Financial Mathematics
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Lecture - 58
Introduction to Life Insurance

Welcome to the lecture on Introduction to Life Insurance. So basically we are going to talk about conditions for life insurance policies in this lecture and we know that you know life insurance policies are very important part nowadays people used to take these policies and normally the payment is made you know after the person's death basically. So this is to so that trauma which is there already with the family of the persons.

So they get some amount so that the unexpectedly you know loss of the persons and inability to meet certain expenses because of the absence of person that can be taken care of. So the difference between the you know life insurance and the life annuity is that.

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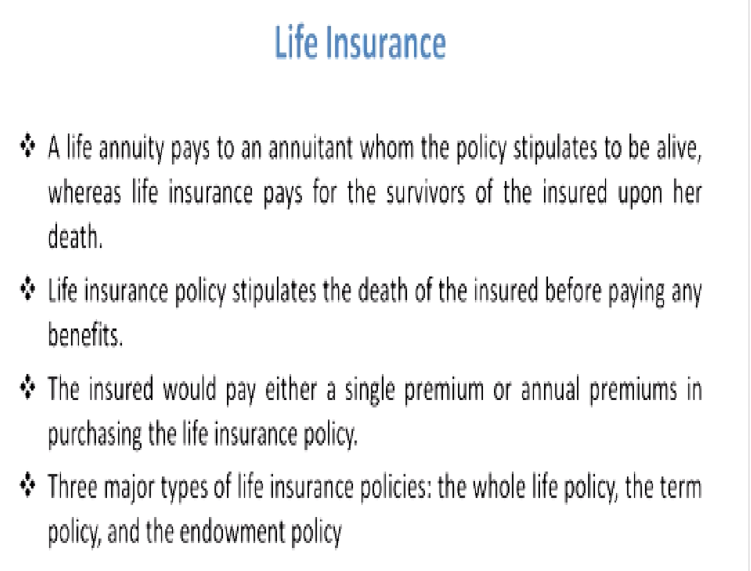

In life annuity you know you are paying a stipulating that it is the person is alive whereas you know for the life insurance it is paid to the survivors after the death of the person who is insured. So that is a difference life annuity paid to an annuitant whom the policy stipulates to be alive whereas life insurance pays for the survivors of the insurance upon his death. Now life insurance policy stipulates that of the insured before paying any benefits.

The insured would pay either a single premium or annual premium you know in purchasing that life insurance policy. So he has to purchase that policy and in that he may pay single payment or he may pay the payment you know in annual terms and you know there are different types of these life insurance policies and in that you have the whole life policy the term policy and the endowment policy. So we will talk about these policies one by one.

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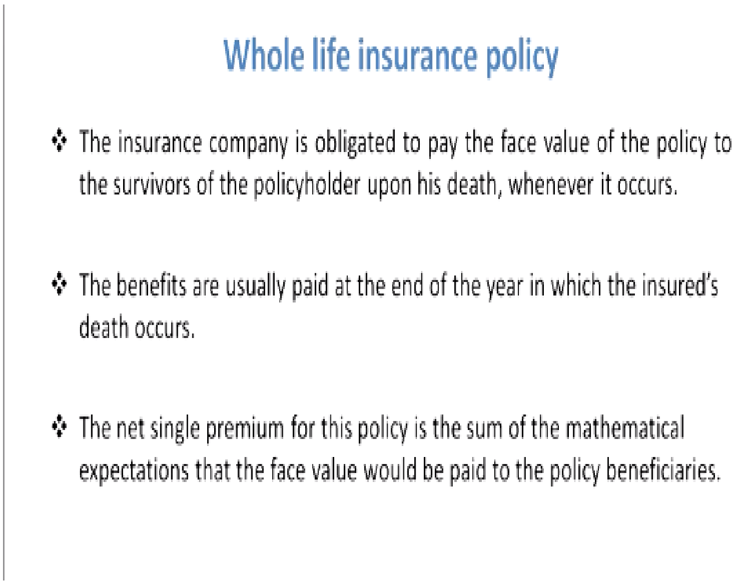

Now whole life insurance policy this insurance policy is obligated to pay the face value of the policy to the survivors of the policy holder upon his death whenever it occurs. So after the death you know the survivors of the policy holder the family will be paid after his death and benefits are usually paid at the end of the year in which the insured's death occurs and in this case you may have you know to calculate the net single premium.

And in this case it is the sum of the mathematical expectations that face value would be paid to for the policy beneficiaries. So you will have you know this expectation is nothing but you have the probability of the that the insured's will be dying and with that you will be multiplying with the present value of the policy benefits.

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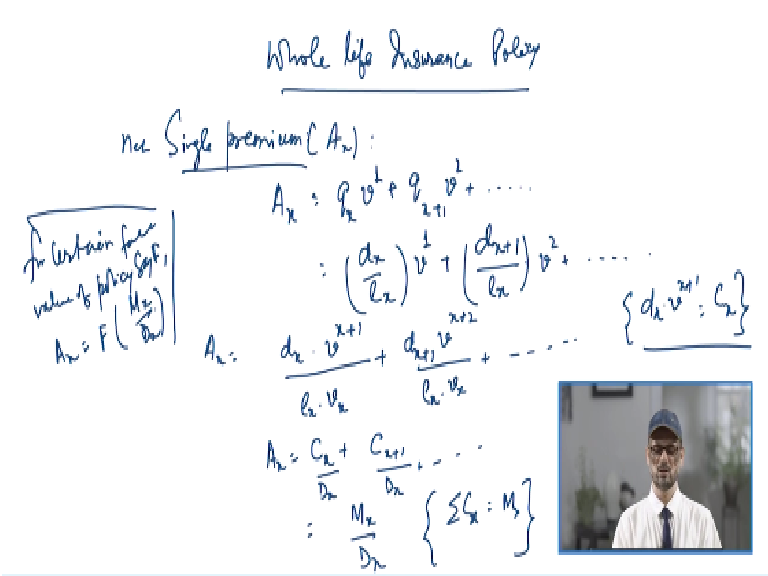

So if you try to calculate for the whole life insurance policy. Now in that case if you are talking about the single premium cases. So in the case of single premium if you want to have a single premium. Now in that case the ax so that single Ax so that is net single premium is so your net single premium is suppose Ax so Ax will be as we talked that it will be the summation of the qx so that is v1+qx+1 v2 like that so it will be going.

And in this way you are going to calculate the value of the Ax so its present value will be there in that case. So qx is as we know that qx is found by the number of death/number of persons who are alive. So qx will be dx/lx and that will be multiplied with v1 similarly it will be you know dx+1/lx and then it will be multiplied with v2 and so. So it will be going like that and then you can further find the values.

So you are multiplying you know the numerator as well as the denominator with vx. So you can write Ax if you multiply the numerator as well as the denominator with vx it will be dx* v raise to the power x+1/lx* vx. Similarly, dx+1 v raise to the power x+2/lx * vx so this way it will go. Now we know that dx * v raise to the power x+1 so that is becoming as Cx. So we know that dx*vx+1 it is nothing.

But Cx is computation term that we have derived earlier. Now this becoming an lx*vx is Dx capital Dx. So you know Ax will be you know this will be Cx then Cx/Dx+ it will be Cx+1/Dx so it will be going like this and this summation of these Cx is becoming Mx so you can write this as Mx/Dx because the summation of this Cx is becoming as Mx. So you know so in that case this you know premium net single premium which should be paid.

You know for that value will be you know Mx/Dx/ So taking that \$1.00 as the value now if you are calculating for the you know certain face value of the policy. So for certain face value of policy if you are taking something different you will be multiplying this you know with that F. So for certain face value of policy say F so your net single premium Ax will be basically F times Mx/Dx.

So by using this mortality tables if you have certain phase value so this face value can be computed that will be F*Mx/Dx.

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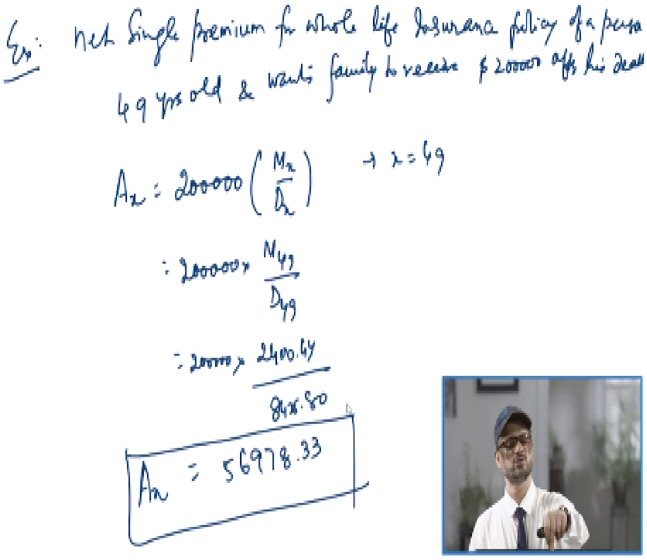

So if you can understand with certain examples say like a person you know he has to calculate the net value of the single premium. So net single premium you know you have to calculate and for whole life insurance policy and you know of a person. Now this person is 49 years old and wants family to receive a \$2,00,000 so okay so that after this death. So suppose a person who wants to do the insurance he wants to give a single premium.

He is 49 years old and he wants that his family should get \$2,00,000 after his death. So in that case it is a single premium payment and we know that in that case Ax will be face value face value is 2,00,000 so you will be 2,00,000 and then it will be multiplied with the factor Mx/Dx so x is 49.

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0	100,000	1,260	0.012600	100,000,000	1,992,208.86	1,200.00	5,132.91	74.4
1	98,740	90	0.000932	94,038.10	1,892,208.86	83.45	3,932.91	74.3
2	98,648	64	0.000649	89,476.64	1,798,170.76	55.29	3,849.46	73.4
3	98,584	49	0.000497	85,160.57	1,708,694.12	40.31	3,794.18	72.4
4	98,535	40	0.000406	81,064.99	1,623,533.55	31.34	3,753.87	71.5
5	98,495	36	0.000366	77,173.41	1,542,468.56	26.86	3,722.53	70.5
6	98,459	33	0.000335	73,471.62	1,465,295.15	23.45	3,695.66	69.5
7	98,426	30	0.000305	69,949.52	1,391,823.53	20.31	3,672.21	68.5
8	98,396	26	0.000264	66,598.29	1,321,874.01	16.76	3,652.90	67.6
9	98,370	23	0.000234	63,410.18	1,255,275.73	14.12	3,635.14	66.6
10	98,347	19	0.000193	60,376.53	1,191,865.55	11.11	3,621.02	65.6
11	98,328	19	0.000193	57,490.35	1,131,489.02	10.58	3,609.92	64.6
12	98,309	24	0.000244	54,742.13	1,073,998.67	12.73	3,599.34	63.6
13	98,285	37	0.000376	52,122.63	1,019,256.54	18.69	3,586.61	62.6
14	98,248	52	0.000529	49,621.92	967,133.91	25.01	3,567.92	61.7
15	98,196	67	0.000682	47,233.95	917,511.99	30.69	3,542.91	60.7
16	98,129	82	0.000836	44,954.03	870,278.04	35.78	3,512.21	59.7
17	98,047	94	0.000959	42,777.58	825,324.01	39.06	3,476.44	58.8
18	97,953	102	0.001041	40,701.49	782,546.43	40.36	3,437.38	57.8
19	97,851	110	0.001124	38,722.96	741,844.94	41.46	3,397.01	56.9
20	97,741	118	0.001207	36,837.55	703,121.97	42.36	3,355.56	56.0
21	97,623	124	0.001270	35,041.03	666,284.42	42.39	3,313.20	55.0
22	97,499	129	0.001323	33,330.02	631,243.39	42.00	3,270.81	54.1
23	97,370	130	0.001335	31,700.88	597,913.37	40.31	3,228.81	53.2
24	97,240	130	0.001337	30,151.00	566,212.49	38.39	3,188.50	52.2
25	97,110	128	0.001318	28,676.85	536,061.49	36.00	3,150.11	51.3
26	96,982	126	0.001299	27,275.29	507,384.63	33.75	3,114.12	50.4
27	96,856	126	0.001301	25,942.72	480,109.35	32.14	3,080.37	49.4
28	96,730	126	0.001303	24,675.21	454,166.63	30.61	3,048.23	48.5
29	96,604	127	0.001315	23,469.59	429,491.42	29.38	3,017.61	47.6
30	96,477	127	0.001316	22,322.60	406,021.84	27.99	2,988.23	46.6
31	96,350	130	0.001349	21,231.64	383,699.23	27.28	2,960.24	45.7
32	96,220	132	0.001372	20,193.32	362,467.60	26.38	2,932.96	44.7
33	96,088	137	0.001326	19,205.35	342,274.28	26.08		
34	95,951	143	0.001490	18,264.73	323,068.92	25.92		
35	95,808	153	0.001597	17,369.06	304,804.19	26.42		
36	95,655	163	0.001704	16,515.54	287,435.13	26.80		
37	95,492	175	0.001833	15,702.29	270,919.58	27.41		
38	95,317	188	0.001972	14,927.15	255,217.30	28.04		
39	95,129	203	0.002134	14,188.30	240,290.14	28.84		
40	94,926	220	0.002318	13,483.83	226,101.85	29.76		
41	94,706	241	0.002545	12,811.98	212,618.02	31.05		
42	94,465	264	0.002795	12,170.83	199,806.04	32.39		
43	94,201	288	0.003057	11,558.88	187,635.20	33.66		
44	93,913	314	0.003344	10,974.80	176,076.33	34.95		
45	93,599	343	0.003665	10,417.24	165,101.53	36.36		
46	93,256	374	0.004010	9,884.83	154,684.29	37.76		
47	92,882	410	0.004414	9,376.36	144,799.46	39.42		
48	92,472	451	0.004877	8,890.45	135,423.10	41.30		
49	92,021	495	0.005379	8,425.80	126,532.64	43.17		
50	91,526	540	0.005900	7,981.41	118,106.84	44.85	2,357.27	28.4

So now you can refer to the table you refer to the table 49. So 49 his in the end and if you look at the Mx table it is you know 2400.44 so it is here so this will be 49. So if you come to 49 it will be 2400.44 and then/Dx. So Dx is coming as 8425.80. So you can calculate this is so this will be 2,00,000/M 49/D 49 so this will be as we calculate it is 2400.44/8425.80 so it becomes= 56978.33. So he must pay the premium of $56,978.

Now, so that after his death you know the person the survivor or his family member they receive 2,00,000 of dollars so that is how this is the single premium. Now the thing is that not every time you know you are in a position to pay this amount in a single stretch at one time. So what you do is you like to pay this amount you know for larger period of time say for your whole life like that. So in that case what to do. So that case becomes so as we know that in that case you have a situation like the annual premium.

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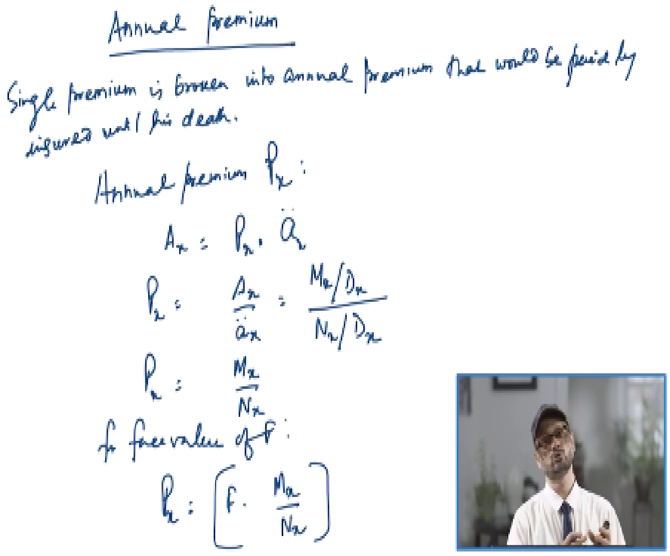

So you can pay the annual premium for the whole life basis. Now in this case so the single premium will be broken into the annual premium. So single premium is broken into annual premiums that would be paid by the insured until his death so you know so it is something the case of the whole life annuity due. So in this case you know you have to find these annual premium.

So this annual premium which you will be calculating for the whole life insurance that is Px. So for the it will be according to the you know formula this Ax will be Px* so this is a annuity due cases. So it will Ax that is a double dot. So that way you can calculate this Ax*=Px* a double dot x. Now Px will be basically Ax/a double dot x. So as we know that this Ax is we have calculated this is Mx/Dx.

And we have calculated this a double dot x earlier that is annuity due for that cases. Now in this case it will be Nx/Dx. So what we get Px as Mx/Nx so this is basically for the annual premium which he will be paying that is annual premium Px till his death he is paying and this is for $1.00 payment so this is Px=Mx/Nx this is for the face value of 1. If we have the face value for F so for face value of F you will be having this annual premium=F*Mx/Nx.

So you had earlier when you had calculated for the single premium it will be Mx/Dx whereas for the annual premium you will be calculating by the factor Mx/Nx and that will be divided with the face value that will be multiplied with the face value. So suppose for the previous problem where you had the single premium value was coming close to 56,900 or so. Now in this case if he is willing to go for the annual premium then how much he should pay.

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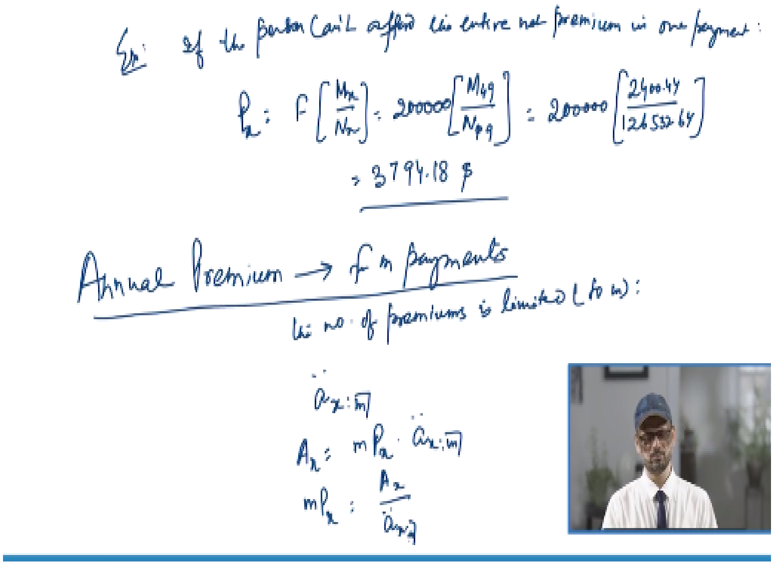

So for that suppose so if the person you know cannot afford the entire net premium in one payment so in that case you know he is going to have the value of Px and Px will be for the face value as we know that Px will be F* Mx/Nx. Now in this case F is 2,00,000 and Mx was M 49 and Nx was N 49. Now N 49 can be you know computed from the table and if you look at the N 49 values it is 1,26,532.64.

So it will be you know 2,00,000 multiplied by this is 2400.44/126,532.64. So that way you can calculate these values and it will be coming as 3794.18. So basically what you see that you know for earlier cases where you had to you know pay $56,978 which was quite a huge amount single time. Now if he wishes to pay for the whole life in that case he has to only pay 3,794.18 every year.

So that he can be sure that after his death the survivor will be getting Rs. 2,00,000 so that is how it goes for such cases. There will be another case for the you know m payment basis. So basically what happens that you know the person wants to pay only for certain period and then he does not want. So that is known as the annual premium and m payment basis for m payments. So that is means the insured person wishes to pay only m times.

He will be paying so the number of premium is limited you know to m in spite of I mean instead of paying till his death he wants that he should pay m number of times certain premium amount and then he should stop and then after his death so you know the benefit should be going to his survivors. So in that case it is for the m payment basis. Now the m will be the number for which the premium should be paid.

And it will be something like a case of temporary you know life annuity due. So in that case you will have the situation like the temporary life annuity due will be so this will be m. So a double x m that is for that m payments you will be doing and an annual premium you know will be m Px. So ax is basically will be m Px* then you have a double x you know m so that is how you have to calculate.

Now in this case you know m Px what you get for the m payment you know m Px will be Ax/a double dot x so for m payment. So now you have to simplify this expression.

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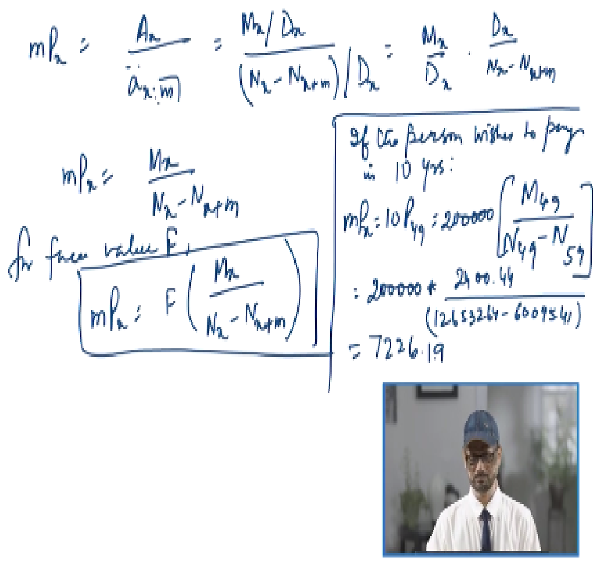

So if you see that your m Px will be Ax/a double dot x m. So what you do is you have this expression m Px=Ax/a double dot x you know for m payments. So you can further write it as Ax can be written as Mx/Dx that is what we have earlier calculated and then this Ax m so that we had earlier seen in the earlier lecture this was Nx-Nx+ m and then it will be/Dx. So that you should refer you know in the earlier lecture.

So here we could not do that much of a deviation but you can find it Nx-Nx+m/Dx. So it will you can further write it as Mx/Dx* Dx/Nx-Nx+m. So you know if you cancel the Dx out you can write m Px it will be Mx/Nx-Nx+m. So this is for the normal face value. So for face value F you will write you know m Px will be F*Mx/Nx-Nx+m. So this is how you are going to calculate the value in case of you know when you have to only pay for m payments.

And you want certain face value. So suppose in the earlier example again as you saw that when you have single payment that time it is coming close to 56,000 or 57,000. Then if you have the whole life payment it is coming out close to 57,000 or so. Now if you want to pay so if the person so if the person wishes to pay him 10 years now in that case what happens that you will be finding this m Px that is m is 10 so that is m Px that is 10 P 49.

Now this will be 2,00,000 multiplied by you know Mx so m will be 49 and then/N 49-N of 59 N59. Now you can refer to the table and if you look at the table M49 so M49 will be 2400.44. Similarly, you have Nx table is there so from here you can refer to the values and you will get the values like 200,000 multiplied by M49 is coming out to be a 2400.44 and/N49 is 126532.64-60095.41. So this way you are getting 7226.19 means the person is.

And this is for the 5% interest rate let us know that you know that let us be sure about it that this is for a certain interest rate R. If the R is changing these values will all be changing. So it means that is the person is willing only to pay for 10 years assuming that after that he will not pay any premium and then he has to ensure that after the death the family should get some amount in that case he may pay $7226.19 for next 10 years so that he gets Rs. 2,00, 000 in the end.

So that is what the case of the you know annual premium which is basically you are giving only m payments. So only m payment after that you are not paying the premium.

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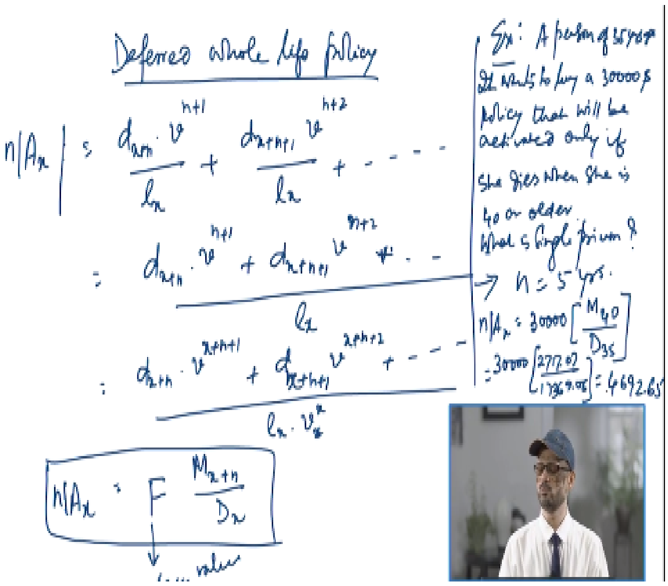

Then another case which becomes important is the deferred whole life policy. Now deferred whole life policy here basically what happens that it is based on the notion that the insured person will not die until certain period of time and the net single premium of this policy will be basically determined by that. So he will be assumed that he will be alive for certain period n and in that case you are calculating the net single premium.

So that time for which it is thought that he will not die that is known as a deferment period and the single premium which is calculated that is denoted by you know n/Ax. So that is the n is the deferment period in that case and it will be calculated as so this will be it is given like this. So that will be dx+n vn+1/lx. So basically what we had done started with you know n as 0 in earlier case where we had single premium it was dx/lx*v1.

So here you are assuming that for n years it is deferring the payments after that he will pay. So dx+n/lx* vn+1. Similarly, dx+n+1 vn+2/lx and that will go. So you know in that case you have to compute the values. So what you will get you can further have dx+n vn+1+dx+n+1 vn+2 like that. And in the denominator as you know it will be only lx. Now you again in this case what you are going to do is you are going to multiply with the term v raise to the power x.

So if you multiply in the top and the bottom with vx so it will be dx+n and vx+ n+1. Similarly, dx+n+1 vx+n+2 and so on and then since you had multiplied with vx on the bottom also it will be lx*vx so that is dx basically that we know that lx*vx it will be you know vx basically it is so it will be dx. Now this summation will be M of x+n so this will be Mx+n and this will be/Dx.

So this nAx this value basically will be coming as Mx+n/Dx and if you are going for the face value of F so that will be the value of the single premium you know when it is deferred for n periods face value. So suppose if you have to now see that you have to calculate this cases. Suppose one example is there that you know a person is 35 years' age. Now it wants to buy so they want to buy basically a $30,000 policy.

So now that will be activated you know only if she dies you know when she is 40 so that will be activated only if she you know dies when she is 40 or older. So what will be the single premium so what is single premium? Now in such cases a person who is of 35 years' age and it wants to buy a 1000 policy and it will be dying. So that will be activated only then. So in that case the period of deferment is here 5 years.

So n is 5 years' period of deferment and in that case if you find the nAx so it will be face value that is 30,000 and then it will be multiplied with Mx+1/Dx. So it will be M of 35+5 that is 40/D of 35. So you can further refer to the table M 40/D 35 and if you calculate these values you are going to get 30,000 and you refer to the table it will be 2717.07/1739.06. So basically you are getting 4692.65.

So this way you know you can calculate you know the net single premium when the it is deferred for n periods and you get the $4,692.64. So we will also discuss about the different you know cases which fall under the insurance policies in our coming lectures. Thank you very much.