Financial Mathematics
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Lecture-09
Introduction to Compound Interest

Welcome to the lecture on introduction to compound interest, so in the lecture which has been completed we discussed about interest and especially the simple interest was introduced. And in that basically we also discussed about the discount rates and how to calculate the simple interest and all that. Now we will have the introduction about the compound interest, now as we see that unlike the simple interest method in the compound interest method.

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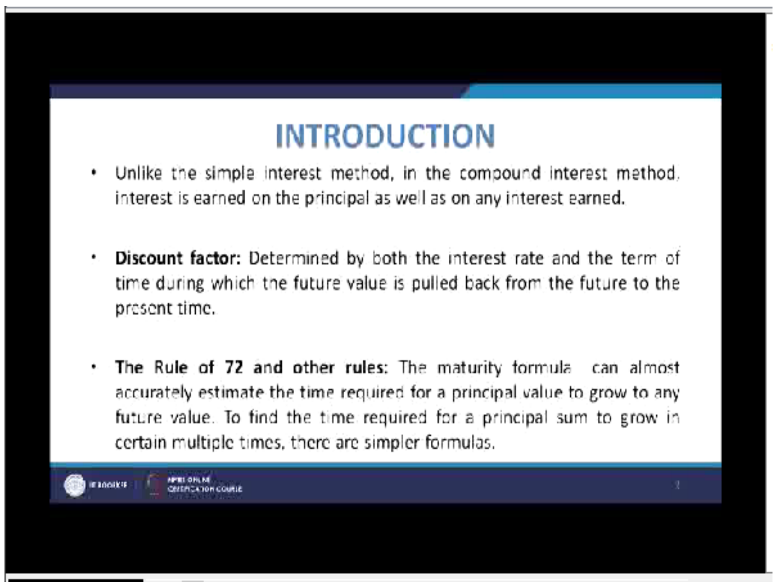

Interest is earned on the principal as well as on any interest earned, so that is what happens in the case of compounding that as the time moves on the interest amount does not remain fixed which is the case in the case of simple interest. Because in the simple interest basically it will be depending upon the first value and there will be certain percentage of that is the rate of interest. And that is multiplied with the time however when we talk about the interest earned even on the interest.

So, then it is defined as the compound interest. Now if you talk about you know about for example if you have suppose 1000 rupees you have invested. And if it is simple interest, so this way you can see that if you have suppose Rs.1000 is there.

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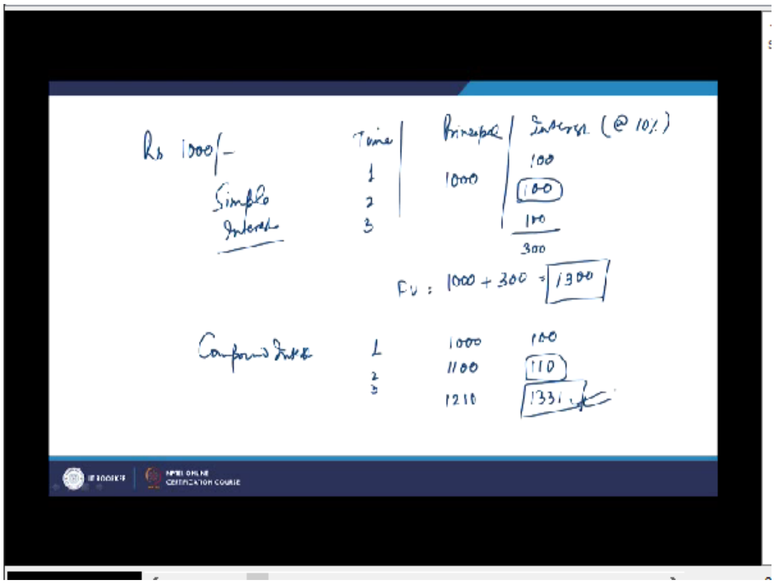

And if you take the simple interest you know method then if you know time is going like 1, 2 or 3, so your principal amount is basically elements to be you know 1000. So this is and then if you see the in the first year your interest will be if the rate is 10% suppose, so this will be basically 100, second year also 100 and third year also 100. So, ultimately your total interest becomes 300, so this is simple interest and you have to pay 1000+300 that is 1300.

So, you have to pay 1000+300, so this will be your future value and that will be you know 1000+300 that is 1300. So that is there in the case of simple interest, now the difference between the compound interest is that you have in this case your time goes like 1, 2, 3. And in this case the principal at the first year, so first year it is 1000 and interest earned is certainly at 10%, so interest will be 100.

Now in the 2nd year basically this 100 is added and 1000+100 that is 1100 becomes your principal amount. And then that on that basically you are getting at the rate of 10%, so it will be 110, so in the beginning of the 3rd year your principal amount becomes 1100+110, so it will be 1210. And again on this you have 10% of interest earn, so it will be 121, so it becomes 1331. So, this is the difference you get when you are basically calculating even the interest on the interest earned.

Because this is the interest on 100 and in the case of compound interest basically you this is the interest is calculated on this 100. So, in the earlier case basically you are interest earned was only 100 whereas in this case interest earned is 10% of 100 also extra that is 10. So, 110+10, so that way you know the things change and then again +11 next time also. So ultimately 221 is the you know extra that way, so 331 comes, so this is 1331 comes like that.

So, this is how the difference between the simple interest and compound interest you know can be understood now many a times what we see is that in the companies if you try to see the you know look at the value of these money. So, what happens that the future value will be the current value+r*current value, so if you try to see that how they are changing graphically basically you can further see that if you have any amount.

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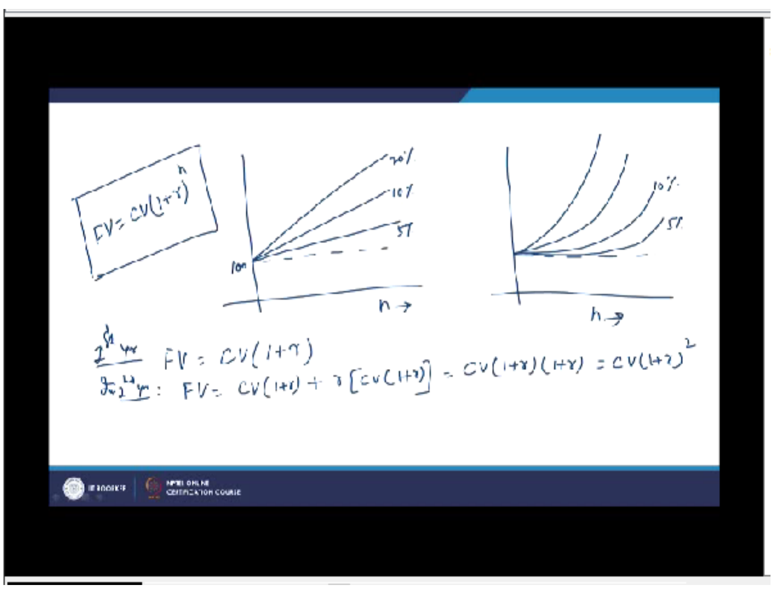

So it will be a changing and when we take of the simple interest rate, so if the n is moving like this and you have invested some amount suppose at this point. So, now when you have 5% interest it will move like this, so this is going to be the case when you have the simple interest cases like in 5% or in 10% or in 20% or so. So, that way it will move whereas if you take the example of the compound interest, now in the case of compound interest this value basically is going to change like if you have n going like this.

So this 5%, so this move like his than 10% will move like this, 15% will move like this then so this way your 5% or 10% or so. So, this way you can see that this is straight increase it is a proportions slope which is I have fixed one when as in this case as the time you know increases that goes on increasing. So, that is what is the change in the value as we move with time.

So this is its value as it changes with time, so what we can see that in the case of simple interest basically we were calculating like F=CV*1+rn 1+r*n. Now that was the case in the case of the simple interest, now if you take the compound interest what happens that you have FV is as we know that if you take for years. So, FV initially at the end of year CVV*1+r. Now this is going to be in the 1st year. Now when you go to the 2nd year then what will happen?

So, in 2nd year, so in the first year it will be happening in 2nd year what happens that your future value, so this will be your present value. So, CV*1+r+ basically your rate is r%, so r*CV*1+r, so what you see that you get CV*1+ r is common and then comes 1+r. So, it will be CV*1+r raise to the power 2. So, in the end of the second year your future value becomes FV is CV*1+r raise to the power 2.

So this way your future value will go on changing and basically at the end of the n years there are n years, so you have a general formula that FV, so if the you know term of maturity is n years. In that case it will be CV*1+r raise to the power n, so that is how you are basically calculating this future value. Now so this is in the case of the you know compound interest. Now you can have the problems based on such like someone has invested for some years.

Then what will be the future value you know at some rate of interest when it is compounded annually. So, when that term is mentioned annual compounding is mentioned. In that case you have to use these terms you know use this equation, so if the current value is given and r is given and n is given, so you have to use them. And that way you have to use with this factor 1+r raise to the power n.

So, basically this factor will be you can also get it, so there are factors. This will be also present r available to you in the form of tables, so simply you have to multiply with that factor and you can directly get the future value. So, that is how you get these values in the case of you know this compound interest now similarly, so in this case you are getting the future value you can also calculate the current value also.

So, if you just the reorient the equation you just try to find current value it will be nothing but FV/1+r raise to the power n.

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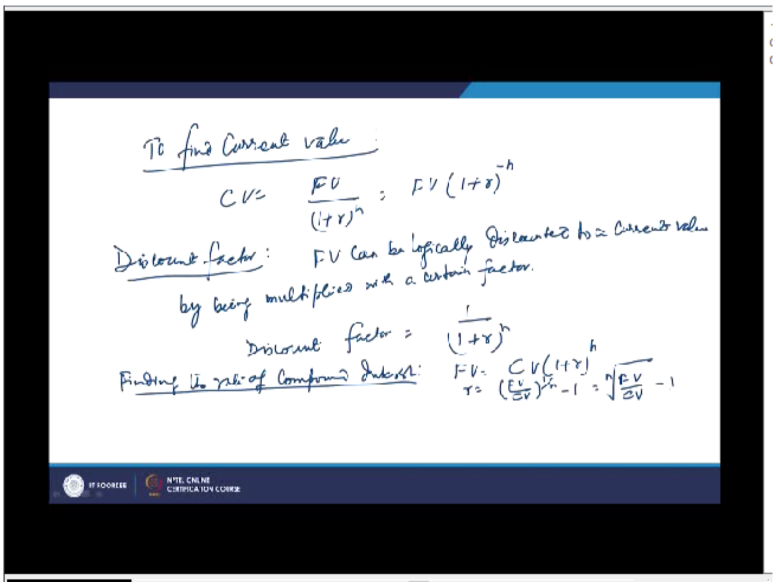

So, basically when you have to find the current value to find how to find current value, now in that case basically you have to use the similar equation. So, you have the current value will be FV/1+r raise to the power n, so this is CV will be FV/1+r raise to the power -n or you can also write FV*1+r raise to the power -1, so this is the factor which will be basically multiplied 1+r raise to a - 1 this is nothing but the reciprocal of the factor which have been used to find the future value from current value.

So it is just a reciprocal of it and this is used to find the you know current value from this future value which is available to us. So, again if you know the future value someone has to achieve some future value in certain time and the rate of interest that the compound interest is known. In those cases you can use this formula and you can get it suppose somebody has to be to inherit some amount in certain years.

So how much you know put he should put now in cash, so that is at certain compound interest per annum he should get that amount in 5 years suppose. So, in that case 1+ that rate which is given that value divided you know raise to the power -5 and that should be multiplied with that future value which is to be inherited. So that will if the amount which should be invested now in the bank.

So, this way you can solve such problems easily then similar to what we discussed about the discount factors you know in any what we saw in the case of simple you know interest. Here also you must know what is the discount factor, so again if you go to the definition of the discount factor which were discussed in our earlier case. So, you know so you know I mean this that factor which you know current value, so that is to be multiplied.

So, that you know you get it you get this current value, so that future value has to be multiplied with that and that is known as the discount factor. So, you know this is it is known as that future value can be an logically discounted to a current value ok, so basically you are discounting this future value to the current value what should be the current value which will gives you that future value.

So that is discounted to the current value by so you are multiplying this future value with certain factor that is your discount factor. So, by being multiplied with a certain discount factor, so that is the factor which is known as the discount factor and it will depend upon the rate of interest and also that time of maturity. And that is how so that is basically 1+r raise to the power you know -nor 1/1+r raise to the power n.

So, your discount factor becomes as we discussed that this discount factor becomes 1/1+r raise to the power –n. And this factor is known as the discount factor, now we will see that how do you know if such problems are coming like you know how to find the rate of compound interest.

There may be, so you have just to reorient these expressions and you can find the value of the rate of compound interest, so finding the rate of compound interest.

Now you see we have the expression between FV and CV and we know that FV=CV*1+r raise to the power n. So, FV will be CV*1+r raise to the power n. Now if you have to find the rate of interest that is r, so 1+r raise to the power n will be FV/CV. So, if you try to get the expression for r basically r will be you know 1+r raise to the power n will be FV upon CV. So, r will be again 1 will be subtracted, from that so FV upon CV raise to the power 1/n will be 1+r.

So, it will be FV/CV and raise to the power 1/n-1, so this will be the rate of interest, so there may be cases when you want to know what that rate of interest you should invest. So, that in certain time you should have the you know amount being multiplied to certain you know certain factor being multiplied by certain factor like it should be 2 times or 3 times or 4 times. So, how much you will be the rate of interest you know, so that it becomes it has certain ratio.

So, FV upon CV it also can be written like FV you know and by CV and then it is nth root-. So, you can also get such kind of expressions and you can find this rate of interest, so suppose there is you know offer that or there is a question that you want to buy a car for suppose say in 50 lakhs rupees or maybe 50 lakhs in certain 5 years suppose. And you want to start investing now, so and you have some amount which you can invest.

So, at what rate you know what should be interest rate that will give you that much amount after 5 years. So, in such cases simply r can be found by FV/CV and then you can have this value computed. So, that will give you this rate of compound interest which should be you know applied which will give you certain you know FV/CV ratio similarly the another thing which can be you know further required is finding the compounding term.

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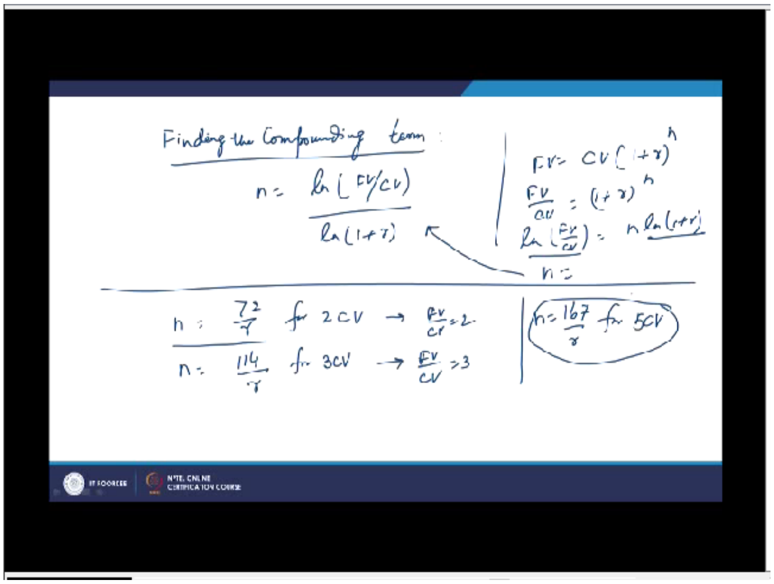

Now, so only one thing is left that is n and again now if you want to find n, so n will be basically lnFV upon CV/ln of 1+r. So, basically you have again we know that FV=CV*1+r raise to the power n, so FV/CV will be 1+r raise to the power n, you have to take a log on both the sides ln of FV upon will be n*ln of 1+r. so, this way n becomes equal to, you know n comes equal to here, so n will be this upon this.

So, this way you can get this compounding term that is your ln of FV/CV/ln of 1+r, so that way you know you get, so suppose you have to you know the rate of interest and you know the future value and the current value. So by using this expression you can directly get the in how many number of years in how many number of years you will get a certain ratio of FV and CV at a particular rate of compound interest.

Now many a times what we see is that we need to know that normally we are interested to know that when our money will be doubled or when our investment amount will be doubled or tripled or so. So for that there are very standard rules and so we discuss about this is discount factor, so it is determined by both the interest rate and term of time and during which the future value is pull back from future value to the present time.

So, that is how discount factor was discussed and then there are rules of 72 another rules, so we are going to discuss about the formulas which are used basically which will be estimating the time required for a principal value to grow to any you know factor. So, what happens that many a times we are interested to know that when our investment amount will be doubled or tripled and for that there are simple formulas.

And the formula is that this n the time during which your investment amount will be growing by factor, so it will be 72/r for 2CV. When the future value has to be 2CV, for 2CV means the future value has to be to 2CV, so FV/CV will be 2, so for that n is 72/r. So, for any rate of interest suppose if r is 12% it means n will be 6. So, when the rate of interest earned is 12% in that case if you are putting an investment amount now.

Suppose 1 lakhs rupees we are investing now then and the rate of interest earned is suppose 12% compounding annually. In that case in 6 years we are going to get the 2 lakhs rupees of that, so that is what it is telling and this is known as the rule of 72 and if suppose we want to go to triple our amount, so if 1 lakhs is to be you know converted to 3 lakhs. So, for that the rule is n=114/r, so this is for 3 times are you know the current value.

So, future value/current value ratio will be 3 in this case FV upon CV is 2 or in this case FV upon CV will be 3. So, these are basically the rules if you are going further for 5CV, so n will be basically 167/r for 5CV, so these are the rules basically which are used you know to calculate that for a particular rate of interest in what amount. So suppose somebody has to double the investment amount in that case at particular suppose 10% is the rate of interest, in that case it will be 7.2 years or in this case for 3CV 10% is there, so 11.4 years like that.

So this is the meaning of the you know the rule of the 72 years and in that there is a further we can have suppose what happen that you know sometimes the we have to see that we want to have the you know this our investment amount to be increased to 34. So for example, so we can have that suppose we want to have investment grown by 30 times.

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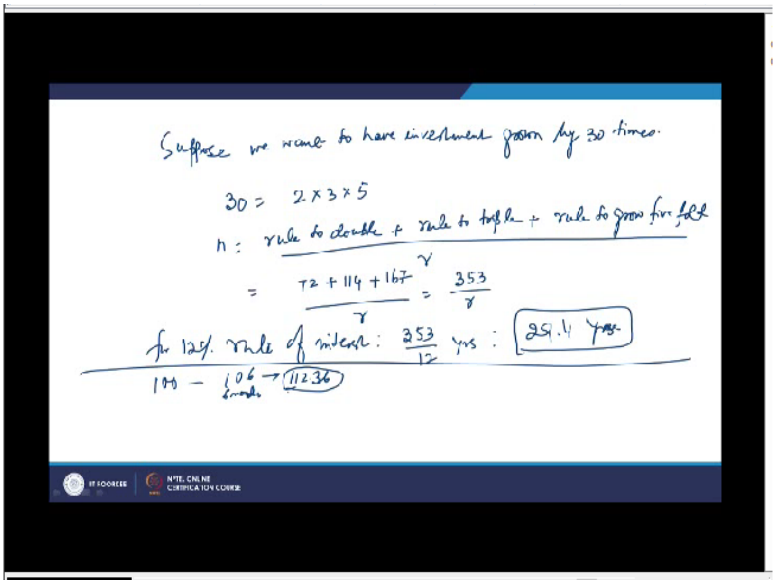

Now in that case what we do is that normally we have to see that you know we have to convert this 30 into factors and we have to break down this factor into, so that is into multiples and then there are rules by which you calculate. So suppose this 30 if you calculate, so it is to be 30 will be put into the number of factors it is as 2*3*5. So, you know that 72 basically we know that 72 is the number of months you know that way no that is different.

So, you have 30=2*3*5 now the rule is that and n has to be rule to double, then rule to triple and then rule to grow 5 fold. And this divided by r, so this is what the rule becomes. So, as we know that in earlier case it was 72 or n=72/r. Now in this case you have rule to double rule triple and rule to grow 54. Now as know the rule to double is 72, then you have you know rule to triple for that you require 114 is the you know in the numerator.

So, you have 114 and for 5fold you have 167/r, so if you add them it will be 186+167 it will be 353/r. So, now if you know if you look at that if your rate of interest is supposed 10%, at that 10% what will be the time required when your investment will be 30 times, so it will be 35.3 years or if it is supposed 12% then it will 353/12. So, for 12% rate of interest it will be coming as 353/12 years, so it will be something like 29.4 years.

So, this is how you are calculating those number of years basically in which the amount will be basically you know changing part from that when we do talk about compounding. So in that compounding there is also that normally by default we feel that the compounding is done annually. However compounding can be done at any interval, so you may have compounding at frequent intervals of time and that may or not necessarily be 1 year.

So, compounding maybe after 6 month, so you have for the you have annual compounding so that is you know, so you are doing compounding once you have you know semiannual compounding which you are doing the compounding twice. You have the you know compounding, so quarterly compound means you are doing the compounding 4 times, so basically you when you do the normal compounding annually.

So in that case your interest rate is fixed you know that 12% is the interest rate for a year. But when there is compounding for 2 years 2 times. In that case 6% is used after 6 month, so **so** for example if you look in at that case, so 100 will become 106 after 6 months and then after 6 months further you will have multiplied this with again 1.06. So, that way you will have 112.36, so basically your effectively you are getting the interest more.

So, if you are being charged you are paying more or if you are receiving then you are receiving more, more than 12%. So, that is basically the effective interest and this is because of the number of times this compounding is carried out. So, because of that you have a different terminologies like you have the effective interest rate and in this case as you see that the effective interest rate becomes larger and it depends upon how many times the compounding is being carried out.

So, accordingly the effective interest rate will be you know calculated and that gives us that what is this effectively you are being charged the interest rate. So this is about the compound interest and we will in our coming lectures will discuss about different type of compounding factors. And we will also deal with those situations how depending upon the in the payment at different times of different amounts or so how this compounding is being done, how the you know this cash flow diagrams look like for such cases, so that will discuss, thank you very much.