Financial Mathematics
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Lecture – 60
Reserve Funds, Property and Casualty Insurance

Welcome to the lecture on reserve funds and property and casualty insurance. So you know we talked about the life insurance policies in the lecture and we will talk about the reserve fund and the terminal reserve fund basically.

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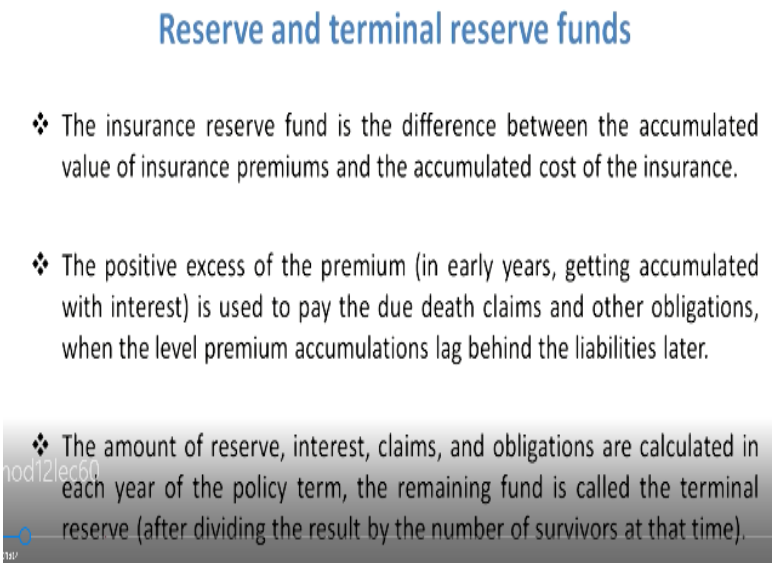

So the reserve fund in the insurance that is insurance reserve fund it is the difference between the accumulated value of insurance premium and the accumulated cost of the insurance. So this basically this is what you have the commodity the value of the insurance premium and also the cost of insurance because you are giving the you know you have to give the penalty you have to pay the benefits.

So you are basically it is difference will be the reserve fund now so what do we have seen earlier the when we talked about the level premium you know calculation. So in that case now what we saw that you have you are paying initially the excess of the premium and basically that access which you are paying now that you know gets accumulated and also you are the earning the interest on that.

So you are basically increasing the you know you know the amount as the time increases but in the later years certainly the necessary premium amount is larger. So in that case that will be you know taking into account you know the lags which is there in the later years and also to pay the due death claims and also other obligations when the level premium accumulations lag behind basically the liabilities.

So in the later years basically your or whenever you would have to give the liabilities you have to pay the liabilities that time basically it will be used to pay that. Now the amount of reserve interest claims and obligations all of these things so actually they are you know calculated in every year and that of the policy term and the remaining fund which we calculate so that is known as the reserve fund.

And when we had a divided it using by the number of survivors then we call it as the you know terminal reserve funds. So for getting the terminal reserve fund you know that we have to divide the reserve fund./the number of survivors. So when we talk about the terminal reserve fund so in that basically you know it will be talking about the maximum amount the policy holder will be getting.

So for suppose if he goes for the discontinuation of the policy at the years end. Or if he wants to have a loan how much loan he will be getting so that also will be depending upon the terminal reserve fund. So that is how it is calculated so we will see that the how these you know so there are basically 2 ways by which these terminal reserve funds are calculated. 1 is a retrospective method and another is the perspective method so prospective methods.

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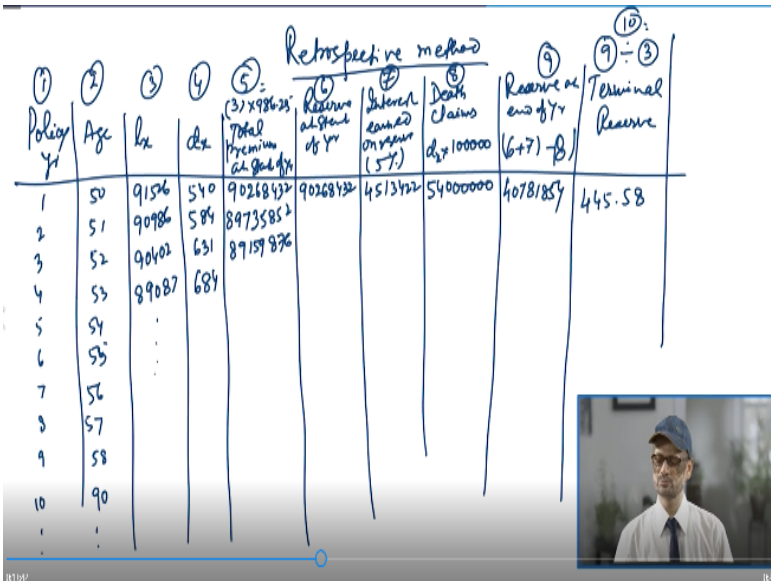

So you know when we talk about the Retrospective method now that can be understood by referring to the table. In that basically in every year you are going to calculate the you know you have to calculate these reserve funds. So how they do it basically that we can see now if you if suppose if 15 year term policies of 1 lakh is you know sued to a 50 year old person. So in that case how you are going to calculate you know these terminal reserve funds.

So for that basically you will have the you know policy year so policy year can be kept from to 1 to 15 so that we know that value will be changing so this will be it will be going like 1 2 3 4 5 6 7 8 9 10 like that it will go. Then next we will be having the age so age we have we talked about the 1 lakh policy for a 50 year old you know person. He age is 50 so we will just start from 50 51 52 53 54 55 and 56 57 like that it will go.

So it will be going for the 15 years then what you do is you have to find the lx so lx you will be you can get it from the you know the mortality table and you know that how these mortality tables are seen.

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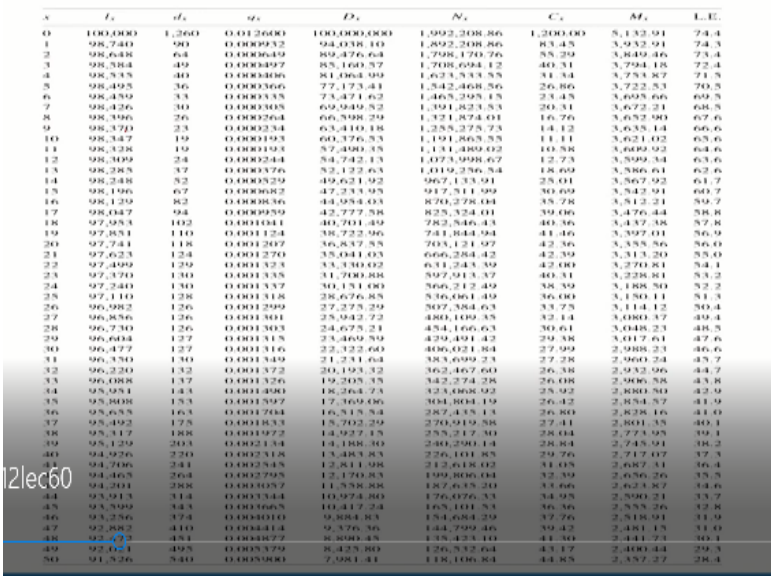

x	l_x	d_x	q_x	D_x	N_x	C_x	M_x	L.E.
0	100,000	1,260	0.012600	100,000,000	1,992,208.86	1,200.00	5,132.91	74.4
1	98,740	90	0.000932	94,038.10	1,892,208.86	83.45	3,932.91	74.3
2	98,648	64	0.000649	89,476.64	1,798,170.76	55.29	3,849.46	73.4
3	98,584	49	0.000497	85,160.57	1,708,694.12	40.31	3,794.18	72.4
4	98,535	40	0.000406	81,064.99	1,623,533.55	31.34	3,753.87	71.5
5	98,495	36	0.000366	77,173.41	1,542,468.56	26.86	3,722.53	70.5
6	98,459	33	0.000335	73,471.62	1,465,295.15	23.45	3,695.66	69.5
7	98,426	30	0.000305	69,949.52	1,391,823.53	20.31	3,672.21	68.5
8	98,396	26	0.000264	66,598.29	1,321,874.01	16.76	3,652.90	67.6
9	98,370	23	0.000234	63,410.18	1,255,275.73	14.12	3,635.14	66.6
10	98,347	19	0.000193	60,376.53	1,191,865.55	11.11	3,621.02	65.6
11	98,328	19	0.000193	57,490.35	1,131,489.02	10.58	3,609.92	64.6
12	98,309	24	0.000244	54,742.13	1,073,998.67	12.73	3,599.34	63.6
13	98,285	37	0.000376	52,123.63	1,019,256.54	18.69	3,586.61	62.6
14	98,248	52	0.000529	49,621.92	967,133.91	25.01	3,567.92	61.7
15	98,196	67	0.000682	47,233.95	917,511.99	30.69	3,542.91	60.7
16	98,129	82	0.000836	44,954.03	870,278.04	35.78	3,512.21	59.7
17	98,047	94	0.000959	42,777.58	825,324.01	39.06	3,476.44	58.8
18	97,953	102	0.001041	40,701.49	782,546.43	40.36	3,437.38	57.8
19	97,851	110	0.001124	38,722.96	741,844.94	41.46	3,397.01	56.9
20	97,741	118	0.001207	36,837.35	703,121.97	42.36	3,355.56	56.0
21	97,623	124	0.001270	35,041.03	666,284.42	42.39	3,313.20	55.0
22	97,499	129	0.001323	33,330.02	631,243.39	42.00	3,270.81	54.1
23	97,370	130	0.001335	31,700.88	597,913.37	40.31	3,228.81	53.2
24	97,240	130	0.001337	30,151.00	566,212.49	38.39	3,188.50	52.2
25	97,110	128	0.001318	28,676.85	536,061.49	36.00	3,150.11	51.3
26	96,982	126	0.001299	27,275.29	507,384.63	33.75	3,114.12	50.4
27	96,856	126	0.001301	25,942.72	480,109.35	32.14	3,080.37	49.4
28	96,730	126	0.001303	24,675.21	454,166.63	30.61	3,048.23	48.5
29	96,604	127	0.001315	23,469.59	429,491.42	29.38	3,017.61	47.6
30	96,477	127	0.001316	22,322.60	406,021.84	27.99	2,988.23	46.6
31	96,350	130	0.001349	21,231.64	383,699.23	27.28	2,960.24	45.7
32	96,220	132	0.001372	20,193.32	362,467.60	26.38	2,932.96	44.7
33	96,088	137	0.001426	19,205.35	342,274.28	26.08	2,906.58	43.8
34	95,951	143	0.001490	18,264.73	323,068.92	25.92	2,880.50	42.9
35	95,808	153	0.001597	17,369.06	304,804.19	26.42	2,854.57	41.9
36	95,655	163	0.001704	16,515.54	287,435.13	26.80	2,828.16	41.0
37	95,492	175	0.001833	15,702.29	270,919.58	27.41	2,801.35	40.1
38	95,317	188	0.001972	14,927.15	255,217.30	28.04	2,773.95	39.1
39	95,129	203	0.002134	14,188.30	240,290.14	28.84	2,745.91	38.2
40	94,926	220	0.002318	13,483.83	226,101.85	29.76	2,717.07	37.3
41	94,706	241	0.002545	12,811.98	212,618.02	31.05	2,687.31	36.4
42	94,465	264	0.002795	12,170.83	199,806.04	32.39	2,656.26	35.5
43	94,201	288	0.003057	11,558.88	187,635.20	33.66	2,623.87	34.6
44	93,913	314	0.003344	10,974.80	176,076.33	34.95	2,590.21	33.7
45	93,599	343	0.003665	10,417.24	165,101.53	36.36	2,555.26	32.8
46	93,256	374	0.004010	9,884.83	154,684.29	37.76	2,518.91	31.9
47	92,882	410	0.004414	9,376.36	144,799.46	39.42	2,481.15	31.0
48	92,472	451	0.004877	8,890.45	135,423.10	41.30	2,441.73	30.1
49	92,021	495	0.005379	8,425.80	126,532.64	43.17	2,400.44	29.3
50	91,526	540	0.005900	7,981.41	118,106.84	44.85	2,357.27	28.4

So if you see the mortality table so in in 50 years this lx is you know this is 91526 and then from 51 onwards it is 90986 90402 so that way these values will be coming in the place of lx so it will be 91526 90986 we have 90402 89087 and so on. So this way you have you can use it from the you know table and then after you get these lx from the table you will be also getting the value of dx from the table.

So that is also 540 584 631 then 684 and so that way that dx will also be you know changing so its values are in this column. Now so total premium suppose the total premium which is calculated at the start of year okay so you know you know that you are going so we have seen that when we calculated for the level premium for the insurance then for level premium we know that we had calculated this to be 986 you know 0.25.

So the total premium at start of the year will be the lx multiplied by the premium amount. So it will be basically so if we take it as this is as 1 this as 2 this as 3 this as 4 and this as 5. So 5 will be nothing but lx and multiplied by 986.25 so it will be 3 so it will be =3*986.25 986.25 we have calculated it was the level premium which we calculated. So because level premium is the fixed amount which is represented by the straight line horizontal straight line.

And your natural premium will be going initially it is a below and then later on it as the you know age increases it will be going on increasing. So that 986.25 will be multiplied so it will be

our neutral premium at the start of the year so your resolve at the start of year will be same. So if you multiply this lx with 986.25 in that case you get 90268432 say you are getting so if we multiply this with 986.25 you are going to get something like this.

So similarly you are further going to multiply 986 multiplied by 986.25 so that way you will be having 89735852 similarly 89159876 like that so it will go. Then if you want to find so reserve at the start of the year will be 90268 and 432no before that and after that once you have the reserve of the start of the year then you will be finding the interest earned. So on reserve so that is interest earned on reserve we are taking out 5% .

So what do we do this a so it will be 6 so once you have a reserve on that you will be having the results? So it will be nothing but 0.05 multiplied by this quantity whatever you have reserve so this interest will be there and once you earn the interest then next is the death claims. Now this will be depending upon the number of deaths and you are giving so a face value of the policy is 1 lakh so you are paying 1 lakh.

So now for that you are multiplying this dx multiplied by 100. So that will be you know dx* not 100 it is 100000, so that will be your death claim and once you have the death claim then now this is your reserve fund and on that you accrued the interest of 5% of this so it will be coming as 4 5 1 3 4 2 2 and death insurance will be 540 on that basically you have 1 lakh so it will be you know this much so that will be the death claim.

So now reserve at the end of the year will be your total reserve that is your reserve was there at the year beginning + interest accrued interest earned on that fund so that is your total reserve fund and then you are subtracting the death claims. So it will be basically so your reserve was 6 and interest earned is 7 so totally 6+7 and claim you are giving as 8 so 6 + 7 - 8 so this will be you know the reserve at the end of the year then it is it a final reserve.

So if we look at if you have to add these 2 and then you have to subtract you know this amount in from this so that will be coming as 40781854 and then if you are further you know finding the terminal reserve absolutely would try to find the terminal is of no for as the finding the terminal

reserve so this is 9 not terminal reserve always basically the reserve which you have at the end of the year divided by the number of policy holders.

So it will be 9 /3 so this is basically 10 and 10 it will be equal to 9/3 so if you divided a you are going to get 445.58. So basically you are using you know you are going to calculate these terminal reserve fund. So you can feel you know once you know these values you can feel you can use the more you know you can you use the xl sheet and you can calculate these values you know for the terminal reserve at any year or so.

This method of retrospective method so basically you are in this case you are you know calculations are made at the end of each year of the policy and that is you are going you know by looking backward to the beginning of the year so you are looking backwards so that is why it is known as the retrospective method. Another method which is you know used is the you know a prospective method where you are looking forward.

Rather than looking backward you are looking forward and in that you are going to look for the future you know benefits and future liabilities of the policy and based on that you know you are going to calculate these reserve funds so that is known as the prospective methods. Now we are going to talk about the you know how much insurance you should buy. So what should be you know how much insurance you should aim of.

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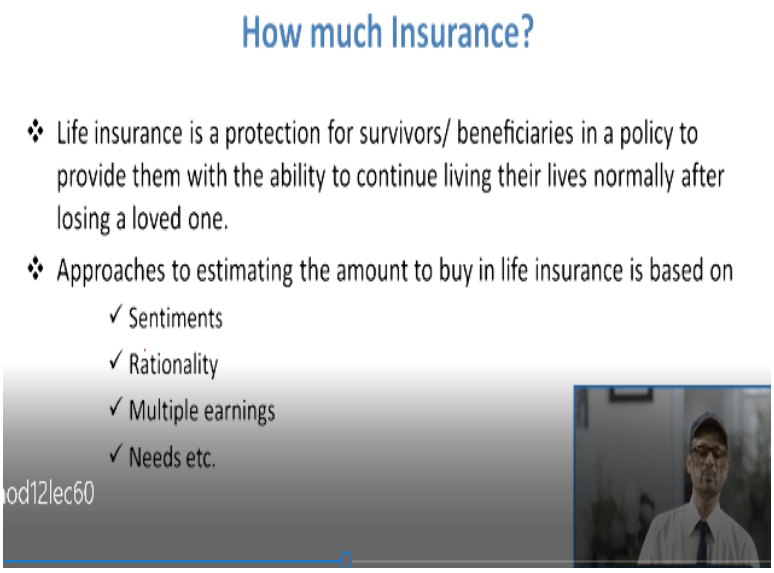

You know buying so what do we know by life insurance so it is a protection for the survivors or beneficiaries in the policy to provide them with the ability to continue living their lives normally after losing a loved one. So that is what the meaning of life insurance is here you are thinking that you are going to you know provide you know your survivors to be to have a protected life this should have a definite standard of life if the you know person is not here.

Because of so any reason and in that case this will not suffer much so basically you are you know survivors or beneficiaries for them for their protection and for their livelihood you normally you know go for insurance. Now how much insurance you should you know provide that is very you know that is the question which is always there in the mind of the you know investor that how much you should buy many a times.

So they have approached which is used for the for estimating the amount to buy in life insurance. So it will be depending basically on the so first is the sentiment so this is a highly subjective approach and you know it will be depending upon the emotional you know emotional values how much you know he places on his survivors how much emotionally he is attached so based on that you know based on I mean respective of the real needs.

Basically normally he will always be thinking you know more than the real needs of the family and so he will be talking about you know he will be using his sentiments only to help his

survivor or the family. Next is the rationality so there is a rational approach you know he will have you know a certain criteria based on which he is going to decide that what should be the amount for which he should go for the investment.

So in that case you know he will be talking about the income insurance income and then he will be talking that how much if he is not there then how what will be the requirement how much you know what will be the needs of the survivors. So based on that he will be talking about he will be calculating these you know amounts. So basically under that you have again you have methods to approach.

And the two methods which is the one is the multiple methods so that is a multiple learning methods so that is basically a part of the you know rational approach so in that what we do is we take a certain multiple of you know so basically it will be depending upon the insured income and you know as a basis of the calculation. So he will say that you know it should be 5 or 10 times the final amount.

You know which will make the insurance so that will make the life of these you know his dependence easier. So based on that he will be calculating the you know insurance amount so the amount for which he should get the insurance. So it will also be depending upon how much you know premium is there so and also if suppose for example suppose at 50 year a person is there and he wants to have a 25 year term life insurance.

So in that case and he will use the 5 times the annual income of 75000 so he will go for a 5 times 75000 that is 375000 now his premium so if you look at so that will be 3750000/100000 that is 3.75 and if you look at the table so by referring to the table you can find the you know for the 1 lakh you know the annual premium is coming out to be 762. So multiplying by the 3 times or 3.75 times there will be 1 premium amount calculated that will be something like 2857.

So he should take a premium of 2800 something like that and based on that he should take that much amount of insurance for which for his you know he should get the insurance or he should get insured for the survivors. Next is the calculation of the needs so he will be talking upon he

will be discussing about what will be basically the need of now you know the person after his death.

So he will be taking into the main needs like a child's insurance or you know the family debt and all that will be taken of and based on that he will be calculating that for what amount he should do the insurance so for that normally he will have you know the calculus and the life insurance amount calculation and that amount of calculus and will be based on you know certain factors and that will calculated by a formula.

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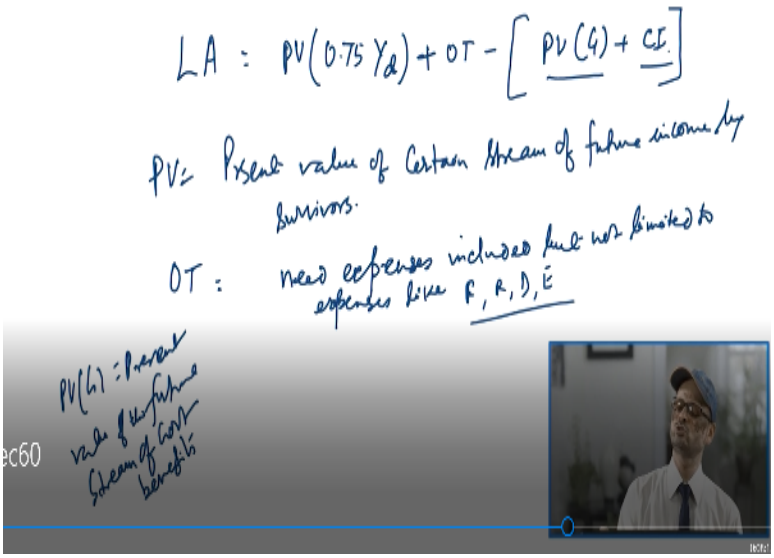

That will be you know amount of life insurance that is LA that will be PV*0.75Yd and the you know +OT and -you know PV*G +CI. So that way he is going to calculate these life insurance amount now this is something PV is the present value of so PV will be the present value of certain stream of future income by survivors. So what is necessary is that they will need at least a 75% that you know of the insurance disposable income that is Yd in his working years.

And then it is basically assumed that 25% you know will be the share of the family incomes that is right 25% is basically not taking into and the 75% is taken then the OT. OT is basically the vector of needed expenses included but not limited to expenses like. So there are different type of expenses like you know there are many expenses after the death of the person like you have the funeral expenses so in in that you can have that is categorised as a F R D and E.

F is normally for the funeral and burial you know of the diseased then you know you have the readjustment expenses that is the R then D will be the death expenses what he has so that is to be paid and he is basically the educational expenses. So these are the different types of expenses so that he needs to have that is why it is added so 75% is 25% is said to be something like that the family will manage so 75% comes here then these are the need expenses after his death.

And then it will be subtracted so this amount you know PVG+CI so this is the PVG is basically present value of the benefits which you will be getting. So the government benefits there are many benefits it will be getting because of the insurance he has you know taken you know social schemes are there he has taken. So for that these you know so the PVG will be the present value of the government.

You know present value of the future the stream of government benefits well as schemes and so. So that will be added and also you know CI is the current insurance policies because in that you know case he will be getting certain amount. So certainly that will be also be a part which will be subtracted from what is the need. So once you get that then you add these 2 and then add these 2 and subtract.

So whatever amount comes basically that is a genuinely an amount for which you should think of being insured. So that is how you know you calculate the amount for which you should go for the insurance. Now we are going to go discuss about the property and casualty insurance so as we know that apart from the personal insurance you have also for property and casualty insurance policies which are covering bought the property and the liability risks.

By you know the reimbursing the policyholder the financial losses due to damage or destruction or loss of property owned or controlled by the policy holder. So in that case it also reimburses the property damage and bodily injuries maintained by others but for which the policy holder is responsible so that is known as the liability insurance. Now what we do normally when we talk about you know the reimbursement.

So there are basically some principle based on which you know these reimbursements are made and the first thing which we talk in such cases is that the financial losses which are going on which have to so they have to be result you know of pure versus speculative type of risk. So you know you will not be having you know you will not be ensuring the insurance you know where you have also some benefits expectation.

Like in the case of gambling suppose you can lose but you can also gain so that cannot be ensured you know when you have the potential for both you know risk and loss. So when there is only potential for the loss so I know potential for gains so that is known as the you know pure you know so that is pure risk basically and a speculative risk is that you know where you have a potential for both my gain and loss. So there is no chance of having that you know insurance then also the second principle is that.

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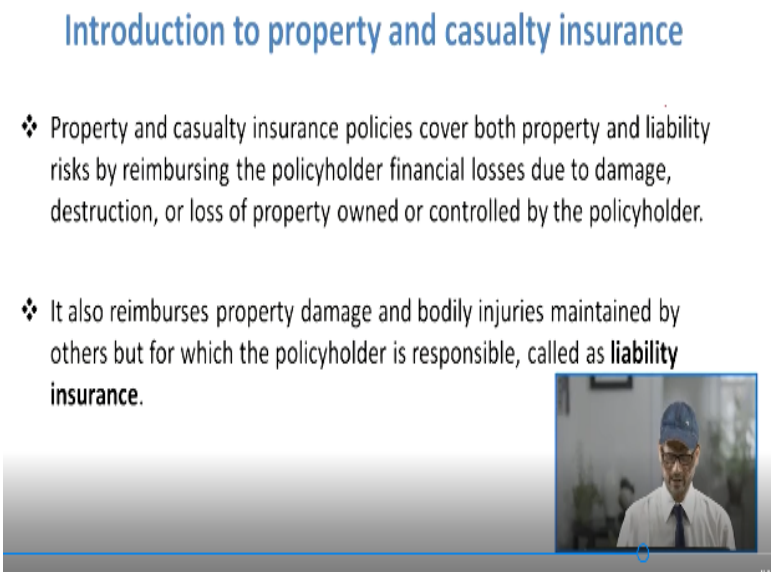

The insurance will not pay more than the actual financial loss. So this is known as the principal of indemnity so it will look at the actual financial loss and then only it will be paying. So the challenge is that how to assess basically what is the actual financial loss. So what we do is the insurance companies normally find the actual cash value you know so as opposed to the original cost.

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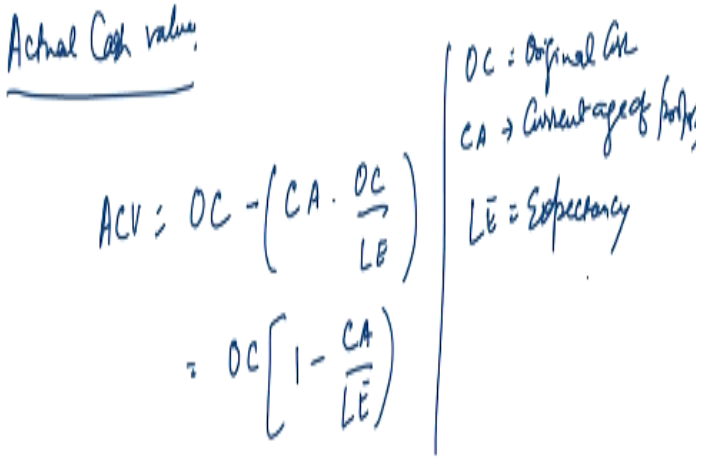

So they will be finding the actual cash value of the property. So you know in that what is done is it will be considering the depreciation of the property due to (()) (29:12) and market conditions and it will be so it will be calculated by taking the depreciation in fact away from the original cost and that depreciation effect also it will be depending upon the current age of the properties that is CA and its also life expectancy what is the life expectancy of the product.

So actual cost to value if you see that is defined as the OC that is the original cost - CA that is your current age of the property and that is multiplied by OC/LE. So LE is the life expectancy so if you take OC out it will be 1-CA/LE so if you try to define these terms ACV is the actual cash value OC is the original cost then CA is the current is of property. And LE is the life expectancy so once you have these data then you can find the actual cost value.

And so if suppose you purchased an item you know a few years ago for certain amount and the life expectancy suppose is you know 9 years so you have seen that and a so in that if you want to have that how much there should be a reimbursement to the policy holder. Then in that case the value is supposed 1500 and you know current age is you know 4 so you have purchased 4 years ago so current age is 4 .

Life expectancy is suppose 9 so you will be multiplying 1500 with 1 – 4/9 that is 5/9 so you will be getting certain amount and that will be the actual cash value of the you know product. Now

we also calculate basically the replacement value and the replacement value is basically the you know original cost and the change in the original cost and the original cost will be there and it is change in there.

Because of the manufacturers like inflation or the based on that you can also calculate the replacement values. So depending upon the inflation rate you can calculate the replacement values also which will be used for the calculation of the insurance amounts. So accordingly you have you know calculation of these parameters are there with the different formulas you can use and you can calculate these property insurances in such cases.

And normally you have also another type of insurance which is a are in the case of the health insurance and there also you have the reimbursement and for that you know there are many things other things also required like reinsurance percentage and then you have the financial losses so and the deductibles which are also the concept which are used for finding you know the reimbursement. Health insurance reimbursement that is required in the case of health insurance.

So basically it is parameters we must to be acquainted with so to calculate you know the premiums or the new benefits in the case of the insurances. Thank you very much.