Financial Mathematics
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Lecture-06
Interest and Interest rate, Time Value of Money

Welcome to the lecture on interest and interest rate and time value of money. So, we will discuss in this lecture about the interest which is chased by the financial institutions by the investors you know this transaction of money whenever occurs. Because the money has value which is function of time, so as you can all say, so we will discuss the time of money later. And because of this time value of money the interest you know comes into picture. So, what happens that you know what is interest, so if you look at the definition of interest.

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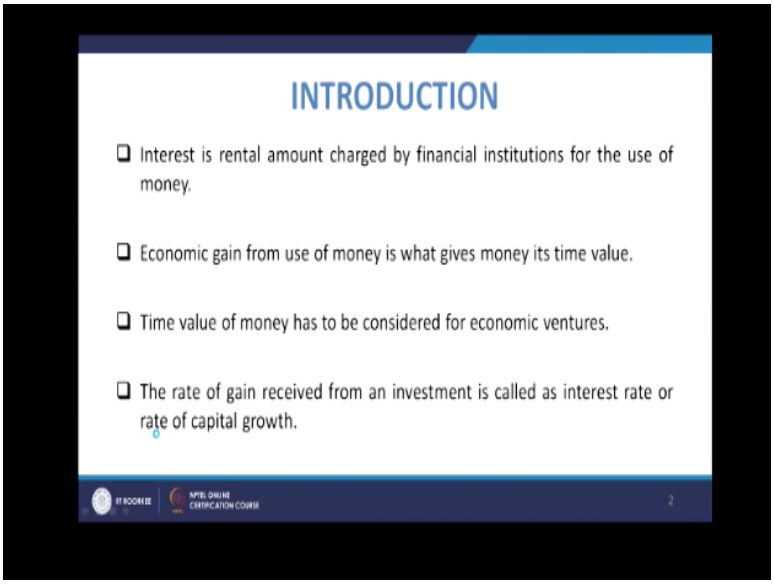

Interest is the rental amount charged by financial institutions for the use of money. Now the money which is used for satisfying the you know your wishes it gives you happiness, you are able to purchase something. So you know it is able to satisfy your wishes you are getting satisfaction and you are able to get those you know assets which has the utility for you. So whenever you use that if you have that much you can purchase it, but you have not then you can lend it to someone.

And they can have their you know wishes being completed, their needs can be fulfilled, so but for that the person who is lending. He is charging a rental and that amount which is charged basically is known as the interest, now economy gain from use of money is what gives money its time value. So as you know that you know with time, so whatever economy gain you get from the money in times, so you have got that much advantage.

Because of that you know money, so that is why if we have money at that at now its value will be more at a later time. So, that way these value of money at a particular time, so time value of money is coming important. Now time value of money has to be considered for economic ventures. You have to consider it many a times we also compromise many a things you have you know you have things of uncertainty also there many a times.

And also you can see that you have inflation, inflation is also there which basically will be making it you know devalued. So, whatever many you have today you cannot purchase the same thing after 1 year. Because its value is decreasing, so that way the time has rolling it and also the uncertainty. So, there are few things which basically are you know e into it. So, if you we will discuss on these things.

Now rate of gain received from investment is called the interest rate or rate of capital growth. So, at what rate you are basically having that the receiving, so that percentage is basically defined and that is known as the interest rate.

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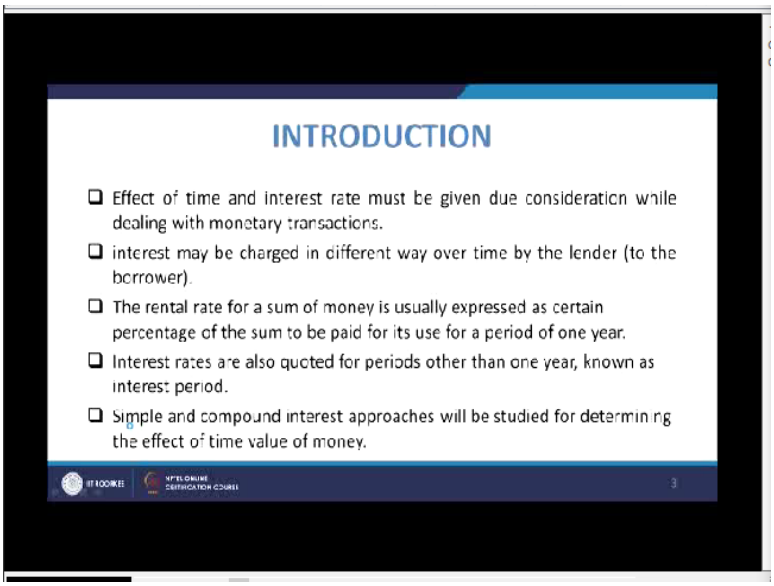

Now if you see that effect of time and interest rate must be given due consideration while dealing with monetary transactions. So when you are dealing with the monetary transactions you have to see that your time and the interest rate both must be given the due consideration. And effect interest may be charge different way what time by the lender to the burrower. So, if you look at the history of the use of interest.

So in the earlier days the interest was interest might be in the form of other units it maybe in the form of grains it maybe in the form of the land or so but in today's world in the since the transactions are mostly done in the monetary ways. So, you have the things are basically things have developed you know earlier there were transactions in other way now we have we are doing mostly the transactions in the term of money.

Now as we discuss that the rental rate for sum of money is usually expressed as certain percentage of the sum to be paid for it is used for a period of 1 year. And that is how the rate when you defined for a particular period most conventionally we defined for 1 year. And then that is why it is we define it as interest rate per year, so suppose you are lending someone 1 lakh rupees and you want to see that you get 10%.

So, it 10% interest, so its ultimately what will happen that at the end of the year is value will be 100000, so 100000 what you receive that is the interest and the 10% is known as the you know

interest rate. So, that is the rate at which you are you know getting the you know amount because of its use during that period. Now you know there are many ways by which these interests are computed and you have simple interest and the compound interest.

So as you know that when we talk about the you know calculation of interest or simple interest. So in that the calculation is that whatever is the you know present amount or the principal amount the interest earn will be for a particular time. So, it will be you know for based on the rates, so if the rate is specified on year basis. So for first year if the rate is suppose 5%, so 5% of any amount that will be the interest for 1 year.

So, in the case of you know a simple you know interest the interest rate will be there define and it will be always the percentage of the principal amount. And for a particular periods for 1 year for suppose 1 and half year you have to again multiply with 1.5, so that way in terms of that you calculate the simple interest, now in the case of compound interest what is supposed, now in this case the interest remains fixed however we also have the case and their has been and.

If this case is very in much in normal practice that the interest rate which is accrued after a certain time that becomes a part of the principal amount. And then in the next period your interest rate is charged on the principal amount+interest rate accrued interest amount accrued. So, this way there is compounding done, so you are basically doing the interest of the interest amount also. So, you are not every time doing the interest of the principal amount also. So, basically what is happening that in the so if we happy as here and so in the case of simple interest.

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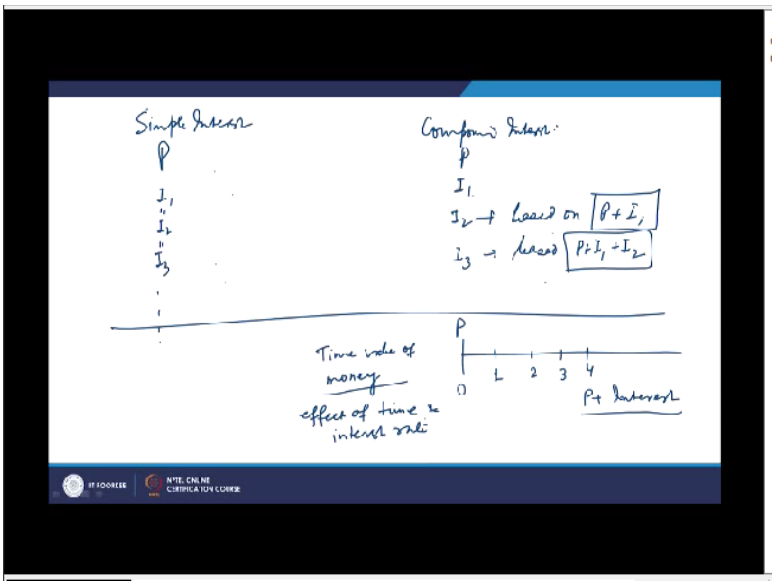

So, in the simple interest you will have P and you have years going on and interest amount will be you know calculated every year I1, I2 and I3 and so. So, this way your I1, I2, I3 will be going on certainly P is added some place where as if you have a compound interest, now in this case basically the I1, I2 or I3 they are you know fixed and this fixed because it is s fixed percentage of this amount P if this time period is you know same where as in the case of compound interest what happens that in the first year the interest is I1 although.

But in the second year it will be you know, so here these all are same. So, I1, I2 I3 they are all same however in this case I2, I2 will be you know what you get it will be based on P+I1. So, similarly the I3 which you get it will be based on P+I2 P+I1+I2. So, what you would see is that the interest which is computed in the case of compound interest in that basically you are also calculate the interest of the interest which is there in the earlier time.

So, that is why it is known as the compounding of the interest, so you have 2 types of interest simple interest and compound interest. And normally we will discuss in our coming lectures that there are many ways by which this compounding is done. And there are many kind of compounding frequencies of interests all that, so but we must know that this is the 2 type of interest which is normally available defined.

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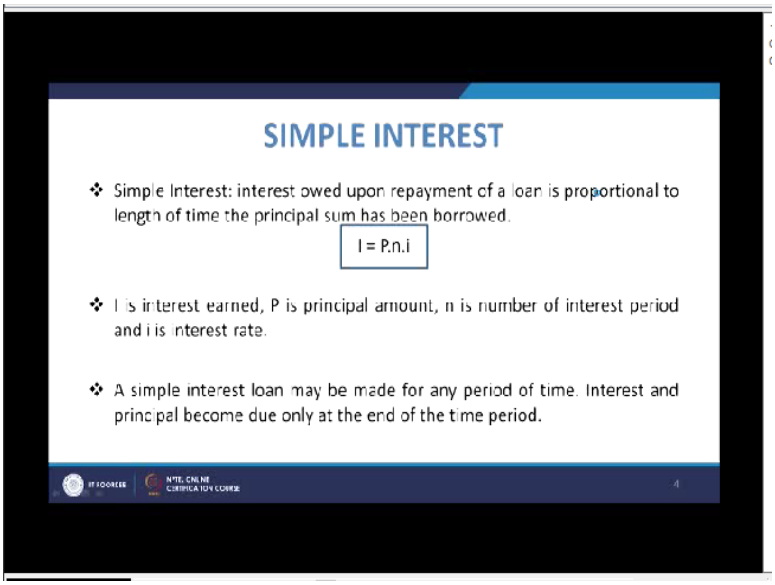

Now, so as we discuss that when we talk about the simple interest the interest owed upon repayment of loan is proportional to length of time the principal sum has been borrowed. So, the interest amount interest rate earned it will be P, P is the principal amount n is the number of interest period and i is the interest rate. So, based on that we calculate the simple interest and as you know that is the i is fixed suppose you have i is the interest rate for a particular interest rate.

If n you are calculating for a particular period, so whatever be the you know whenever you are calculating this for i and n, n means first year, second year or third year. So, this is fixed it will be the same where as in the case of compound interest what we done is that this interest amount which is generated every year that will be very, so in the first year it will be Pni. Second year it will be i*n*P+I1.

So, that way it will be going on, a simple interest loan may be made for any period of time interest and principal become due only at the end of the time periods. So, when we talk about the simple interest, in that case you say that the interest and also the principal amount they are due only at the end. So, your interest amount will also be summed up and principal amount in will be summed and that is and that is the you know last amount which is to be paid by the borrower. So, that is how this simple interest is defined, now when we talk about the compound interest.

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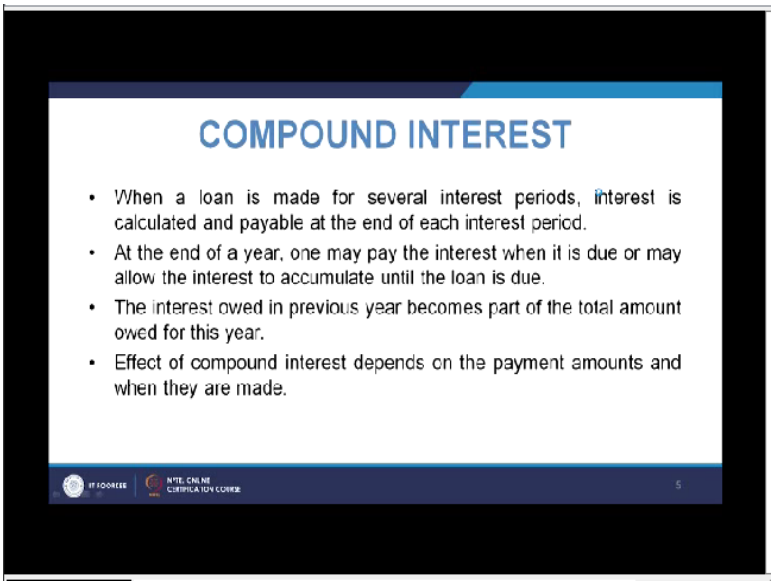

Now in these cases the loan is made for several interest periods and interest is calculated and payable at the end of each interest period. So basically you are suppose to pay the interest at the end of the each interest period and if not if you are paying the interest it is fine. Otherwise you will have the interest calculation on the interest also and this interest we will be basically a part of your principal amount.

So, it will be accumulated and the interest owed in previous year becomes part of the total amount owed for this year. So, that way if you are paying the interest every year then its fine but if not then that becomes a part of your you know principal amount. And this interest amount which is calculated for the year it will be added to the principal amount. Now effect of compound interest depends on payment amounts and when they are made.

So, also when you are making, so mostly what happens that when the compounding of interest is done then the time at which you are paying any amount. So, that defines that what will be the next amount of you know interest which will be charged for you. So, that is the remaining amount, so on that the rate of interest will be charged, so that way you know it is compounding is being carried out.

Now what we see is that now we have discussed that this interest which is charged it has many view point. So, that is it may be from lender's view point or it may be from the burrower's view

point, from the lender's view point basically if you look at a person who has lot of money, so basically by looking at its money itself is quite happy. So, the happy which is getting by looking at his money.

Now if we gives to someone then certainly he is compromising with that happiness, so for that he is he must be paid something. So, that is what he is earning that is you know interest, so that is why charging interest. Otherwise he can have it in his you know with him and then he can most of having you know morally or you know in socially also he may be quite comfortable that way.

So, from the lender's view point there are many points by which you can justify the use of you know the use of interest is being charged. Similarly fro barrower because burrower may has the inability to meet certain you know demands made by his family members or he has to do some work. But it does not have the capital to do that or he has to start a business to you know to sustain, so in that case he has to get the money from someone.

And his wishes are fulfilled but for that he has to pay some extra amount and that is what the interest is, so this way you have the definition of interest for different you know purposes. Now what we see is that there is cash flow over time but before that we will talk about the time value of money. Now what we see is that what we have seen that when we talk about the interest so now suppose you are added 0 time.

And you have amount of P now if you go to first year you go to second year or so if you go to the third year, fourth year or so. Now this value P it will not be the same you know as it is now at any point of time, so that is why the money because here you will have its value as P+interest, so this is the you know the value addition to this amount P now. And that is how its value is basically you know changing.

And that is why as you see that it is you know value is changing with time and that is why it is known as the time value of money. So that is why we call it as the time value of money, now for knowing this time value of money you must know the interest and the interest rate, so basically

the time and the interest rate, so you have the effect of time and interest rate. Now what will be the value of the money at a particular time.

So the time you must know and what rate the you are trying to earn on a certain amount, so that basically is the interest rates. So, once you know the interest rate and you know the time and in that case you can predict about its value at a particular time. Now what we try to say that when we do the financial transactions when we talk about the economic transactions, then what we suppose you are taking the example that you are taking a loan at now and you will be paying it paying back this loan amount in the coming years.

So, if you are taking a loan of 1000000 certainly if you are returning the 100000 now itself that is fine. But if you are keeping this amount then this value of 100000 is going to be increased slowly, so and you will not be almost you know in most of the cases when you take a loan you since you are you know you cannot you know manage that much large amount at once. So, you have to barrow it from certain financial institution now that is and since being a larger amount you will try to repay it at different times, so you will try to repay that at different times.

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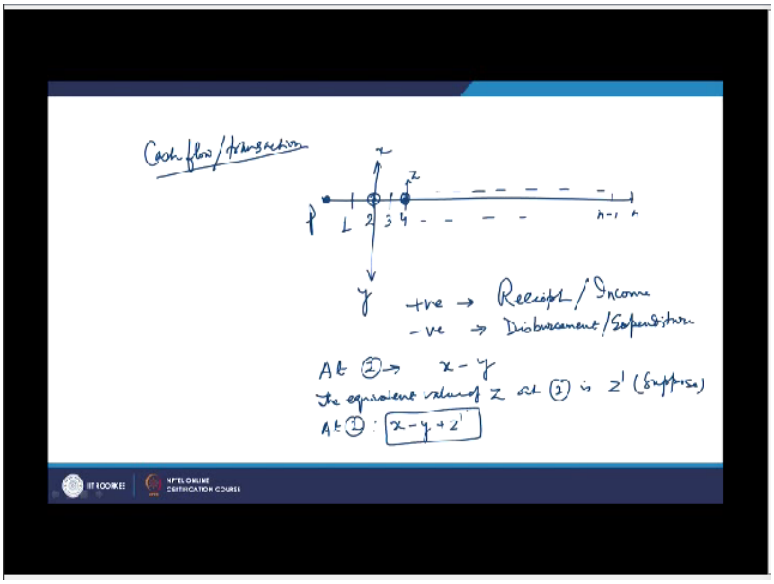

So, you will have P now and you will be paying maybe you have many you know installments. So, suppose you are to you want to pay in many installments, so you the time period at which 1, 2, 3, 4 and this is suppose going to n-1 and this is going to n. Now when we talk about the time

value of money that time we know that you have 2 things rate of interest and the time. Now there may be you know return of the money at this point to certain amount or at this point of certain amount.

Now these are the transactions monetary transactions or we also call it as the cash flow or transaction. Now what happens that as we see that when we find, so at this point suppose the may be case that you have so for any when we deal with the financial transactions of any organization or so. Now at any point if you have something as income and something also as you know as the you know you have to pay.

So, when you are going in the positive side that is known as receipt and if you are going to negative then it is known as disbursement. So, basically it is nothing but income and this is expenditure, so what we see is that in such cases at this point if we you are doing the transaction at a particular time. Then you can add then however if suppose you are doing certain transaction here and you want to analyze of some transactions.

Suppose this is x this is y and this is z, now at the point at this particular point at 2 you can have since the time is same the rate of interest is anyway fixed that way also changed. So, once that will change that also is to be accounted into. Now rate of interest being same and time being same if the both the things are same then basically the value at this point you can some or you can subtract.

So in this case the net you know receipt or disbursement net amount which is transacted will be x-y. But if the z is also a transaction but it is at 4. So in that case its value must be calculated at a point where a 2 because it is a different time. So, that is why we discussed about the time value of money, the value which is z here. It is at the time 4 and it is value will not be same as that a 2.

So if you want to have the effect of z, now for that you have t find its value equivalent value at time 2. So, depending upon the rate of interest the now rate of interest and time both will come into picture and you will use that rate of interest. And with that and time is also you can see that

there are 2 interest periods 3 to 4 and 3 to 2. So, this amount is to be you know its value, so if you see in a time since it is moving in a forward direction.

The value basically its value you know as the progression depending upon the it is lender or it is the burrower. So, it is value has to be taken at 2 or else you can have its net value also being mapped at 4. So, that way this you know its value or equivalent value, so that is known as the equivalent value of z at this point. And then so suppose the equivalent value of z at 2 is zprime suppose.

Then in that case you can have further you can have the you know net transaction at 2, so at 2 you can say that your net transaction is x-y and then +zprime. So, this basically gives you concept about the time value of money because how this you know its value is changing with time. So, that basically gives you about the time value of money. Now if you go to this slide, so here it is told about the cash flow over time, what is cash flow?.

Basically the flow of the transaction which we do since we are doing in monetary terms, so we call it as the cash flow, now as we discuss that you will have the receipts.

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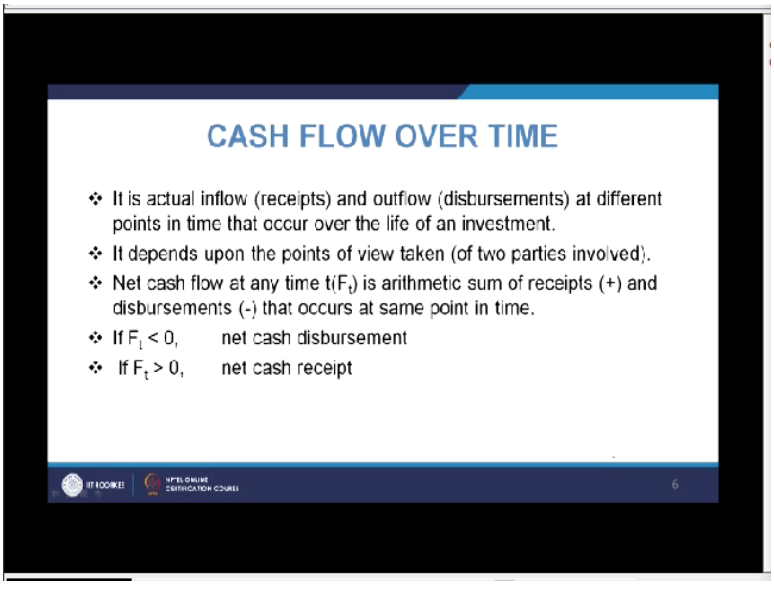

And you will have the disbursements, so receipts means inflow, so when we talk about the flow of the you know money or the transaction along a certain you know time line. Then wherever you have so now there may be cash flow.

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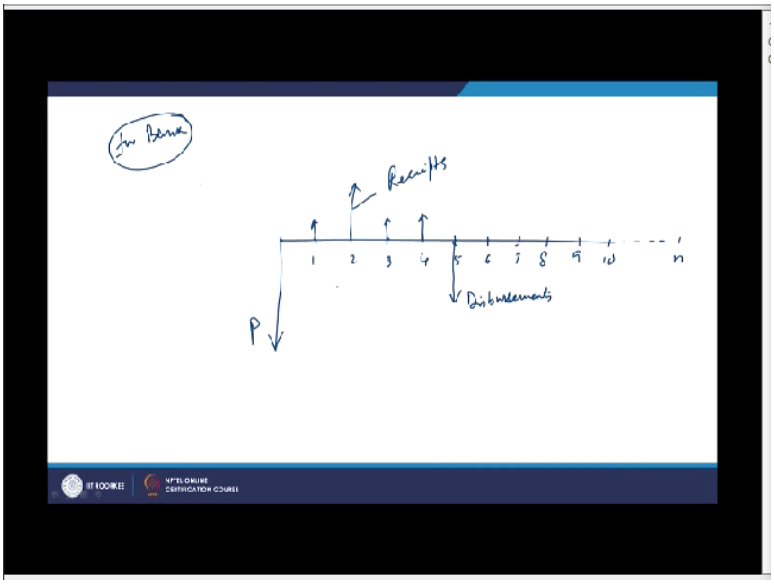

And if you look at any particular time zone, so now suppose there is you have taken some amount from the loan. So this is suppose you have taken some amount from the loan some amount as a loan from the bank. So, you are giving this as the negative because since it is disbursement you know now for the bank. Now for the bank, so for bank, bank has given you, so bank. So, this will be negative and then at all interest periods you will be paying like that it will go and it may go up to suppose n periods. So, if you are you know giving some amount here you may be giving some amount here you may be you may give you know the amount like that.

So, basically these are and suppose at some point again you are taking some amount from the bank suppose in a same way. So, these are known as the receipts and these are the disbursements. Now this is for the bank because you are paying back to the bank. So, that is receipt for it and this is the since the bank has given you, so it is going in the negative direction and when you are paying the money to the bank it will be receipt for the bank.

Now for the barrower it will be different for burrower the same thing will be P will be up. Because his receiving the now from the bank and he will be paying that amount you know as a

time progresses, so since he is losing that money. He is paying that money, so that will be his disbursement. Now depending upon the rate of interest, so ultimately for the time horizon it has to be seem that you know the net receipt should be equal to net disbursement.

And the rate at which it will be satisfied that will be the interest rate, so that is how the interest rate is defined. So, it depends upon the point of view taken or the part is involve, so as we discussed that we may have either the lender's view point or the burrower's view point, for lender, so this you know this what we have drawn now for any transaction if you see. So, what you will see in the case of financial you know.

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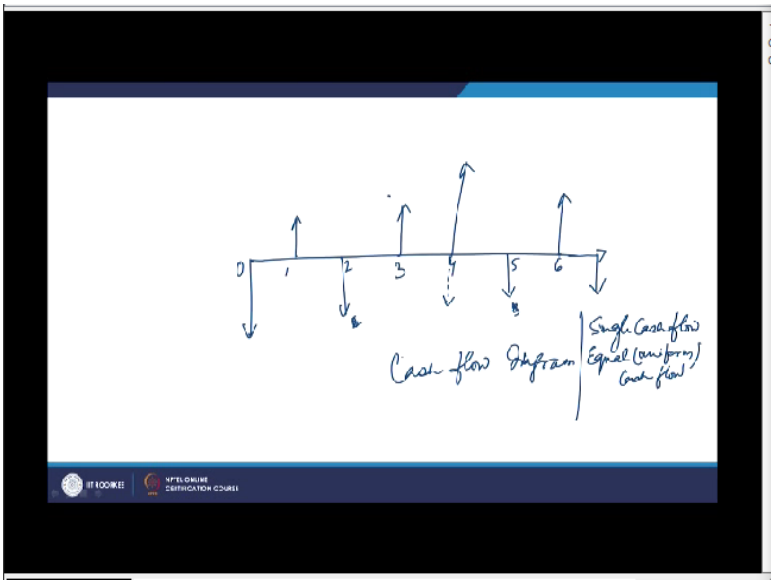

When we talk about the transactions you may have different type of you know sometimes you have receipt disbursement or so. Now with time when you have such transactions going on, so this is known as a cash flow diagram. So the cash flow diagram basically will talk about the different points, so this is 0, so, this will be 1, 2 this is 3, 4, 5 this 5, 6, 7 years. So, this is time now 2 here but this is 2 and you will have different you know monetary units here.

So, this diagram you get this diagram is known as the cash flow diagram. And the net cash flow at any time t is arithmetic sum of receipts and disbursements that occurs at the same point in time zone. So, that is what we told you have to find the net cash flow at any point, now suppose there may be some case where the there is also some disbursement also at this point. So, there will be

net cash flow at this point of time you have to find arithmetic sum of the transaction which has taken place at this point.

So that is known as the net cash flow, then if the Ft is less than 0 it is the net cash disbursement and if Ft is more than 0 it will be net cash receipt. So, this is about a particular time now in this case you can see the is this is smaller amount as compared to the receipt. Then you have net cash receipt of at that particular time. Otherwise net cash disbursement if it is more and there is small receipt here in that case it is net cash receipt.

So, this way you will have a diagrammatic representation of the cash flow over time and that is known as the cash flow diagram. So we will deal with these things we have to discuss when we talk about this cash flow diagrams and you have many kinds of cash flow diagrams like you may have single cash flow or you may have equal uniform cash flow. Then you may have the gradient series like what we do many a times that you take loan and you are paying the amount in a increasing orders.

So, you except that every year you will have increments, so you will be paying larger amount of you know loan repayment will be done by you. So, that is the gradient series by certain amount linear gradient, then if you are increasing in terms of percentage and it is geometric gradient series like that. And there may be irregular series also like this, so you will have irregular series of payments which is done in that cases.

So, this is the cash flow and you know you can do the analysis based on finding the equivalent values at one particular time and then you can have the net cash flow or cash you know net cash flow you can find it and ultimately it has to be 0 for a particular interest rate. So, that is how they interest rate is defined and that gives you the (()) (33:28) or so there are many terminologies that will come across as we move in analyzing such problems, thank you very much.