Financial Mathematics
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Lecture-07
Simple Discount, Focal Date and Equation of Value

Welcome to the lecture on Simple discount focal date and equation of value, so in this lecture we are going to spend our interaction which we did in the last lecture regarding the simple interest. So what we studied in the last lecture it was about the interest amount which is to be paid and what we saw that normally you have the value which is at present with you that is your principal amount or that is also known as the current value CV.

Then so current value which is used for calculating the interest and for that you need to have the rate of interest which must be known to you and also the period of time.

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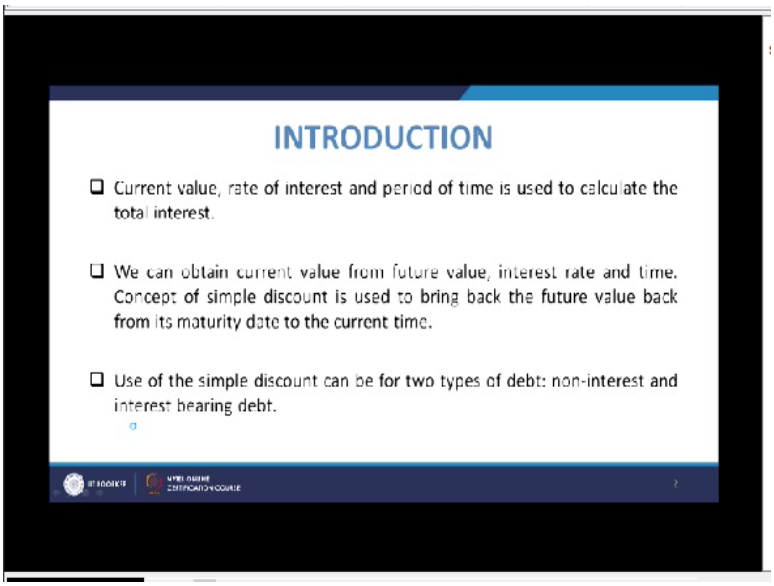

So, once you know the current value and the rate of interest and the period of time, so you can calculate this total interest which is basically you know to be accrued on. And then accordingly you can also calculate the future value, so if you can obtain this current value from future value further interest rate and time. So, once you have an expression that expression can further be you know used in many ways.

Many a times you may required to find the interest rate or you may required to find the period of time or you may required to find the current value. So based on that basically you can have these values like suppose you have to find the interest amount that is I.

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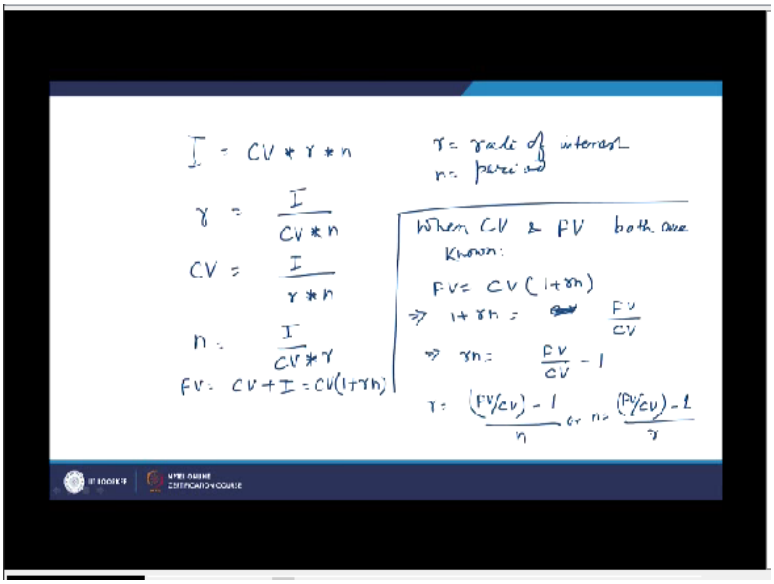

So, I that is total interest and this will be basically a current value, so this is CV we represented through the term CV. And then we have r that is your rate of interest and then you have this n that is your period of time. So that is how you have if you have any principal amount that is your current value it will grow over the time because you will have the future value that will be the current value+the interest amount which is acute. So r is rate of interest and n is basically the period, so certainly rate of interest will be specified.

And that is normally for 1 year, so if the interest is to be you know used for smaller period then accordingly you have to adjust that n. So suppose for 6 month then n will be 1/2 and if you provided interest is defined rate of interest is defined as 10% per annum. So, per annum by default we consider it as per annum, so then from this expression this expression can be further used for calculating suppose you may be required to find the rate of interest.

So, from this expression itself if you have and if you know the total interest, so in that case I upon CV that is current value*n. So, this way you can find the rate of interest similarly if you have to find the current value and you know the interest and interest rate and also the time for

which the interest is you know calculated. In that case you can find I upon r*n. So these are basically the you know different expressions which can be used for finding the you know one of the parameter.

If you know other 2 parameters, similarly what we have when we may have to find n. So, n is you can find as I upon CV*r. So, n basically is known as term of maturity, so that is you know that is you know that is business term which is used. So, n is the term of maturity and for by that you can have it these values. Now many a times what happens that your current value and future values both are known.

So when current value and future value both are known now before that we must know that once you know the period of maturity. Then you know that will be basically current value+interest, so you know that in the current value total interest will be added that will give you the future value. And since I is CV*r*n, so CV will be common and you will have 1+rn. So this expression will be mostly used when you have to find the future value.

Then you can and you know the CV then in that case you can find use this expression to find the future value. Now many a times if you know the future value and the current value and you may have to find the either r or n suppose in those cases. So, once you know that, so as you know that CV*1+rn, so from this expression FV equal to CV*1+rn. So, what you get is 1+rn will be basically FV/CV.

So, this will be FV upon CV, so rn will be the so this will be FV upon CV -1 and this will be rn. So, from this you can find either r or n. so, if you want to find r suppose it will be FV upon CV-1 upon n or if you have to find the term of maturity. In that case n will be FV upon CV-1/r, so this way when your these you know both the FV and CV is known and one of the r or n is to be calculated another is known.

In those cases you can use this formulas to find the other one like you have to know r we provided n is given or you have to find n provided r is given. So, you can use these formulas, now so you may have female you may deal with this different type of questions once you go to

the you know that lecture where we solve the problems based on these concepts. Now there is another concept that is your simple discount.

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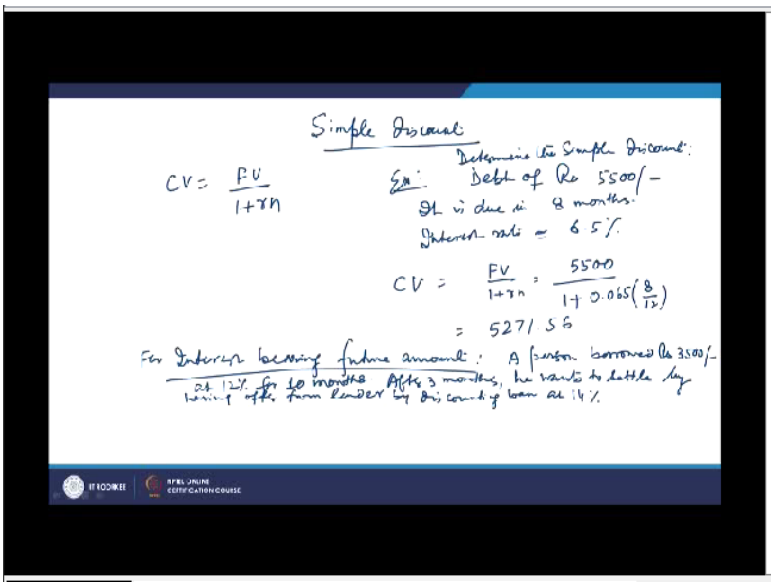

Now what we see is that basically if we are reversing this process of obtaining this future values. So, and we obtain this current value from the future value, so that is through this concept of having the simple discount. So, basically we know that FV CV*1+rn, so when so in that case your CV will be FV upon 1+rn. So, the current value that will be your and the difference basically will be known as the simple discount.

So, what happens that you know we are getting this current value using this simple discount formula and if the you know is its nothing but basically we are finding the future values may equivalent at present time. So, that way we use it and you can say for we can understand it by example and now what happen is that normally we are dealing these cases with 2 types of cases suppose it maybe non-interest debt or so that is what it is?.

Now we can obtain this current value from the future value interest rate and time and concept of simple discount is used to bring back the future value back from its maturity date to the current time. So, that is how the simple discount and the difference basically termed as the simple discount and it may be for 2 types of cases or debts one is non-interest debt and another is

interest debt. So let us take the example of some case like a non-interest you know bearing type of future amounts.

So suppose you may have one example where suppose you have a debt of rupees 5500, let us say and it is due 8 months ok. And interest rate is basically 6.5%, so unless otherwise specified it will be 6.5% per annum now in this case if you look at, so you have to find the simple discount and the you know in those cases what you have to see is that what will be it is you know simple discount. Now if you try to find this simple discount what we see is that the current value if you find from here what will be there.

So what you see is that current value is future value/1+rn, so that is how you have to find. So, basically debt of rupees 55000 is there, so that becomes the future value. And so you will get FV upon 1+rn and so you basically you have to find the determine the simple discount. Now for such case you have to find an FV is known time is non-interest rate is known so you look at that it will be 5500/1+r is basically 6.5%.

So, it will be 0.065 and it is 8 months so 8/12 years, so you will have 8/12, so this comes out due to the calculation it will be coming to be 5271.56. So what you see is that you can have the discount of the amount that will be 5500–5271, so this discount which you are getting that basically is known as the simple discount. Now this is for a case when you have the this is non-interest bearing you now case.

Now if you talk about one interest bearing future amount when you have, so for interest bearing future amount now what happens that there may be a situation suppose now in that case someone has borrowed, so a person borrowed rupees 3500 suppose, now he has borrowed at 12% you know for 10 months. Now what happened is that after 3 months now he has to go you know at elsewhere and he wants to return.

So, after 3 months he wants to settle by so basically by having offer from lender now that lender offers him a discount of basically discounting the loan by discounting loan at 14%. Now in this case basically what will be how much she has to pay, so how much she has to pay, now this is

suppose the case where the after 3 month only we had taken the loan you know how much he or she has to pay.

Now what is the problem is that he had taken the loan for 10 you know months and after 3 month only he wants to return it. So basically the lender has given him further he has discounted the loan at 14%, so that we what he has to pay. So for such case what you have to do is that now if he has to suppose he has gone for 10 months, so in that case what will be its future value.

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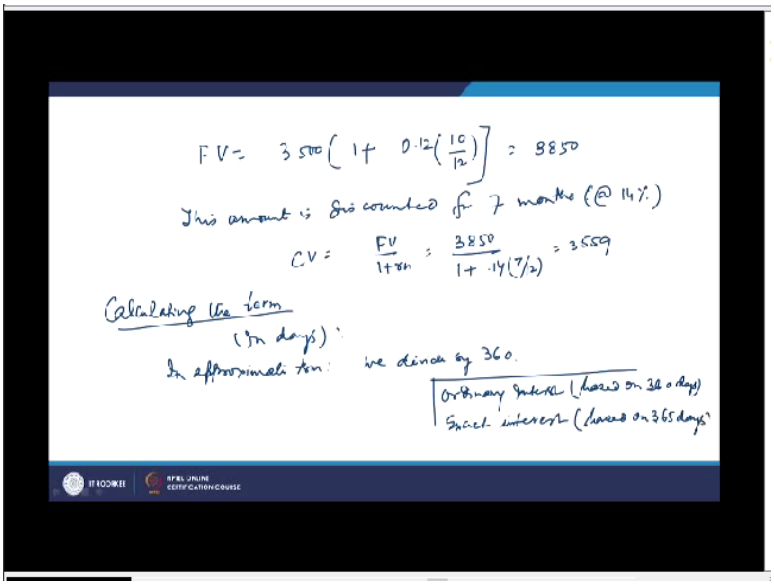

so what will be happening future value will be CV*1+rn, so CV is 3500 and1+rn, so r is you know it is given as 12%. So, it will be 0.12 and n will be 10 months, so 10/12. So, **so** that becomes like 3850, now the thing is that for 6, 7 months that loan is discounted at the rate of 14%. So this amount is discounted for 7 months, so for 7 months you have to find its current value you know at the end of 3 month or 7 month you know period.

So what will you find is for that basically you have to have the CV as V/1+rn, so this will be 3850/1+0.14 and it is at the rate of interest of 14%. So, 0.14* you know 7 months, so 7/12, so if you calculate this amount it will be 3559. So the this person has to pay only 3559 rupees in that case if you look at other way he might have to pay something different. But since he discounted and how we consider that one.

So, that is you know clear from such cases, so this way we can calculate this amounts for the different cases. Now we will we talking about how to calculate the term in days, so you know that if you are given a period is specified like suppose for 3 months or 10 months. So you can have that NH3/12 or 10/12 or so but many times for the business calculations you there you have to calculate that you know term in days.

So, in my 1 month for 15th of September it is given and then you have to calculate maybe on the next year 10th of August or so. So accordingly you have to calculate those dates and normally you take the difference and the thing is that normally what we do is we have we start and we so by taking the difference. And then divided by the number of days in the year, so once you calculate the number of days.

In that case you have you know to divided by 365, so the convention also is that you are excluding that the day first day. So, if you are going from 15th you suppose 10th of September to 15th of September, so in that case 10th is excluded. So, 11, 12, 13, 14, 15 5 days, so basically you take the difference of 15 and 10 and is 5, so if you look at in that way there are 6 days but the first day is excluded.

So, that way it becomes the you know number of days and then you have divided by the number of you know days in the years. So, that will give you know the calculating the term in days ok, so in days you can do it. Now once you find the days then you can divide it by the 365 or so. So, you can divide by 365 or 366 also in the leap years, so that way you have to take , in normal case we do it by divided by 360.

So that is how we do it and in approximate terms, so in approximate terms we divide by 360. So you assume that there are you know 12 months and every month is of 30 days, so you take it that and then you calculate that you know the amount of interest based on that approximate term days. So that is how you do it now what happens that you have ordinary interest as well as you have exact interest or so ordinary interest will be based on 360 days.

And exact interest will be calculated based on 365 days, so that is how you will have you will be getting some difference suppose any amount is there. So, if you have 19 days, so for ordinary interest 90/360 1/4 will be n however and forever for exact interest will be 90/365 it is not a leap year case. So, in cases you will have 365, so basically you can compare these ordinary and exact interest rates, in such cases.

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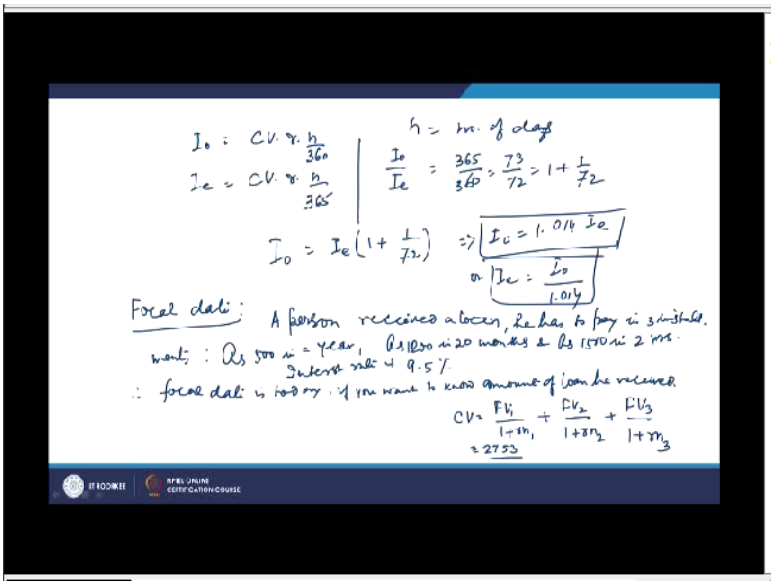

So, what you do you find is that with ordinary interest rate you get as CV*r *n. so, if you know the number of days, so n is will be 360, so n is number of days. And if you look at the exact interest it will be CV*r*n upon 365, so basically there will be you know difference in both the cases and what you see is that from this you can have the you know relation between these 2 terms. So, if you find I0/Ie, at ordinary interest by the you know exact interest.

So, it will be 365/360, so that is how you can further divided by 75, so it will be 73/72. So, it will be 1+1/72, so what we see is you get I o will be Ie*1+1/72. So, that's how you get these values or you will also get Ios 1.014 Ie, now so you can even get the expression for Ie suppose. So, Ie will be I0/1.or Ie will be I0/1.014, so this is how you can have the expression for ordinary interest as well as the exact interest values.

Now what we intent to see the another term that is known as the focal date, now what happen is that money **is** value is fluctuating with time. So, we need to know something about the focal date,

so we have studied about the calculating the term in days exact time and approximate time. Now the focal date is the date at which various funds are to be chosen or to be chosen to be evaluated. So basically what we see that many a times many payments are due at different points of time.

And we know that the money has time value, so with that time it will have different values. Now when you have to compare or when you have to find a value at a particular time. So, that is basically more convenient suppose somebody has for someone some amount is due after 2 months some another amount is due after 4 months. And some third amount is due after 6 months now how much he has due on in the last day.

So, the amount maybe you know a1, a2 and a3, so it will not be a a1+a2+a3 because they are being transacted at 3 different times. So, basically you have to find the value of all these at one particular time and that time or that date basically is known as focal date. So, how to have what is the significance of this focal date and that can be understood by dealing with certain examples suppose a person he has received a loan.

And a person received a loan and he has to pay in 3 installments suppose those installments are like Rs.500 in the year. Then Rs. 2000 now 1200 in 20 months and Rs.1500 in 2 years, so you know what would be the amount of loan received if the you know annual interest simple interest rate is 9 and half percent interest rate it is 9.5%.

Now the thing is that if you want to know that how much basically was the loan because of which he has to pay Rs.500 in the year end Rs.1200 after 22 months in 22 to 20 months and Rs.1500 in 2 years. So, basically what you need to do is you have to find its equivalent values you have to find you have to map all these values to a particular day. And if you want to know the loan amount how much you are taken basically you want to know.

Now, so basically your focal date is today so focal date is today, basically if you know what amount of loan had received if you want to know amount of loan he received. So, that is how so focal means on that particular time you have to find the you know equivalent value of all those

payments which has been made. Now if you look at that in that case the current value basically will be current value component of Rs.500.

So, this will be first future value, so FV1/1+rn1 and money is so V1 is 50 n1 will be 1 year. Similarly FV2 will be 1200 and 1+rn2, n2 will be you know the second installment at which it was paid. And that is given as 20 months, so it will be 20/12 years and similarly every 3/1+rn, so every 3 will be 1500 and n3 will be basically it is 2 years, so 2 n will be 2 n3 will be 2. So, if you find this value, so 1+r will be.0 095 and then if you calculate all these values what you will get if you will get something values close to 2753.

So, what do you find that by finding this focal date you know you can you have decide what should be the focal date. So, based that you will have to convert either it may be the future value or it may be the current value or so. So, you have to compute the you know respective amounts and then you have to calculate it and tell and then you can summit up. Because it is happening at a particular time.

So, you can know very well summit up and find what will be the value ultimately what is the amount of loan he had to pay. Now also we must know something like, so what we saw is that none of focal that can be changed also you can have a focal date in the future also. And in that case all the transactions which has been done the it is future component, so in that case you will have will be knowing current value.

So, you will be multiplying with 1+rn and there are many transactions that all transactions component will be you know calculated and then they will be added like that it will be done. So, basically when you are obtaining one payment at the focal day for 2 obligations basically if you have 2 obligations and I am sure if you have turning that one you know payment at the focal day.

Then and basically they are also due for different you know dates then this method by which you are obtaining that is known as the equation of value. So, basically what you do is suppose at this point you have to find, so if you are talking about focal date particular time it is for the you know current value and then you have a future value. So, if you are adding finding the current value

and the future value , so in this case if you are doing at so if any in this case we are always calculating the current values.

But we may deal with situation where you have the transaction at a particular point another transaction is your its component is to be calculated. So, in that case what you do is you calculate the you know FV+CV and you add them and that way you get x=FV+CV which is known as the you know x discount equation of value. So, what we are doing basically in the equation of value is we are basically we equate the unknown single payment.

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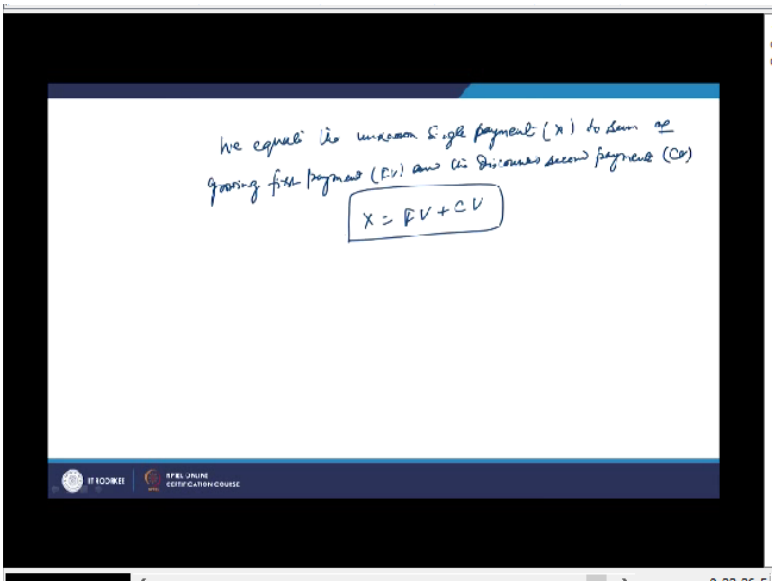

When you have 2 obligations, so we are basically equating unknown single payment you know that is x to sum of growing first payment that is you know, so that will be FV. And then you will have the and the discounted second payment that is your CV. So, in that case we are having the expression like x equal to FV+CV and this is known as the equation of value. And this is what is used, so this is the mathematical equation.

So, this is the mathematical equation which will be expressing the equivalence of these you know amounts are original obligations which is due on specific dates and the interest rate is given. And this equation is known as the equation of values which will be used in our when we solve we can also see when we solve the problems based on these expressions, thank you very much.