Robotics and Control: Theory and Practice

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Lecture – 01
Robotics and Control: Theory and Practice Introduction

Dear students, welcome to this course on Robotics and Control: Theory and Practice. This course is divided into two parts, each having 20 lectures. I will take the first part which is about the fundamentals and mathematical concepts pertained to robot manipulators, biped robots and their control. The second part is about the practical applications and implementation aspects of robotics which will be taken by Professor Felix. So, the content of this course is as given below.

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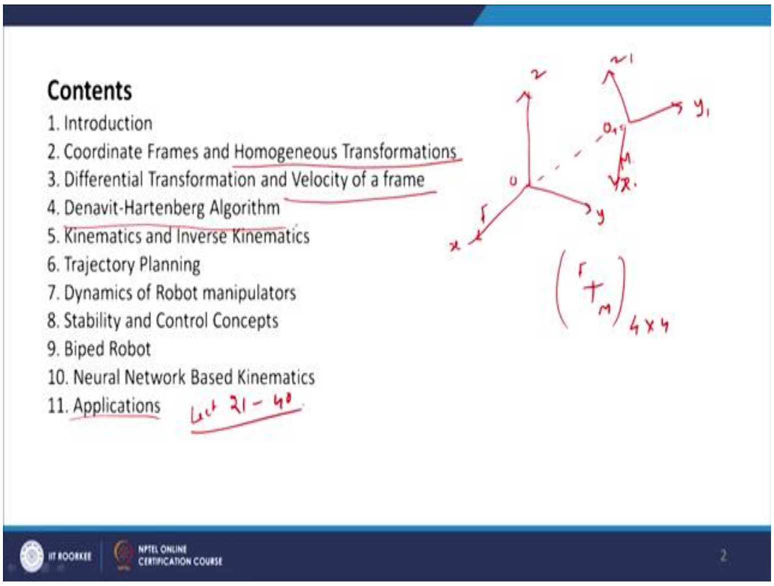

So in this first 20 lectures we will see about the coordinate frames homogeneous transformations, differential transformation and velocity of a frame. So, here we will consider a coordinate frame x, y, z and the origin. So, it is a fixed frame and another coordinate frame with x_1 , y_1, z_1 whose origin is called o_1. So, we will consider how to represent the frame M we call it as M frame. So, how to represent the frame M with respect to the frame F, its position and orientation. So, the position means how the origin O 1 has shifted from the origin o and the orientation represent how x_1, y_1, z_1 axis are represented with respect to x, y, z axis.

So, this will be represented in terms of the homogeneous transformation which is denoted by $^{F}T_{M}$; T represent the transformation and F and M are the coordinate frames. So, this is read as the frame M with respect to frame F is denoted by a 4 by 4 matrix called the homogeneous transformation. So, this will be done and the velocity of a frame with respect to another frame. So, if the frame M is moving in space with respect to the F frame how to represent the velocity of the M frame with respect to F frame.

And, then the Denavit-Hartenberg algorithm. So, this is with respect to a robot manipulator. So, this algorithm gives how coordinate frames can be fixed at each joint of a given robot manipulator and the kinematics and inverse kinematics, trajectory planning, dynamics of robot manipulators and stability and control concepts and biped robots. So, this will be described during the course of the lectures. The last portion application part is from the lecture 21 to 40 will be taken by Professor Felix.

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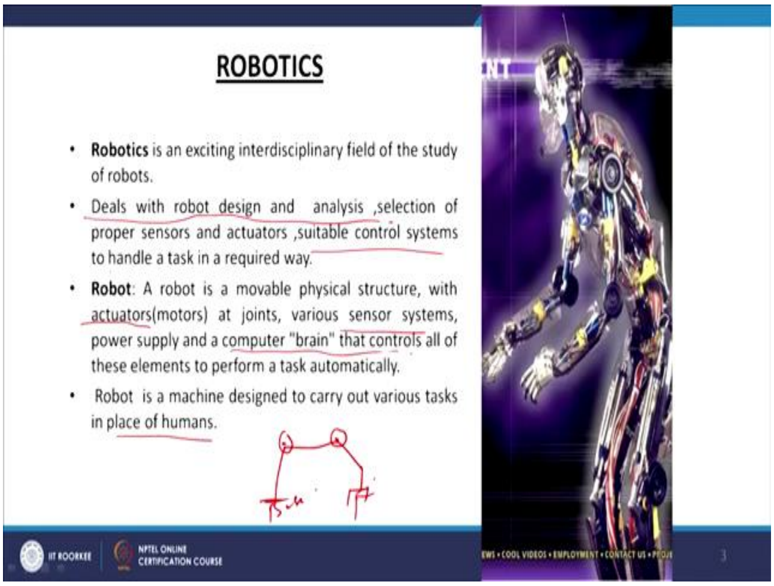

So, this the definition of a robot broadly we can say that it is a machine which can perform a given task automatically using artificial intelligent and without human intervention. So, with in the place of human it can perform a task automatically. So, it is a machine with links and joints we can say that broadly a robot manipulator is having various links and the joints at the connection of the links and there is a hand or the end-effector of the manipulator and at this and it also will have actuators. Actuator is the motors attached at each joints and sensors like position sensor velocity sensor or the vision sensor like cameras etcetera and a controller which using the computers it can act as the brain of the robot manipulator.

So, this broadly represent a robot and robotics is a study on the design and analysis selection of proper sensors and actuators and suitable control systems to perform a given task efficiently. So, this is the study of robotics

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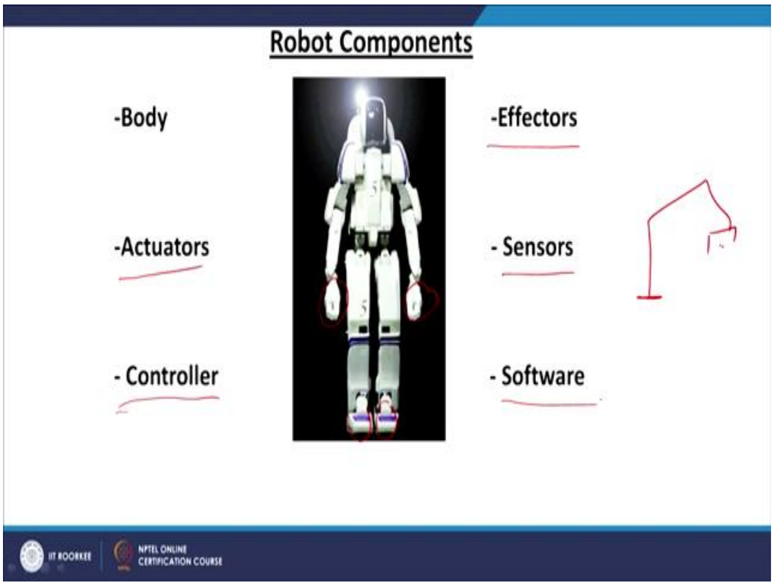

So, here the picture shows a humanoid robot, but a robot need not be in this particular shape it can be a crane like of like a structure or it can be insect like or say several types of robots which can perform any task automatically are all called a robot.

So, here one particular robot is shown. So, it has a body and it has effectors end effectors. So, there are here two hands are there and two legs, there are four end effectors are there for this particular robot manipulator. In this picture you have only one end-effector and a base here.

And, then actuators at every joint sensors like vision sensor or various other touch sensor etcetera can be attached with a robot and the controller which controls a robot to perform a particular task. So, this is with a mathematical analysis one can design various efficient controller to perform a task and the software which will implement the controllers etcetera.

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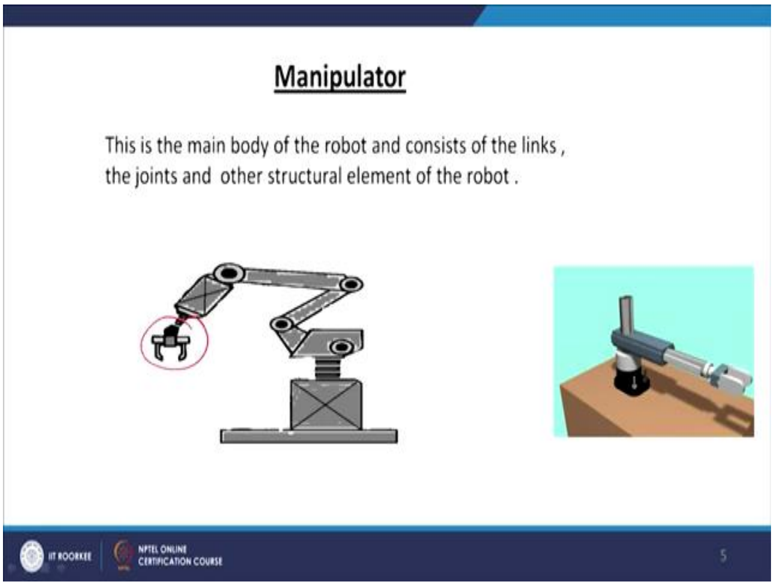

So, a manipulator is the body of the robot. So, here we can see the body and excluding the end-effector. The reference frame.

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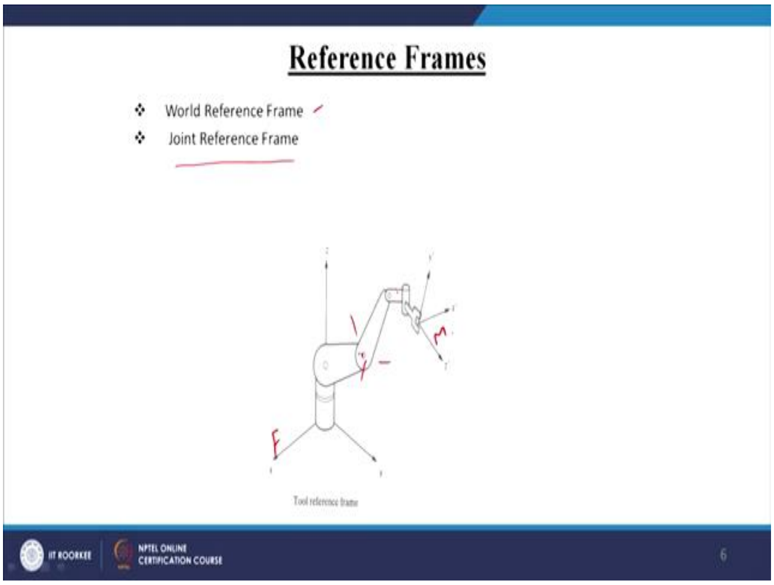

So, to study a robot and control it, we need a reference frame coordinate frame has to be fixed first and then within that coordinate frame all the movements of the robots are analyzed or controlled.

So, here a coordinate frame is fixed here that is a world coordinate frame it is a fixed one and the joint reference frames; if you fix various coordinate frames at various joints then they will not be stable, they will be moving. So, the world frame is a fixed frame and the other joint frames are the moving frame of the robot. So, how to fix various coordinate frames at various joints is an algorithm that is called the DH algorithm which will be studied in the coming lectures.

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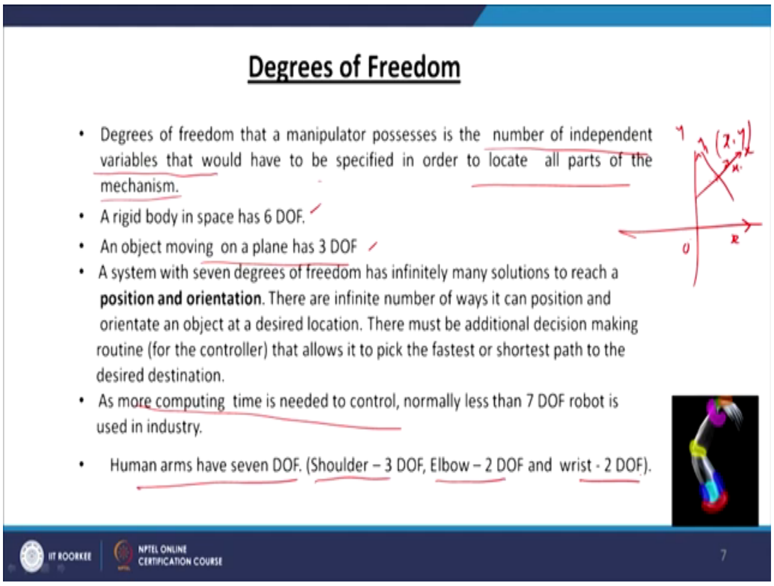

So, degree of freedom is the number of variables which is required to identify the parts of a robot manipulator. So, the number of independent variables that should be specified in order to locate all the parts of a robot manipulator. So, we know that a rigid body will have 6 degrees of freedom, 3 position variables and 3 rotation variables that is orientation variables.

And, a object moving in a plane it has 3 degrees of freedom because a 2-dimensional plane if you have the position is represented by x, y two variables are required and the orientation. The orientation of the object if it is x, y plane x y axis and this is x_1, y_1 axis. So, x_1, y_1 axis represent the orientation of this object with respect to the frame x, y coordinate frame.

So, here there are three degrees of freedom. So, x, y is the origin has shifted from the original origin and the coordinate's frames x_1, y_1 with respect to x, y. So, the two position variables and one rotation variable is representing the 3 degrees of freedom. And, if we have 6 say more than 6 degrees of freedom for a robot manipulator it is called a redundant robot manipulator because to perform a particular task in space we need 6 degrees of freedom at the most and more than 6 degrees of freedom using that we can perform or reach a particular object in an infinite way. So, the such degree 7 degrees of freedom manipulators are called redundant robot manipulators

So, normally as there are infinitely many ways of doing the same job the computing time for selecting an efficient way of doing will be huge and normally it is not preferred in the industrial robots. And, we can easily see that the human hand it has 7 degrees of freedom and 3 degrees of freedom at the shoulder and at elbow 2 degrees of freedom and at the wrist 2 degrees of freedom of all of them are rotational movements only.

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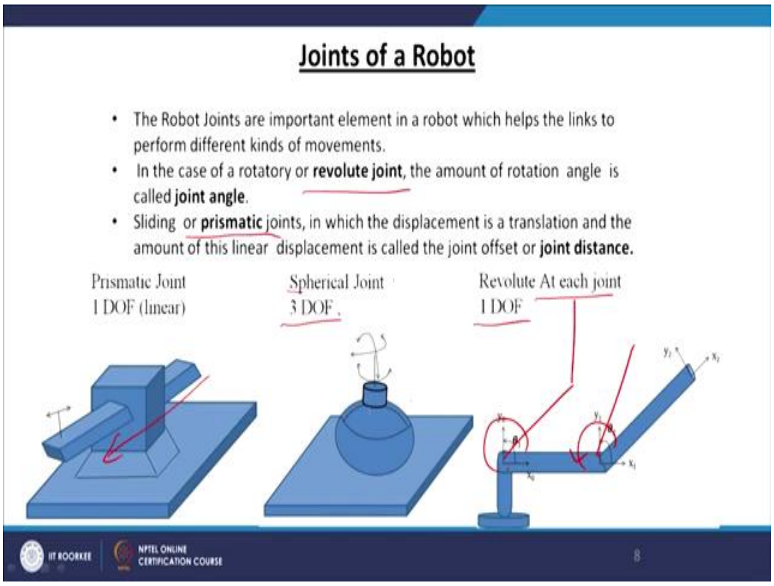

So, the joints of the robot are of two types. One is the revolute joint that is which makes a rotational motion, another is the prismatic joint. So, prismatic joint is. So, this represent a prismatic joint because joint is able to perform as straight line motion only. It will move forward and backward and here the link will rotate like this. So, there is a axis of rotation which is perpendicular to the paper. Similarly, there is a axis of rotation perpendicular to the paper and this joint also can rotate from it is current position.

So, these two are revolute joint. So, here it says that there is a revolute joint and at each joint there is one degree of freedom. So, one rotation is available at each joint. So, at each joint it has one degree of freedom and totally this picture has two degrees of freedom because there are two revolute joints. And, here it is a spherical joint. So, it has rotation in all the three directions, all the three perpendicular directions it can rotate. So, this is 3 degrees of freedom at a particular joint and it is called the spherical joint.

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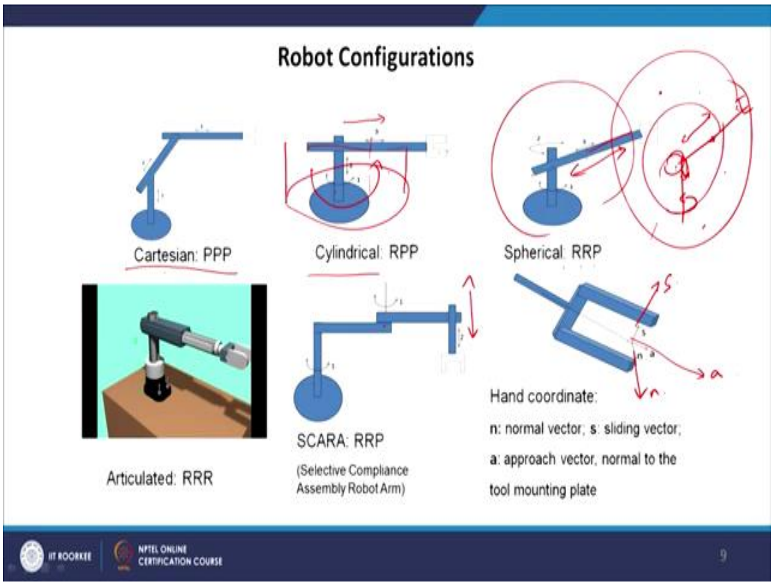

So, here we can see this is a revolute and prismatic motion. So, here the so, the first one is called the prismatic joint; three prismatic joints are there. So, it is a PPP 3 times and it is called the Cartesian manipulator. And, cylindrical you have one revolute joint and two prismatic joint. So, it is cylindrical because the prismatic joint one is it will this link will go up and down like this, and this link again it will move forward and backward and one link is revolute, it will rotate here the base is rotating.

So, the finally, it can be seen that the end-effector can reach points within a cylindrical shape, shape of a solid. So, we can see that the height of the cylinder will be from the base to the total height and it can rotate 360^0 like this and it can move forward and backward. So, in between two cylinders whatever is the point available the end-effector can reach that particular point so, it is cylindrical.

And, here spherical is two revolute joint one prismatic joint here the it can rotate like this and then the last link is it is a prismatic joint, it will go forward and backward and there is a rotation of the second link it will rotate in this particular manner. So, we can see that it will form a spherical type. So, here we can see that the base is rotating with the axis of rotation as the vertical line and the first joint is rotating as the axis of rotation perpendicular to the paper and the third joint is moving forward and backward. So, this will represent a spherical motion. You can see that the sphere will have the center at the joint sec second joint and radius is the total length of the second and third link. And, here we have a revolute joint at the base and a revolute joint at this link and a prismatic joint which will make the link go upward and downwards.

So, the end-effector normally we have to fix a coordinate frame which is given by the perpendicular vector which is called the approach vector and if we connect the two fingers by a straight line this direction is called the sliding vector and the perpendicular to both sliding and approach vector is called the normal vector. So, this three is usually fixed at the end-effector irrespective of what coordinate frames are fixed at various joints.

So, for every robot manipulator at the end-effector we have to fix a coordinate frame exactly in this particular manner; one is the approach vector, sliding vector and the normal vector in this particular order.

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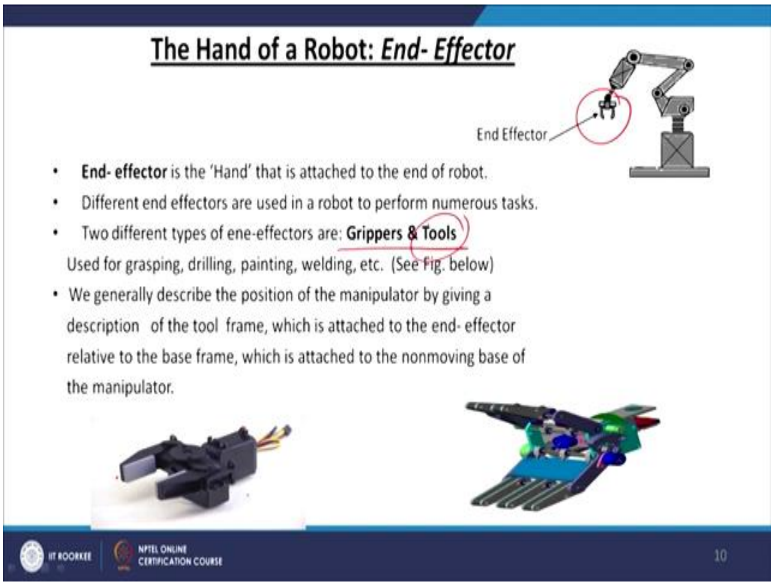

So, the hand of the robot manipulator is also called the end-effector. And, it is of two types; gripper and tools; the gripper is a machine with which has two fingers for example, like this or it can be like a human fingers or it can have simply the tool attached at the end-effector at the end of the robot manipulator that is also called a end-effector here.

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So, in this video one can see a robot manipulator which is having links and joints and the end-effector which grips which is having a gripper and takes an object placed here and then it takes the object and then put it in a different place. So, the task of a robot manipulator is mostly picking and placing in a picking from one place and placing it in another place or a task of a wheeled robot manipulator is taking an object and moving to a different place through wheels and then placing an object in a different place in a factory situation. So, similarly different types of robot manipulator.

The task of a biped robot manipulator or a insect like or animal like with four legs and all. So, they are all utilized for different task in a factory situation or in an entertainment industry etcetera. So, but most of the robots task is only to take an object and place it in another place. And, the control design is mainly for planning a trajectory of the end-effector because it has to take from one place and place it to another. The intermediate path has to be planned properly and how to control the robot for performing the particular trajectory. So, these are the main concerns of a study of a robot, study in robotics.

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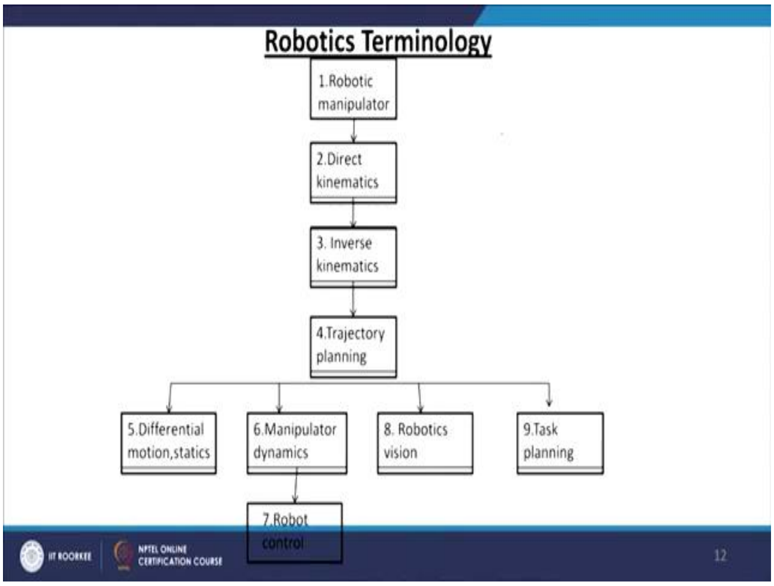

So, here the terminology are like this we have seen a roughly about a robot manipulator and the direct kinematics means how to write the position of the end-effector with respect to the position of the base of a robot manipulator. The equation to be developed so, that is using the DH algorithm.

And, the inverse kinematics it represent how much a robot should for performing a particular task how much angles of rotation is required at each joint. So, the calculation of the angles of rotations at each joint for performing a particular task is called the inverse kinematics problem.

And, the trajectory planning is how to plan a trajectory for performing a particular task for the end-effector and differential motion represent the velocity aspects with what speed the end-effector should move and the dynamics represent what amount of torque or force to be applied at each joint.

So, the dynamic equation is nothing, but the equation of motion of a robot manipulator or any rigid body. So, here you can calculate using the dynamics equation, the torque or force to be applied at each joint for performing a given task. So, that calculation is called the robot control. So, this we will see in the due course.

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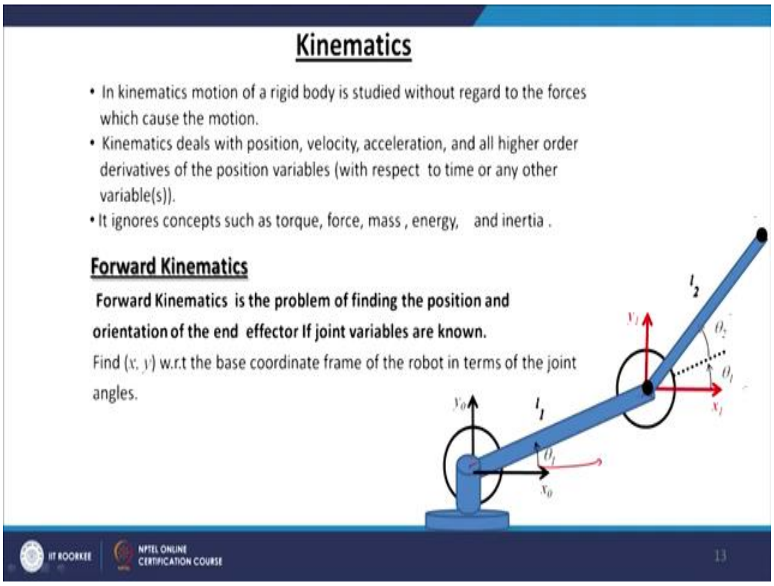

So, for a very simple robot manipulator all this calculations are very easy. We do not require the DH algorithm or anything just a simple geometry will give the kinematics equation and the inverse kinematic solution etcetera.

So, here we see a simple two link robot manipulator and it has two degrees of freedom. There is a revolute joint first one x_0, y_0 is the base coordinate frame and the angle θ_1 measured from the x_0 axis to the first joint is θ_1 and if you extend first link and measure the angle between the first link and second link that is called θ_2. So, we can observe that from the x_0 axis the link 2 is the angle between the x_0 axis and link 2 is $\theta_1 + \theta_2$. So, that is shown here.

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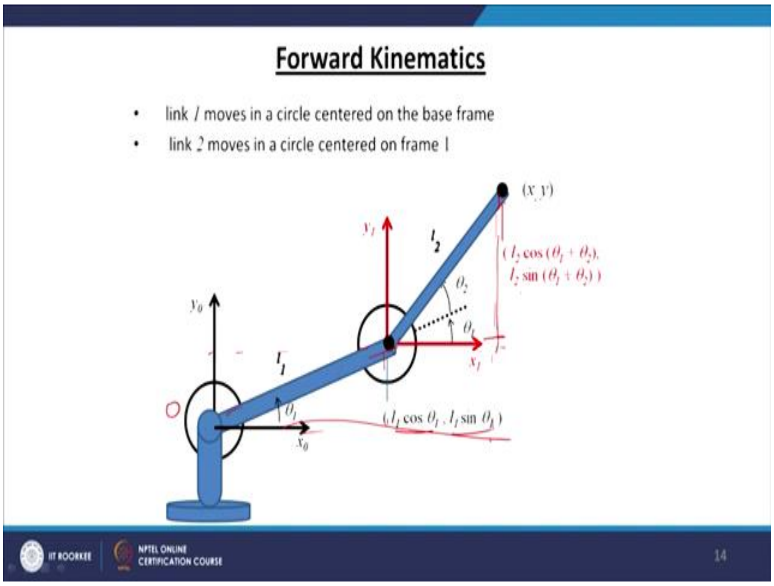

So, using this information and the information that the length of link 1 is l_1 and the link 2 is l_2 we can easily get the equation connecting x, y and the $\theta_1 \; and \; \theta_2$. So, that is very simple to see here.

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

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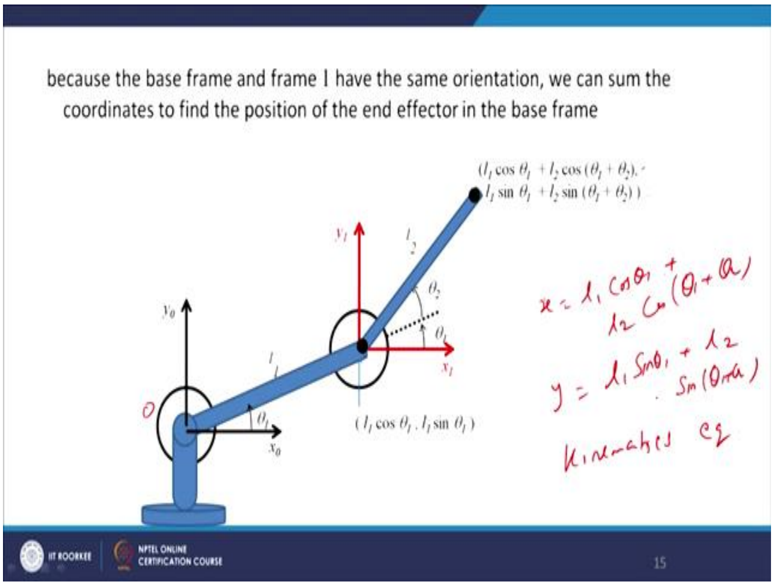

So, with respect to the origin O the x, y coordinates are given by this two formula and which is called the direct kinematics equation.

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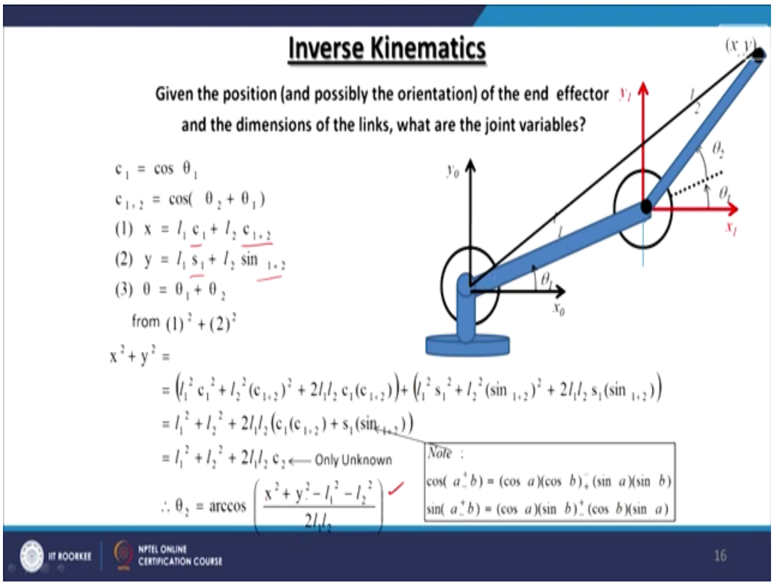

So, this is called the kinematics equation. Now, if $\theta_1 and\ \theta_2$ are given it is easy to find x and y. So, that is called direct kinematics, and if x and y are given find the value of $\theta_1 and\ \theta_2$ that is called the inverse kinematics solution of the problem. So, inverse kinematics can be easily solved by using simple manipulations mathematical manipulations like:

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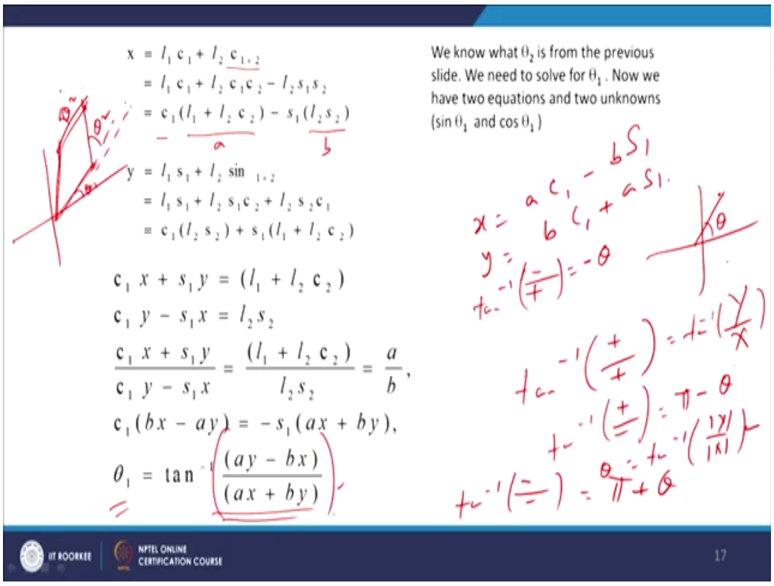

$$
\begin{aligned}
&c_1 = \cos\theta_1\\
&c_{1+2} = \cos(\theta_2+\theta_1)\\
&(1)\, x = l_1\, c_1 + l_2\, c_{1+2}\\
&(2)\, y = l_1\, s_1 + l_2\, \sin_{1+2}\\
&(3)\, \theta = \theta_1 + \theta_2
\end{aligned}
$$

$$
\begin{aligned}
&(1)^2 + (2)^2\\
&x^2 + y^2 =\\
&= \left(l_1^{\,2}\, c_1^{\,2} + l_2^{\,2}(c_{1+2})^2 + 2l_1 l_2\, c_1(c_{1+2})\right) + \left(l_1^{\,2}\, s_1^{\,2} + l_2^{\,2}(\sin_{1+2})^2 + 2l_1 l_2\, s_1(\sin_{1+2})\right)\\
&= l_1^{\,2} + l_2^{\,2} + 2l_1 l_2\left(c_1(c_{1+2}) + s_1(\sin_{1+2})\right)\\
&= l_1^{\,2} + l_2^{\,2} + 2l_1 l_2\, c_2\\
&\therefore \theta_2 = \arccos\left(\frac{x^2 + y^2 - l_1^{\,2} - l_2^{\,2}}{2l_1 l_2}\right)
\end{aligned}
$$

$$
\begin{aligned}
x &= l_1\, c_1 + l_2\, c_{1+2}\\
&= l_1\, c_1 + l_2\, c_1 c_2 - l_2 s_1 s_2\\
&= c_1(l_1 + l_2\, c_2) - s_1(l_2 s_2)
\end{aligned}
$$

$$
\begin{aligned}
y &= l_1\, s_1 + l_2\, \sin_{1+2}\\
&= l_1\, s_1 + l_2\, s_1 c_2 + l_2\, s_2 c_1\\
&= c_1(l_2\, s_2) + s_1(l_1 + l_2\, c_2)
\end{aligned}
$$

$$
\begin{aligned}
&c_1\, x + s_1 y = (l_1 + l_2\, c_2)\\
&c_1\, y - s_1 x = l_2 s_2\\
&\frac{c_1\, x + s_1 y}{c_1\, y - s_1 x} = \frac{(l_1 + l_2\, c_2)}{l_2 s_2} = \frac{a}{b},\\
&c_1(bx - ay) = -s_1(ax + by),\\
&\theta_1 = \tan^{-1}\frac{(ay - bx)}{(ax + by)}
\end{aligned}
$$

So, here we should observe that the $\tan^{-1}$ will have several different values. So, if the numerator is positive and denominator is also positive, then it means if the angle is in the first quadrant the it is like $\tan^{-1}\frac{Y}{X}$ for some $\frac{Y}{X}$ So, if Y and X are both are positive the angle θ_1 which we will obtain is in the first quadrant the that is from 0 to 90^0.

If capital Y is positive X is negative value so, tan-1 of a negative quantity which we will get. So, this will give the angle as $\pi - \theta_1$ where $\theta_1 = \tan^{-1}\frac{|Y|}{|X|}$. We take the positive values of the numerator and denominator and find tan inverse that will give an angle between 0 to 90 and so, $\pi - \theta_1$ value is whenever the numerator is positive denominator is negative we get this value for theta 1.

Similarly, for negative and negative if both of them negative then we will get $\pi + \theta_1$. And, similarly for fourth quadrant if Y is negative and X is positive we will get this is $-\theta_1$.. So, depending on the numerator and denominator values of this expression we can find the θ_1 values.

So, we observed that for reaching a particular position the θ_1 we can find like this after finding the θ_2 values; θ_2 is having two different values. So, you can see that when θ_2 is positive we get this configuration, when θ_2 is negative we get the other configuration because measuring θ_2 is from the extension of the first link.

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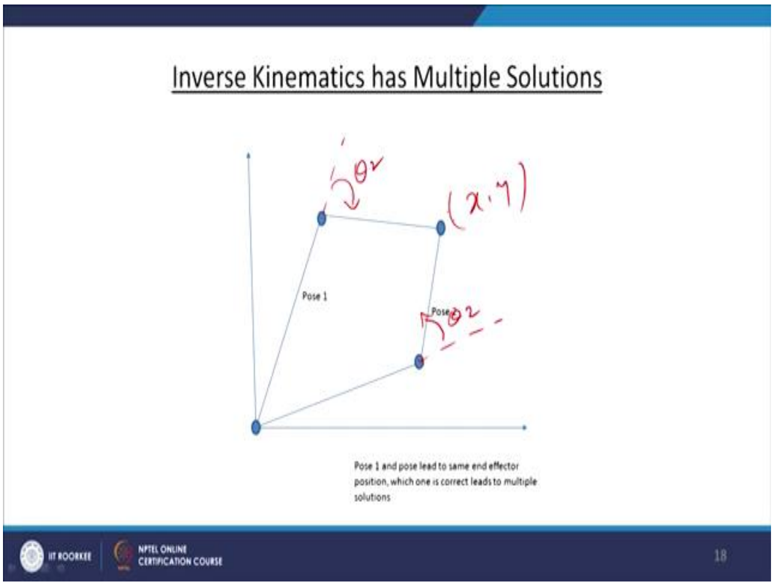

So, we can see that the link one is like this and if you extend it from here we measure the angle θ_2. So, this is in the anticlockwise direction which is called the positive direction. So, if θ_2 is positive we get this configuration and if θ_2 is negative we get the other configuration.

So, for reaching a same position x, y we have two solutions for the inverse kinematics problem. So, if we have several links instead of two link if you have more links then we can see that there will be several solutions for reaching a particular position.

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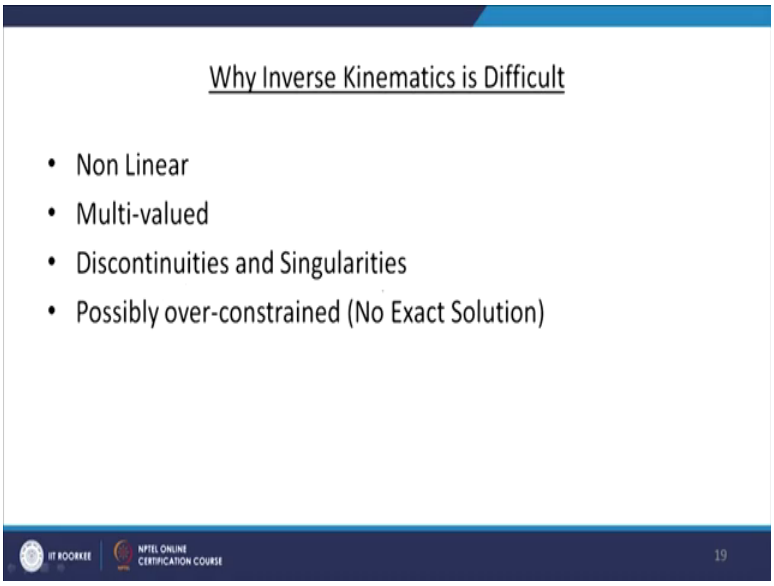

So, in general the inverse kinematic solution will be difficult if we have more complicated robot manipulators and it is multi-valued because to reach a particular position and orientation we can get several solutions and there may be singularities because if we give a particular x, y value the end-effector position, then angle does not exist for outside the range values. So, it means that the position x, y is outside the reach of the robot manipulator.

Or in some different types of robots a particular position may not be the links cannot reach a particular position. So, such difficult positions are called the singularities of the robot manipulators. So, we can see that there may be discontinuities in the path so, which will be dealt in a subsequent lectures.

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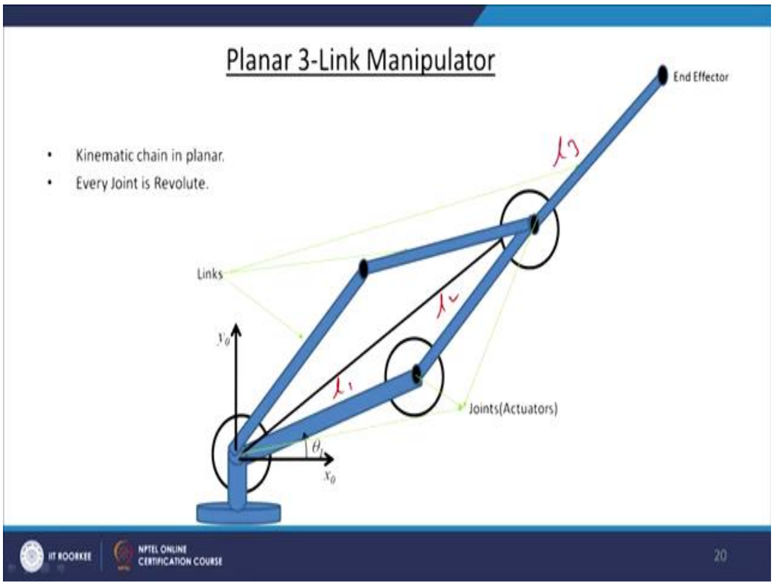

So, here we will see that a planar robot manipulator with three arms. So, here the first link has link l_1, second has l_2 and third one has the link length l_3.

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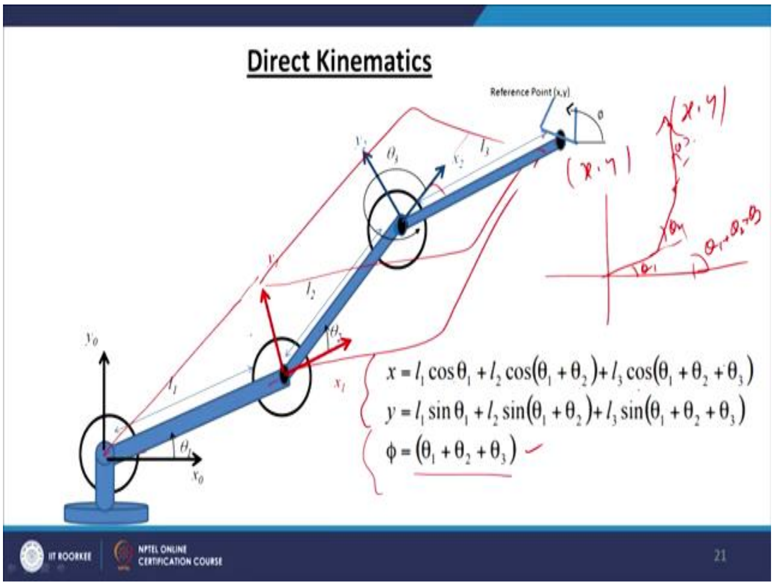

So, the solution the kinematics equation is similar to the previous one two link manipulator if the end-effector position is x, y you can write

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$
$$\phi = (\theta_1 + \theta_2 + \theta_3)$$

So, we get that direct kinematic equation like this.

Now, if you want to reach a object a position x, y using this three arm manipulator, then there is infinitely many solutions for this because there are two equations and we have to find three unknowns $\theta_1, \theta_2, \theta_3$. So, there are several solutions for the a particular end-effector position.

So, for example, we can to reach this particular position we have this two this solution two solutions are available and if you change the first link to this position and this can be again reached by two other positions. So, like that for each θ_1 value you will get two different solutions for link 2 and link 3. So, like that we can get infinitely many solutions for this one.

Now, if you fix this $\theta_1 + \theta_2 + \theta_3$; that means, if you want to reach a particular position we can say that for example, this is our position x, y. So, this angle is θ_1, this is θ_2 and the third one is θ_3. So, this particular angle this is $\theta_1 + \theta_2 + \theta_3$. So, if you fix this value the sum of all the theta values it means the x, y position is approached via this particular orientation of the end-effector.

So, by specifying this value we are specifying from which direction we should approach the point x, y. So, for having that particular orientation we have now three equation and we have to solve the three unknown values $\theta_1, \theta_2, \theta_3$ this will have a unique solution.

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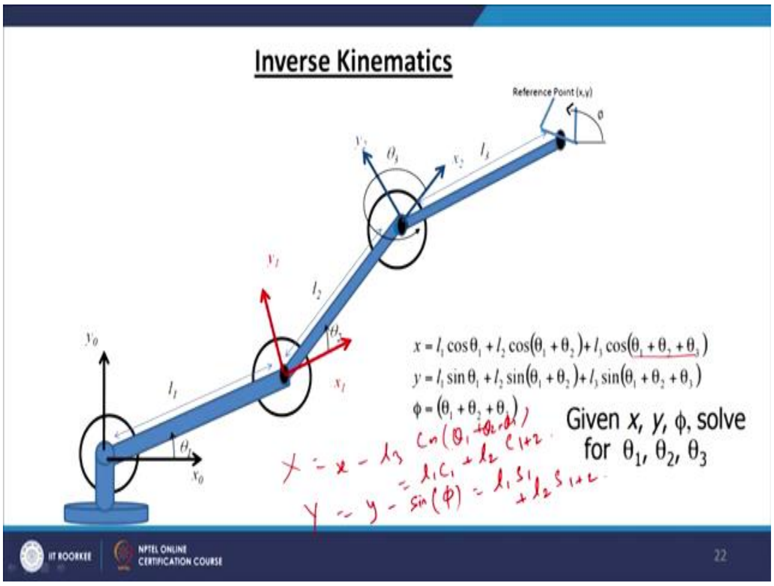

And, the solution can be obtained in the similar way as we have done in the previous because if you take. So, if $\theta_1 + \theta_2 + \theta_3$ is specified by this particular value we can. So, this is known to us. So, what we get is $x - l_3 \cos \theta_1 + \theta_2 + \theta_3$ that is equal to $l_1 \cos \theta_1 + l_2 \cos \theta_1 + \theta_2$.

And, similarly here if you take $y - \sin \phi$ it is equal to $l_1 \sin \theta_1 + l_2 \sin \theta_1 + \theta_2$. Now, if you call the left hand side as some capital X and write the left hand side of the second equation as capital Y so, it is exactly like the two link manipulator equation. So, given this capital X and capital Y we can find the θ_1 *and* θ_2 value.

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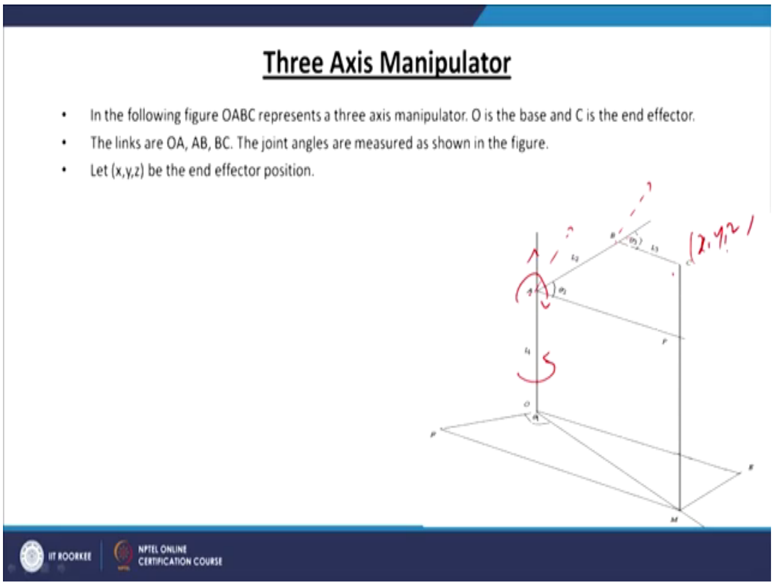

And, if you have a three link manipulator the rotation the first link will rotate with respect to the axis which is vertical line. The second link will rotate whose axis is perpendicular to the paper at a particular configuration it has the axis of rotation parallel to the previous axis of rotation. So, there are three axis of rotation and when the end-effector is to reach a point x, y, z so, how to write the relation between x, y, z and the three angles θ_1, θ_2 and θ_3.

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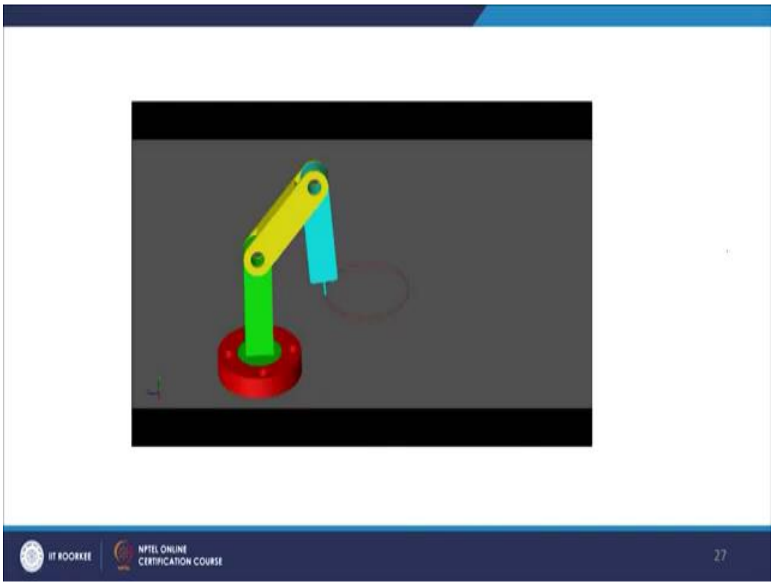

So, that is the three axis manipulator. So, the picture is depicted here and this video shows the exactly the same robot manipulator. The second link is rotating with respect to the axis perpendicular to this paper and the third link is rotating with respect to the axis parallel to the various axis of rotation and the base is rotating with respect to the axis which is vertical line.

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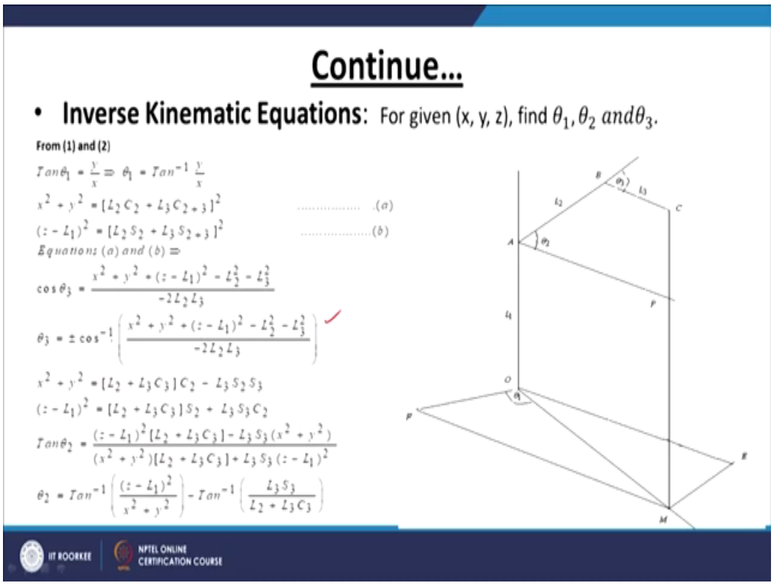

And, after writing the equation kinematics equation we can solve the inverse kinematics equation as we did it for the two link manipulator. So, we can easily see that the theta 2 is given by cos inverse of this expression.

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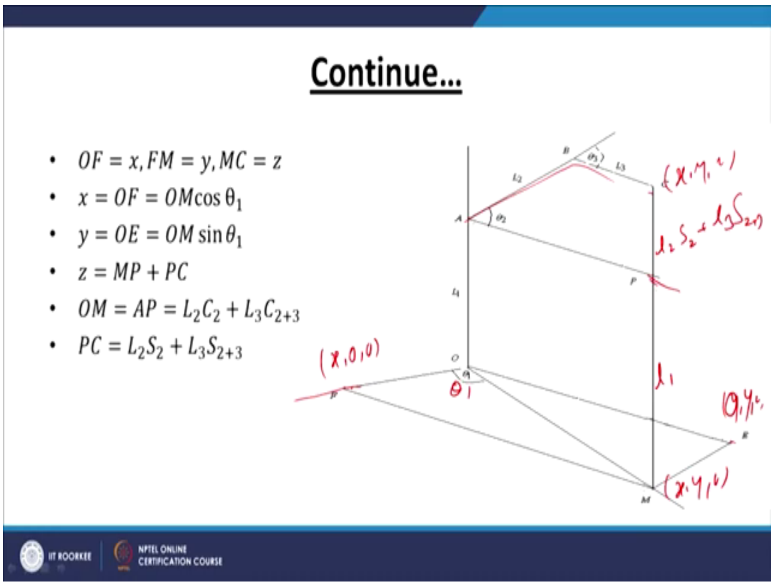

And, so, the inverse kinematics of the previous so, we see that the direct kinematic equation of this robot manipulator is given by the simple geometrical relation as in the equation 1, 2, 3.

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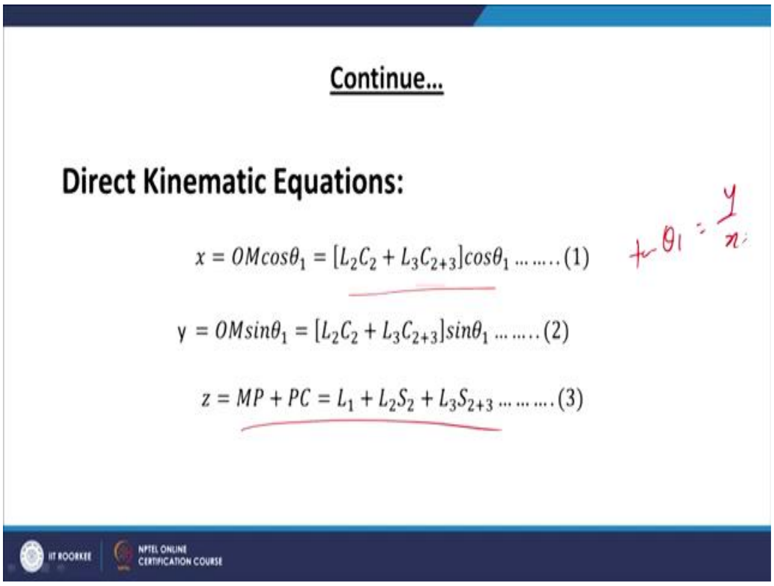

$$x = OMcos\theta_1 = [L_2C_2 + L_3C_{2+3}]cos\theta_1 \ldots\ldots(1)$$

$$y = OMsin\theta_1 = [L_2C_2 + L_3C_{2+3}]sin\theta_1 \ldots\ldots(2)$$

$$z = MP + PC = L_1 + L_2S_2 + L_3S_{2+3} \ldots\ldots\ldots(3)$$

So, this gives the direct kinematics equation from here by just squaring x square and y square we will get a similar equation like the two arm manipulator and we get the inverse kinematics solution first we get θ_3 and then we can get the angle θ_2 *and* θ_1 can also be obtained directly. θ_1 can be directly obtained by just taking dividing equation 2 by equation 1. You will get tan $\theta_1 = \frac{y}{x}$. So, θ_1 can be obtained from this.

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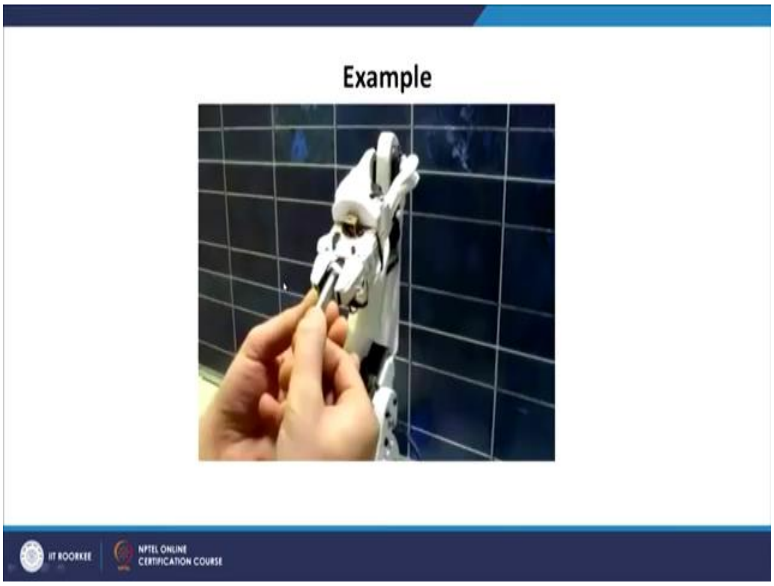

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So, this video shows that the YouTube video shows the exactly similar robot manipulator where the base is rotating like this and the two links are rotating, the end-effector is reaching a given particular point.

So, in this lecture we have seen the simple configuration of robot manipulators with two link and three link and how to write the inverse kinematic solutions for them. And, in the subsequent lectures we will see more complicated robot manipulators.

Thank you.