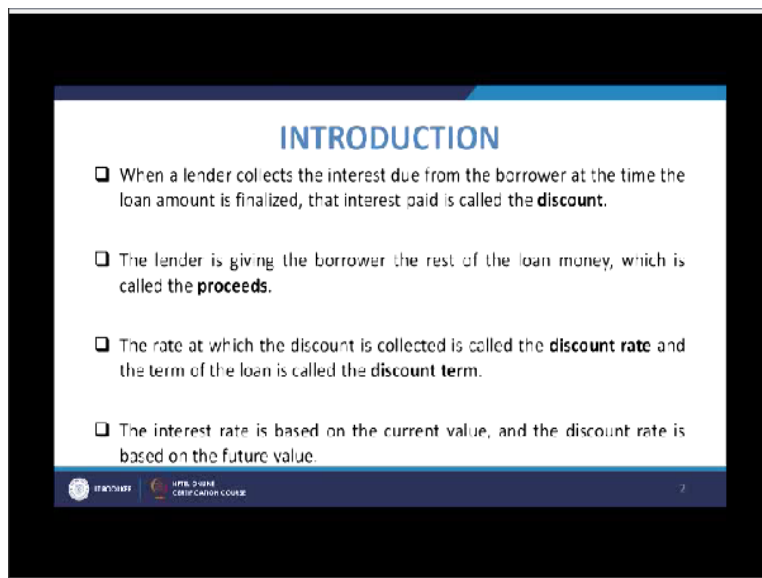


Financial Mathematics
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Lecture-08
Introduction to Bank Discount

Welcome to the lecture on introduction to bank discount, so will have some more introduction about and more knowledge about the discount bank discount basically.

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INTRODUCTION

- When a lender collects the interest due from the borrower at the time the loan amount is finalized, that interest paid is called the **discount**.
- The lender is giving the borrower the rest of the loan money, which is called the **proceeds**.
- The rate at which the discount is collected is called the **discount rate** and the term of the loan is called the **discount term**.
- The interest rate is based on the current value, and the discount rate is based on the future value.

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So, when a lender will be collecting the interest due from the borrower at the time the loan amount is finalized the interest paid is called the discount. So what happens that suppose you are have getting loan and you are supposed to paid, so some amount which you suppose somebody has you know to get a loan of 1000 and 10% is the interest. So he will only get, so interest is basically 100 at the end of the year and basically he will be getting only 900.

So that amount is basically known as the discount, the lender is giving the borrower the rest of the loan money. So, that 900 which will be getting that is basically known as the proceeds the rate at which the discount is collected is basically known as the discount rate and the term of the loan is called the discount term. So this is something unit rate of this discount which is there in the case of transactions and what we find is that when we try to see the interest rate.

Interest rate is basically based on the current value whereas the discount rate is basically based on the future value. So that is how the you know this terms are having the significance.

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discount rate is 10% on a loan of Rs 1000.
 Bank will deduct its discount (Rs 100)
 Borrower gets 1000 - 100 = Rs 900 - proceeds
 C.V. → 900

$$I = C.V. \times r \times n$$

$$D = FV \cdot d \cdot n$$
 D = total discount (bank discount)
 FV = Maturity value of loan
 d = discount rate
 n = term of the discount

Suppose if you look at the you know example suppose that discount rate is 10%. Suppose, so on a loan of 1000, now in that case what will bank will do is the bank will deduct its discount. So bank will deduct its discount and this discount is basically 10% of 1000, so it will be Rs.100. So basically the borrower will only get 1000-100, so borrower gets 1000-100 that is 900 and this is known as proceeds.

So that is what we have seen that in this case the lender is giving the borrower the rest of the loan money and that is basically known as the proceeds. Now you know this borrower he have to pay back this 1000 rupees, so 1000 rupees basically is considered as the future value that is why you know that on that amount basically this discount is computed. So, which is there in the case of discount otherwise in the case of interest you calculate on the current value.

So so in this case that is why 1000 taken as the future value and you have current value of 900. So, if you take the discounted rate, so that is your 10% that is 100 you know by 1000 that becomes 10% and you should not be confused with the interest rate. Because interest rate if you look at you know in this case it will be if you find the interest rate, so it will be 900 and 100.

So, in that case 100×100 , so that will be something like 11.1%, so on 11%. So, that will be the rate of interest, so the interest and the total discount which we calculate they are basically different in that sense. So when we calculate the interest it will be $CV \times r \times n$ and similarly when we calculate the discount total discount it will be $FV \times d \times n$. So, it is a to r and here it is a rate of discount that is D, so what will see is D is total discount and we also call it as the bank discount.

Now FV is the maturity value of the loan and all these terms like d is the discount rate and n is the term of the discount, So what we saw is that basically you know this from the future value, now this discount will be subtracted and that is why in that case what we expression we get is that we get C that is your proceeds.

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The image shows a whiteboard with handwritten mathematical derivations and a numerical example. The derivations are as follows:

$$C = FV - D$$

$$\text{But } D = FV \cdot d \cdot n$$

$$C = FV - FV \cdot d \cdot n = FV(1 - dn)$$

$$FV = \frac{C}{1 - dn}$$

Two boxes are drawn to show the relationship between variables:

- Box 1: $d = \frac{1 - \frac{C}{FV}}{n}$ (labeled "discount rate")
- Box 2: $n = \frac{1 - \frac{C}{FV}}{d}$ (labeled "discount term")

The numerical example is written on the right side:

Ex: to borrow Rs 1500 from local bank for 1 1/2 yrs at discount rate of 9 3/4% How much will be proceeds & interest (?)

$$\rightarrow D = FV \cdot d \cdot n = 1500 \cdot (0.0975) \cdot (1.5) = 219.75$$

$$C = 1500 - 219 = 1281$$

At the bottom of the whiteboard, there are logos for "BDOORIE" and "APTEL ONLINE CERTIFICATION COURSE".

And this proceed will be in FV-the discount amount, so that is how you get you know these proceeds that is C. So, C is proceeds, so this lender will basically deduct this discount amount from the you know paid back value that is future value FV. And that is how you get the proceeds, so now what we found D as, so but we got D as $FV \times \text{discount rate} \times n$ that is how we calculate the d. So what we see is C will be equal to $FV - FV \times d \times n$.

So, it will be $FV \times (1 - dn)$, so you can calculate the proceeds once you know the discount rate and the future value, in that case you can and also the term of the discount. So that way you can find the proceeds, now if you have to find the future value, so you can further rearrange these

equations. And you can find the future value and if you know the proceeds it will be $C/(1-d)^n$, so from the same expression if you reorient these expression you will get this FV .

And you can get the future value as $C/(1-d)^n$, so you can have certain examples suppose that someone has to borrow, so if you have to borrow suppose Rs.1500 from local bank for 1 and half years, now at you know the discount rate of $9\frac{3}{4}\%$. Suppose so how much that person will receive, so what how much will be proceeds, so and how much will be the bank getting as a discount. So, that you can find and if you look at, so and discount also.

Discount by the bank will be how much, so we know that the d will be $FV*d*n$ and so so your d will be basically $FV*d*n$, so it will be 1500. And then the discount rate if you look at, so it is **97**, so it will be 0.0975 and then have one and half years it will be 1.5. so, the discount amount will be approximately 219 rupees, so the proceed which will be getting the person it will be it will be something like 1500.

So C will be $1500-219$, so it will be **2 1281** something like, so it will be close to that, so whatever you are getting here you will be getting the same no competition can be done. So, whatever you are getting this will be you know you it will be checked, so you can calculate from this value and you can have this amount calculated. So, that is how you calculate this you know proceeds or you can calculate the amount of discount which will be you know which you has to be found out.

Now you may also need to find the discount term and the discount rate, so from the same expressions you can have these you know, so if you have to find the discount rate you have to find d from here. So if you say $1-d)^n$ will be C/FV , so you are getting $1-d)^n$ as C upon FV, so from here you can either the value of d and so it is $d*n$. So, you can get the value of discount rate or the term and if you find the d, so d will be $1-C/FV$, so it will be $1-C/FV$.

So, it will come this side so and then divide by n or if you have to find the n, so n will be $1-C/FV/d$, so this way you will have the expressions by which you can compute the discount rate you know discount term form or discount term formulas. This is formula for the discount term

and this is a formula for the discount rate, so that way you can use so suppose any you know for a loan something is given and the dates are given and how much borrower gets.

So you know the C also proceeds also, so in that case you can find the you know and the days are given, so you can find the rate or if the rate is given then you can find the days. So, that is how you can find it, now if look at the difference between this simple amount simple discount and the bank discount. Simple discount we have studied earlier, now what is the difference between the simple discount and the bank discount.

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Difference between Simple Discount & Bank Discount

Ex: Discount Rs 5000 for 6 months at 9% using Simple Discount & Bank Discount.

→ Using Simple Discount:

$$CV = \frac{FV}{1+rt} = \frac{5000}{1+0.09\left(\frac{6}{12}\right)} = 4784.69$$
 Simple Discount = $PV - CV = 5000 - 4784.69 = 215.31$ Rs.

→ Using Bank Discount: $D = PV \cdot d \cdot t = 5000 \cdot (0.09) \left(\frac{6}{12}\right) = 225$
 $C = PV - D = 5000 - 225 = 4775$
 Bank Discount = Rs 225

So, difference now how to you evaluate these simple discount and the you know bank discount, simple discount as well as the bank discount. So, basically what we have seen that there is a procedural difference of collecting the discount amount, so a bank discount when we talk about it has a slight computational difference from the simple discount. And we can understand it with the help of one example.

Suppose you have, so suppose we discount Rs.5000 for 6 months at 9% using simple discount and bank discount. Now if you use the simple discount formulas if you are using simple discount, now as we know that in the case of simple discount what we do is that you are getting the current value based on the future value and you know have r and n known. So you will have CV as $FV/1+rn$.

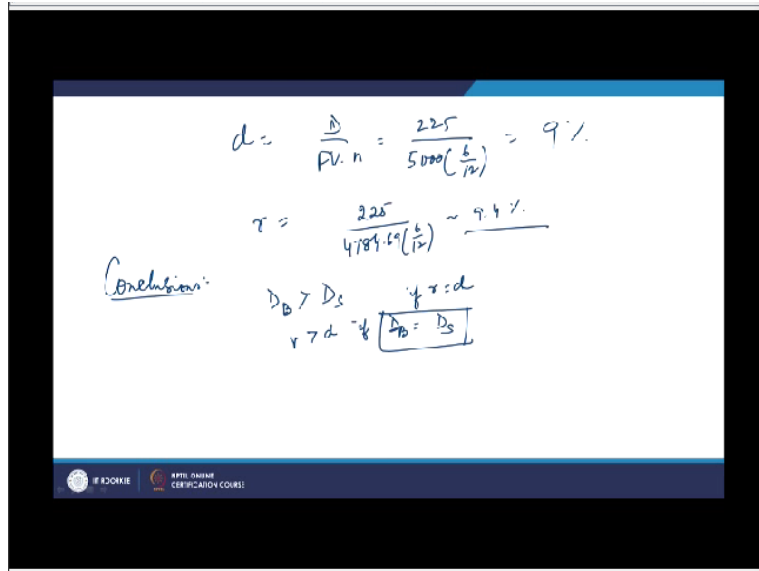
So what we do is that you have FV as 5000 and $1+rn$, so r is 9%, so it will be 0.09 and n is 6 months, so $6/12$. So, that is n , so in this case you are getting 4784.69. So, that is how so what if you calculate the simple discount it will be $FV-CV$, so it will be your FV is 5000 and your CV is 4784.69, so it will be you know 215.31. So, this is the simple discount now if you use the bank discount, now in the case of bank discount you know that day will be $FV*d*n$, so your d will be $FV*d*n$ and in this case FV is 5000 and the d , d is again 0.09 and n is certainly $6/12$.

So it will be 225, now in this case the proceeds will be every $FV-d$, so $FV-d$ will be $5000-225$. So, it will be 4775, so now what you get is that bank discount comes out to be 225, so your bank discount is now what you see is that bank discount method has produced a larger discounted that is 225 as compared to the simple discount method which has given you to 215.31. So because this is the case when you are using both discount rate as well as interest rate of interest same as 0.09.

So that is how that is basically the difference between the simple discount and the bank discount. Now if you try to have the different logic of achieving that using this example. Now what we see is that the discount basically will be d will be $FV/$ so that is the discount rate it will be so discount rate if you have to find it will be $d/FV*n$. So, that becomes equal to 9% whereas if you find the r that is rate of interest.

Now in this case your amount is 225 and then divided by 4784* you know the time and that gives you the rate of interest as 9.4 %. So what we see in normally is that the interest rate for the simple discount process is normally larger as you know look at in this case the 9% will go to 9.4%. If you use the simple discount method in those case. So, that can be seen the what you see is that if your discount is 9% how it is coming.

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Because you are getting $D/FV \cdot n$, so that is how you are getting 225 and this FV is 5000 and this is $6/12$, so that is how you are getting as 9%. Now if you try to have the rate of interest for this. So, rate of interest will be basically you have 225 as the discount and what you have the 4784 and you have the 0.69 and then $6/12$. So 4784 is here calculated and this is the current value because one you are talking about the rate of interest you have to know about the current value only.

So, this comes out to be 9.4 %, so what you see is that when you calculate using the simple discount method. It gives you a larger value larger you know rate of interest that larger value interest rate will be larger using that you know simple discount process. So what we, so we can have a certain conclusions and that conclusions can be you know pointed out and this is that when we use the same rate of interest r and that d .

Then the simple discount amount of the bank discount method would be larger. So that is what we have seen that the bank discount will be larger than the simple discount. So, if r is taken as equal to d , that is what we have seen in this example and r will be d more than d if $D_B = D_S$. So, if D_B will be equal to D_S in that case r will be more than D we have seen that here. So this is what is there the difference between the simple discount and the bank discount, now we can compare this discount rate to the interest rates.

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Compare Discount rate to Interest rate

$$C = PV(1-dn) \quad CV = \frac{FV}{1+rn}$$

$$FV(1-dn) = \frac{FV}{1+rn}$$

$$1-dn = \frac{1}{1+rn}$$

$$\frac{1}{1-dn} = 1+rn \Rightarrow \frac{1}{1-dn} - 1 = rn$$

$$\Rightarrow \frac{dn}{1-dn} = rn$$

$$\Rightarrow \frac{d}{1-dn} = r \Rightarrow d \cdot \frac{1}{1-dn} = r$$

$$d(1+rn) = r \Rightarrow d = \frac{r}{1+rn}$$

$\frac{1}{1-dn} = 1+rn$
 $r = \frac{d}{1-dn}$
 $d = \frac{r}{1+rn}$

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So, we will compare discount rate to interest rate, so we can have the comparison between d and r and what we see is that we have equations like we have $C = FV \cdot 1 - dn$. So that is what we had the 2 equations for this and another equation is CV will be $FV / 1 + rn$, so these are the 2 equations which we have got for the you know for this proceeds and then we have also here for the interest rate this is for the discount.

Now what we see is that we have you know here you have the proceeds and here you have the current value. Now what we see is that $FV \cdot 1 - dn$ it will be basically the current value only, so they will be equal. So what will be happening that CV , so normally what happens that this proceeds becomes same as the current value the maturity amount. So we are equating these 2 and we get $FV \cdot 1 - dn$ will be same as $FV / 1 + rn$ that is what it is for the proceeds which is there at the present time and similarly you are the current value.

Now from this if you are dividing it you know by FV , so what we get is so you are getting $1 - dn$ you divide the whole side by FV . So $1 - dn$ will be equal to $1 / 1 + rn$ and now you can write that $1 / 1 - dn$ will be equal to $1 + rn$. So, you will further write $1 - dn - 1$ will be rn and from there we will write same as $1 - dn$ will be dn upon $1 - dn$ it will be rn . So from here we can have dn upon $1 - dn$, so that is rn or we can further have the expression as $d / 1 - dn$ that is r .

So, your n and n will be cancelling from here and we can get the expression for $d/1-dn=r$ and the interest rate is expressed in terms of the discount rate, in such cases we can also further write as $d*1/1-dn$. This is becoming as the r and now what we have earlier seen is that you know $1/1-dn$ it will be $1+r n$. So, what we see, so this is from here what we have got $1-dn$ or $1/1-dn$ is $1+rn$, so this concept is used.

So it will be $d*$ you know, so d will be, so from here what you get is $d*1+r$ and will be r , so you can get d as $r/1+rn$. So you get 2 expressions, one is calculated in terms of r and n , another is calculating r in terms of d and n . And you can calculate anything, so you what you got is once you first expression you got is the $d/1-dn$ that is r , second is d is $r/1+rn$. So from this basically you can get the values of either d or you know r . And you can compare them accordingly, so we can have some examples of suppose they suppose a bank basically discounts a note at the rate of 3% for 120 days.

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A bank discounts a note @ 3% for 120 days.
What will be equiv. interest rate?

$$r = \frac{d}{1 - dn} = \frac{0.03}{1 - 0.03 \left(\frac{120}{360} \right)} = 0.02 = 2\% \text{ int.}$$

So, that is how so what will be you know equivalent interest rate, so suppose in this case you know the d and also you know the n so in that case you can find r and r will be $d/1-dn$. So, d you know the 3%, so it is 0.3 and then $1-dn$ and then $1-dn$, so $0.03n$ and n is 120 days. So by 360, so this way you will have approximately you know 0.02 or 2% interest. Similarly you may be aware you know in some case a person may be offered something like interest rate of 9.6% on a loan and that was to be planned to pay back in 8 months.

So in that case if you have to find the you know the discount rate, so there again you can use the expression like $d=r/1+rn$, so r you know, n you know, so in that case you can find. So, all this formulas can be you know used for finding the rate of interest or discount rate and further they can be used for further calculations, thank you very much.