Financial Mathematics
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Lecture – 59
Types of Insurance Policies

Welcome to the lecture on types of Life Insurance policies, so we have the introduction about the life insurance in the earlier lecture and we also had started discussing about the types of life insurance policies in that and as we knew in that lecture.

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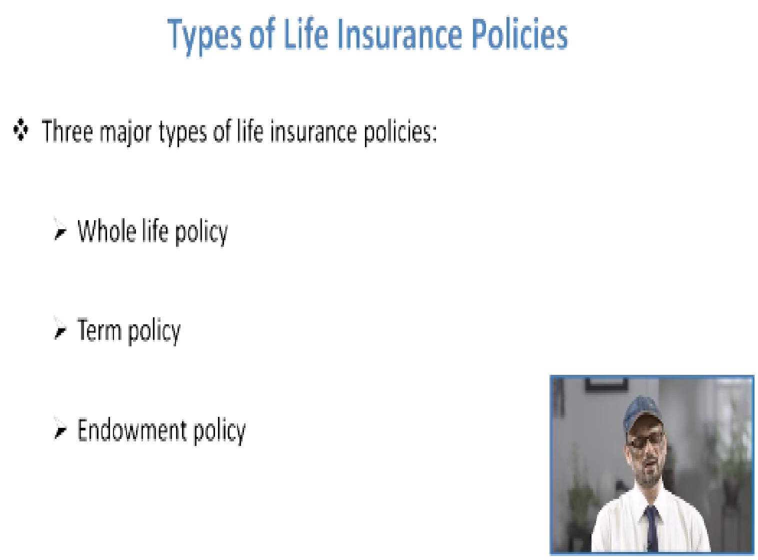

That there are basically 3 types of life insurance policies one is the Whole Life Insurance policy 2nd is the Term insurance policy and 3rd is the Endowment policy. So, we talked about the Whole Life Insurance policies in that basically you can have the net single premium or you can have the annual premium. You can also go for the differed whole life premium you know then a whole life policy.

You can have the deferred annual premium and also the deferred annual premium on m payment basis. So, all these you know are the examples of whole life you know policies so for that the premium calculation as we know that if you are sued for the whole life insurance policy so for that as we knew we had a seen how to derive them. We had derived also for the so when we are interested to find the net single premium.

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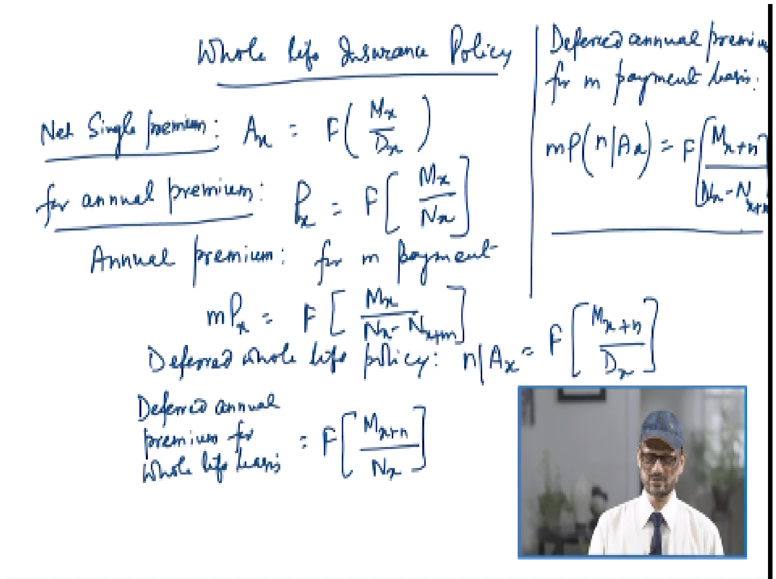

So, that was you know Ax calculation of the Ax and if f is the face value so for face value of F you know and the Ax will be F(Mx/ Dx).So, that was you know the whole for the whole life insurance policy. Now what happens when there will be annual premium so this is the single premium and for annual premium. So, for annual premium means many a times the net single premium amount is quite high for a certain face value.

So, in that case the person who is insured he wants to pay you know annually so in that case you know the amount which is calculated will be Px and that will be for face value F. It will be Mx/Nx that is what we had also seen that how to find it because, in that case x will be Px * A double dot x a double dot x. So, based on that we calculated the value of Px as F * Mx/Nx then many at times if you are doing for the annual premium for embayment basis.

So, annual premium and for m payment basis so in that case the premium amount so we call it as mPx this comes as for face value of F. It will be Mx/Nx- N of x+m so this way you can calculate the you know annual premium for the m payment basis. Now if you want to defer the whole life policy so far the net single premium so far the deferred whole life policy. So, if you go for the net single premium that will be it is known as a n A x on deferment period is n.

So, that way it is denoted as n Ax and it is basically F * Mx+n/Dx that is what you know if you look at here it is n Ax * FX Mx+n/Dx. Here it becomes n Ax is for the deferment period n it will be nAx = F * M*x+n /Dx. Now you know if you find the deferred annual premium and so this is for the whole life policy and you can go for the deferred annual premium and that is for the whole life basis.

So, you can calculate that deferred you know annual premium so deferred annual premium for whole life this is now in that case this becomes you know = F * M of x+n/Nx. So, that is for annual premium it is in the denominator it is Nx so it will be F * M of x+n/Nx that is what it will be written as and it will be F of Mx+n/ since it is Nx here so it will be Nx. So, this is the deferred the annual premium for the whole life basis.

And the next one which remains to be seen will be the deferred the annual premium for the m payment basis. So, if you find the deferred annual premium for m payment basis you are only paying m times. So, in that case you know we denote as mP and then we call it as n Ax this is by for the deferred payments of you know this is deferred annual payment only for you know m payments only.

So, in that case in the same way it was here if you look at the deferred payment so it will be again m F of M of x+n and / you know Nx- N of x+m so that way as you see here. So, it is similar to this it will be Mx and since it is you know for that deferment annual period so it will be for F of Mx+n/Nx – N pf x+m so these are the formulas which are used for the whole life insurance cases. Now the next type of insurance policy is the term insurance policy.

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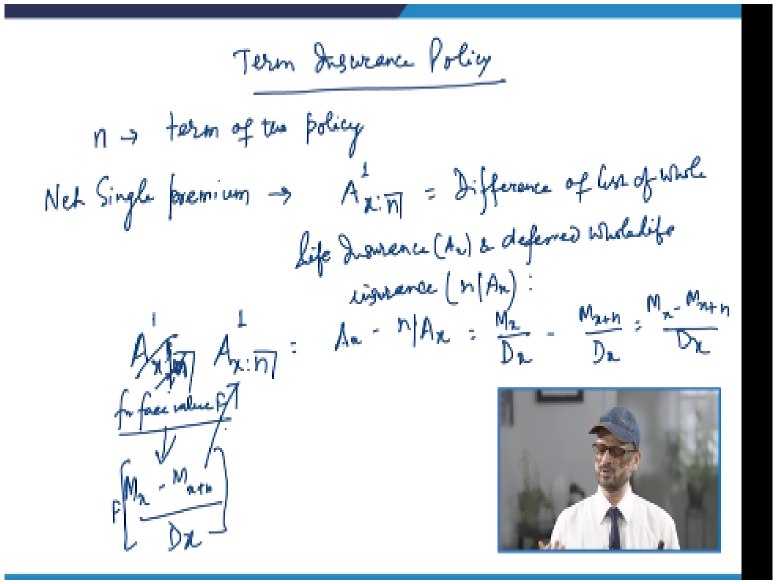

Now what is this a term insurance policy so now you know if the survivor is a person who has insured now in this case the face value of the policy will be paid to the survivor only if he is alive during that the term of the policy that is n. So, that is what the term insurance policy tells and here also again you can have the net single premium or you can have the annual premium also so n is here the term of the policy.

And during that the person has to be survived he has to survive and if not then he will be paid the face value of the policy. So, if he dies then only he will be paid that face value of the policy now in this case again you will be calculating and the values of the premium amount. And it may be again in the similar line for the single premium or for the annual premium or deferment period or so.

So, when we talk about the you know net single premium so net single premium in the case of term insurance. So, that case we denote it as A1 and then we will call it as xn, so this is known as this is the notation for the term insurance policy and this is that is the single you know net single premium. Now this is this will be nothing but it will be the difference of the; so it will be a difference of cost of whole life insurance.

That is x and deferred whole life insurance that is n Ax so this can be viewed as the you know premium which is paid for the cost of the whole life insurance in which you pay once and then if you are deferring. Now you know because up to that if you are alive that you are not paid so it will be as per the deferred whole life insurance. So, it is something like you are going to pay after that period.

So, after after that you are any way now alive so in that case this will be the difference of the two values so you can write xm and that is for the you know so this is n basically. So, you can write A x n and that is be this, It will A x – n Ax so now we have already got the expressions for the cost of whole life insurance. That is your Mx / Dx and similarly the cost of the deferred whole life insurance so that will be Mx+n/Dx.

It is basically a deferred by that period term of th term policy you know years n and so it will be Mx/Dx and then it will be divided by Mx+n/Dx so it will be Mx- Mx+n / Dx so this is for the on the basis of single dollar. And if you try to find it for the face value F so no for the face value F you can write it as Mx- F times Mx- Mx+n and /Dx. This will be your; the value of the net single premium for the you know a term insurance policy.

So, that this can be computed by referring to the table and if suppose somebody wants to suppose for example, if a person wishes to you know buy a 13 year you know term insurance.

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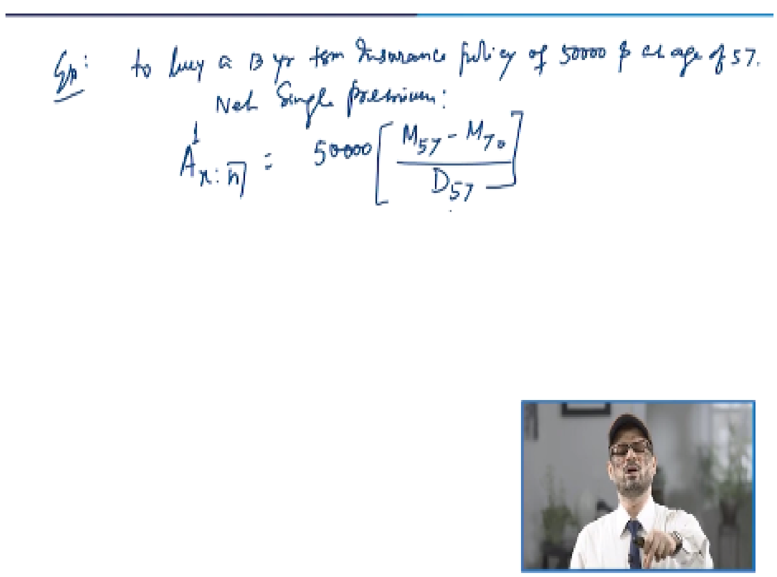

So, for example suppose a person to buy a 13 year term insurance policy of 50000 suppose dollars so at age of 57 so the person is age of 57 and he wants to have a 13 year term insurance policy of 50000 dollar. Now in that case how much the thing they will net single premium it should be. So, net single premium which would be how much now in that case if you find the you know A x n and that is that will be F.

So, that is 50000 and then you have to develop multiply with the factor that will be Mx. Mx will be in M57 – Mx+n so that is 57+ 13 that is 70 and then divided by you have to do Dx so D 57. So, you can get these values from the table.

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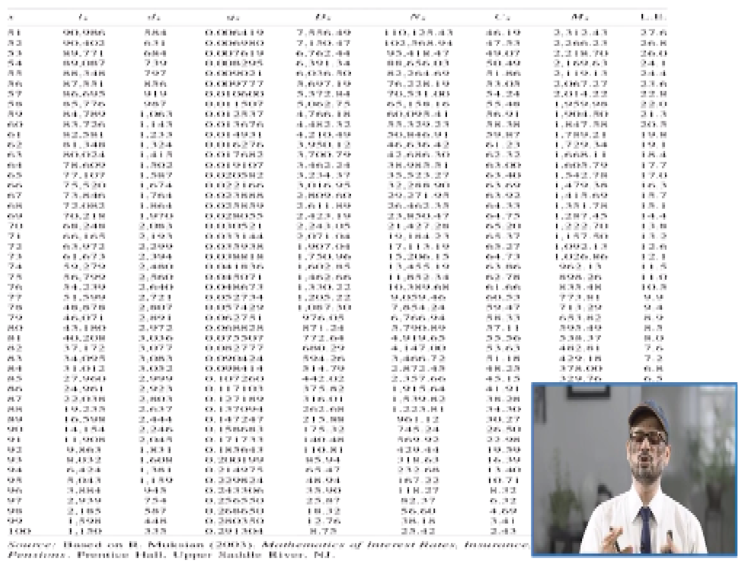

And so for 57 you can see and for 57 Mx values so it will be something like 2014 so that way yes 2014.22 for 70 it will be you know 1222.70 1220 2.7 so that way and then again you can find the Dx d 57 so it will be 5372.84 so this value can be used.

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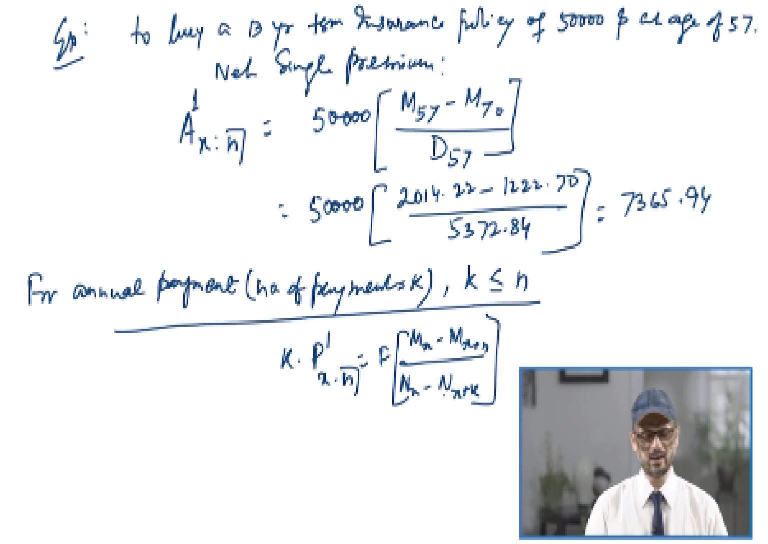

And then you can calculate these values so it will be 50000 *2014.22 – 1222.70/5372.84 so if you do that you will get 7365.94. So, you know this way you can find these net single premium values. Now in this case of term insurance policies you can go for k number of payment and k should be <= n so when you are you are going for annual payments so for annual payments you know with number of payment = k.

So, k has to be <= n so if you want to calculate for suppose k payments in that case annual premium will be calculated and it will be called as and it will be denoted as k * Pxn and so this will that. And this will be calculated as F* Mx – Mx+n/ Nx- Nx+k so this can be you know you can find these as presence and these values can be used for annual payment. You have to make k payments and k has to be in any way maximum k value can be going to n.

So, that way you can get the values then you know there is another case of the deferred annual premium also for the whole life basis.

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Deferred annual premium for whole life basis

$$K \cdot P^1_{x:\overline{n}|} = F\left[\frac{M_x - M_{x+n}}{N_x - N_{x+k}}\right] = 50000\left[\frac{M_{57} - M_{70}}{N_{57} - N_{67}}\right]$$

$$= 959.21$$

So, deferred for whole life basis so we will discuss basically about before that we are going to discuss about the example of the case where the person suppose wants to you know go for the same payment. So, for the same problem if a person wants to pay in 10 instalments say so in that case what will be the net single premium and that can be computed for these insurance policies. So, suppose the person is you know for the same person.

If the person wants to you know pay it off in 10 terms now how this you know how this can be computed so far that, what you will do is you are going to calculate again k*Pxn and so that will be case. Basically 10 now that will be = F * Mx- Mx+nNx- Nx+k so again you are going to use N is 15 and k is 10. So, you are going to calculate these 50,000 and then you will have M 57 and M70/ you know N57- 57+10 so it will be N 67 so this again you can use from this table.

And you can calculate these values and it comes out to be 959.21 so if we has to you know pay in 1 go it is 7365.9444 but if he has to pay in 10 terms in that case he has to pay for 959.21. So, that way you know he can find all these you know instalment values or the premium values for the term insurance. But this is about the term insurance we have yet discussed about the deferred annual premium for the whole life basis and the now we will discuss about the Endowment insurance policies.

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Endowment Insurance Policy

Net Single premium of endowment insurance policy ($A_{x:\overline{n|}}$) would be sum of net single premium for n term life insurance ($A^{1}_{x:\overline{n|}}$) and net single premium for n term pure endowment

$$= A^{1}_{x:\overline{n|}} + {}_{n}E_{x}$$

So, we talked about pure Endowment cases for the endowment insurance policy you know in this case you will be having this will be some off. So, basically the net single premium so in this case you are getting what the benefits if you are so getting the benefits of term insurance policy. As well as so you know I mean the life insurance this life insurance and also the pure endowment was the one which we discussed earlier.

So, in this case you are getting the benefit of both the you know whole life insurance as well the endowment you know deposit. So, it will be in this case the net single premium of endowment. It will be basically so endowment insurance policy and this is basically denoted by Ax n so and we put it under the so this will be 1. So, this will be equal to; so in that case it will be sum of net single premium for n term life insurance.

So, that will be A1 x you know n so and then net single premium for n term pure endowment. So, basically it will be because here you are in the case of pure you know term insurance you are not getting anything. Now in this case you are getting both the benefits so you are what you do get is Axn so that will be n + you have n Ex so that is your pure endowment and that is how you calculate you know the net single premium for these endowment insurance policies.

And if you know if you are calculating the for the face value of f so it will be basically if you simplify it will be F* Mx – Mx+n +Dx +n and /Dx so if you have all the data you can directly compute the you know net single premium for the endowment insurance policy. Now while after discussing this we also need to know something about the natural premium verses the level premium. Now what is this natural premium and what are the level premiums.

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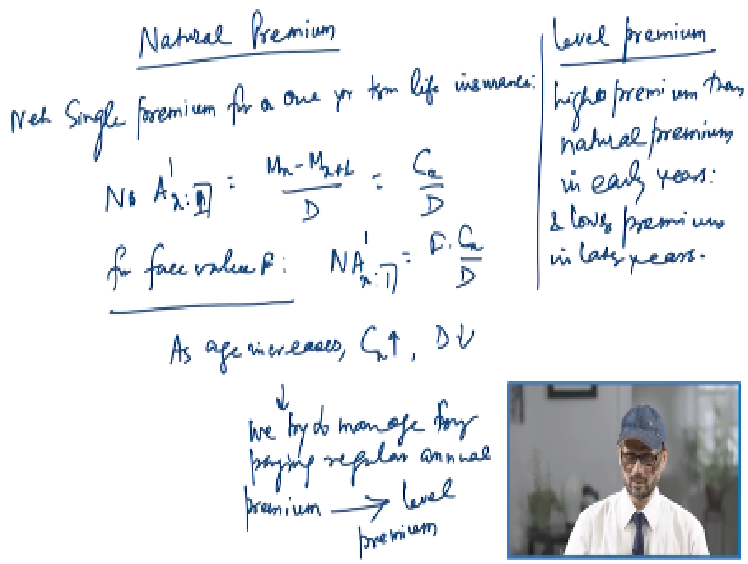

So, this natural premium is basically the you know it is the net single premium for a 1year life you know term life insurance. So, it means that the policy will say that if the insured person dies within a year. Then he will be paying the face value, to the ensured so that way that is you know the natural premium you know so such a premium you can obtain by adjusting you know the regular life insurance formula by making n = 1.

So if you take an equal to one in that regular term life insurance you know formula if you put m = 1 it will be for the net well premium. So net well as premium will be denoted by N A and this is x n so this be you know so n will be 1 basically in this case. Now in this case so it will be M x – Mx+n /D.

So, basically it becomes Cx/D so for the face value D, F for face value F it will be you know it will be = so N A 1 and x 1. It will be F * Cx/D so that is you know net well premium and now what happens that. So, this is your now the thing is that as the age increases this value basically will be going on increasing a lot. So, what happens that as the age increases the Cx value will be going on decreasing.

And that D value goes on decreasing that can be seen from the table that Cx value is basically increasing is on that way. And then so this is an D value. The value will be decreasing so in that case the premium value will go on increasing a lot. So, in that case what you do is that you take one value which is basically you know which will be compensating in the initial part. So it will be taking initially you know more and then we later on it will be less.

So, that way that will be known as the taking the face value so this way we calculate the face value or you know they know that is the level premium. So, this is necessarily premium in that case so that will be your level premium so the justification is that as age increases. So, this you know Cx will increase and D will decrease. So that is why you know the natural premium amount will be going on increasing and it will not be.

It wills no longer to be affordable so what we do is we depend upon the affordable premium that is we tried to give the regular annual premium. So, we try to manage by paying a regular annual premium which basically will cover and so that will be known as the level premium so this is known as the level premium. Now these level premium policies now they have a higher premium so these level premium policies which are used you know.

In those cases, it will try to see that it does not reach so high as the age increases so in that case what it does is it will be you know it has a high premium you know. So, it has high premium then natural premium in early years. And you know so it will be making you some giving you some excess fund so that will be compensating in the later years so and lower premium in later years. So that way you calculate this level premium and a level premium basically will be calculated.

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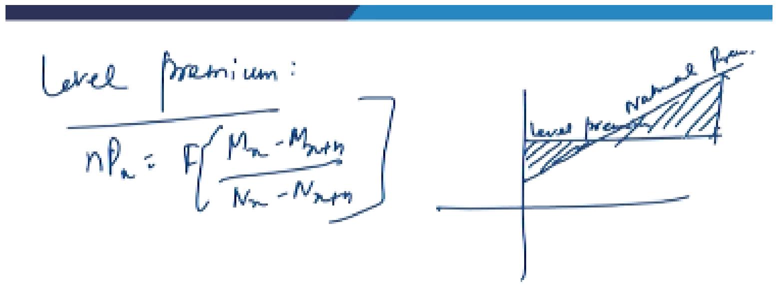

So if you try to calculate the level premium okay the level premium is calculated as a n Px so that will be F of Mx- Mx +n /Nx-Nx+n so that way you know for. Suppose 15 year you know term insurance policy and if it is 1 Lakh * 50year old in that case if you calculate you know these so a level premium and the premium and the natural premium so if you try to calculate the value and if you tried to find the graph.

What happens the graph comes like you know so this will be your level premium that will be calculated at one fixed amount and that is what our premium basically initially it will be less but it will be increasing slowly so this will be graph for the natural premium and this is the level premium so in the initial; the time so what happens that it will be taking the larger amount and that will be covered in that you know later years.

So, that way this concept of the natural premium and a level premium is used in the case of the insurance sectors. So, that is how you calculate these premium values. Thank you very much.