a 6 cross n matrix. So, we will we may not get a inverse of J generally, but we can get the Pseudo inverse; we call it as the Pseudo inverse of the non square matrix we can write.

So, that is given by this is equal to J transpose J inverse into J transpose sorry into J transpose. So, we can see in this example if J is 6 cross 4 matrix; then J transpose J is 4 cross 6 4 cross 4 matrix. So, it is a square matrix and we can find the inverse of J transpose J and multiply it with J transpose. So, this expression is called the generalized inverse of a matrix; whether it is a square matrix or non square matrix.

So, we can calculate the q dot value using this particular formula, the expression given here. So, if the end effector velocity v is given with respect to the end effector frame; then how much of the velocity of the actuators to be given at each joint is calculated by this particular formula. So, similarly by generating the arm matrix and making use of the formula as described in the lecture, we can calculate the manipulator Jacobian for various types of manipulators, ok.

Thank you.