Financial Mathematics
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Lecture – 57
Pure Endowment and Life Annuities

Welcome to the lecture on Pure Endowment and Life Annuities. So we will be talking about pure endowment cases and also the types of life annuities and we have discussed in the last class last lecture in fact about the mortality table and we also had introduction to the concepts of mathematical terms related to insurances. So coming to the definitions or the description about the pure endowment cases or the life annuity case.

We must be conversant with certain terms computation terms and these terms you know these values are given in the mortality table itself if you refer to the mortality table which is basically you know which is this one. So here we know that in this case you have this as the age then as the number of persons who are live at certain age then this is the table lx is that and dx is telling about the deaths which are occurring so based on that.

And depending upon the death you know you can calculate the probability of that deaths so that is qs then you have all these other terms like Dx, Nx, Cx and Mx these are the computation terms and you must know that how they are computed.

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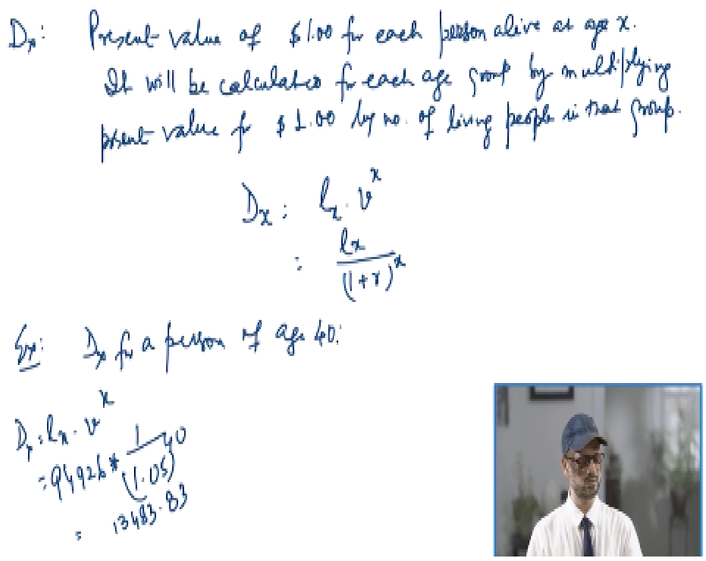

So the first term is Dx so this Dx is shown here. Now this is basically the present value of suppose $1.00 you know for each person at age x. So this is present value of $1.00 for each person alive at age x. So for each person alive at age x what will be the present value. So know that for finding the present value you have to multiply with the present value factor and if you have something in the future.

And if you have to find its present value then you have to multiply with the factor 1/1+r raise to the power n. So n will be the time you know periods where r is the interest. So based on that we calculate this you know Dx so Dx is the present value of $1.00 for each person alive at age x and so it will be calculated. So it will be calculated for each age group by multiplying this is present value so what will be the present value for $1.00 by number of living people in that group.

So what you do is you know the number of living people in that group that is lx and that will be multiplied by the present value of $1.00. So you can find so you can define this Dx as the lx*vx so that is your present value of $1.00 for certain age so that will be vx. So you can also write it as lx and it will be nothing but 1+r raise to the power x. So basically you can if you refer to that table you can see that this table and so you require to have a certain value of r.

And our table which is this one this is based on 5% interest rate. So for 5% interest rate you can calculate the you know Dx for 0% suddenly we know that it will be 1,00,000 * (()) (05:18) so it is same so it will be 1 itself so it is 1,00,000 but otherwise you can calculate it. For example, if you are going to calculate this Dx you know for a person at age 40 so Dx for a person of age 40 so for age 40 if you look at you have to refer to the table.

And you have to find the lx that is number of persons who are alive at 40 age and that is given 94926 so it will be Dx will be lx* vx. So lx is nothing but 94926 and then it will be multiplied with the factor 1/1+r that is r is 0.5% so 1/1+ 0.05 so 1/1.05 raise to the power 40. So that way you have 1/1.05 raise to the power 40. So this way you will get the value of 13,483 and 0.83.

So if you look at the table by looking at the table in this case of 40 if you go to the Dx you know value it will be 13,483.83 that is what you can find these values from the table.

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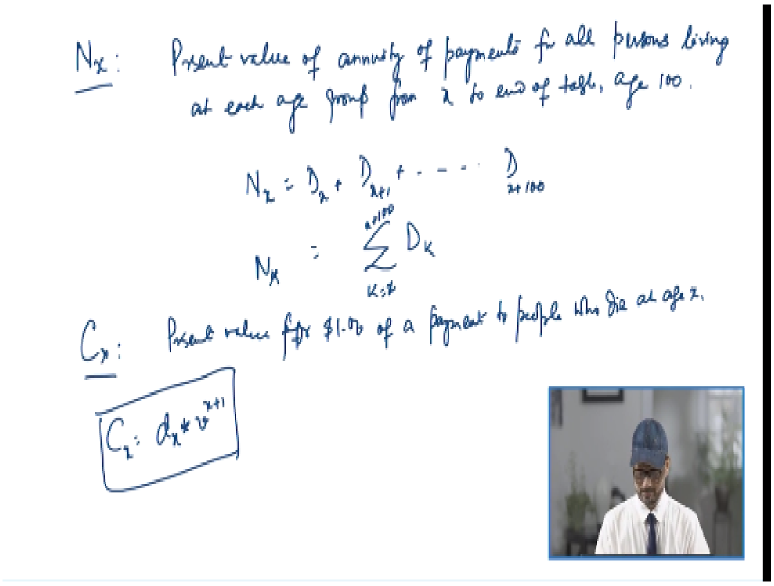

Next term which will be useful you know will be Nx it is the present value of annuity of payments for all persons living at each age group. So basically we are you know taking a summation so we are going to calculate that for the each age group from x to end of table so basically we are going to calculate from x to the end of the table and we know that the table is ending at the age of 100 so we are going to go up to age 100.

So basically it is nothing but the summation of D which has we have computed it will be calculated as the sum of those values. So Nx will be basically a Dx+ Dx+1 like that it will go up to 100. So it is written as something like k=x to x+100 and you are going to find the Dk so this is how you are going to calculate these Nx values like you have you know for every person of the 70 years you calculated annuity of each person is suppose it is computed as 500 per year.

Now in that case what you do we are going back to that age group and we are calculating we are calculating this annuity by multiplying with the N70 and N70 can be computed from the table. So if you calculate from the table so if you refer to the table that will be your. So what you do is basically you are so this is your N and it will be calculate so you have to have the value of all this you know summed up.

So for 70 if you go to the next one. Now for 70 it is coming out to be something like 214 you know 62.35. So it is nothing but the summation of all these you know Dx after from here from 70 onwards. So if you sum them it will be coming as 214 27 we can even get the you know things clarified from this table also and as you see that it will be 25.42+ 12.76 is coming out to be 38.18.

So 38.18+ 18.32 it will be coming as you know 56.60. So basically it is a summation from any value Dx and it will go till age 100. So it will be giving you the sum of all those values then the next computation term is finding the Cx. So what is Cx in that case so Cx will be basically the present value of $1.00 so it will be present value of 1.00 so this is for $1.00 of payment you know to people or beneficiary who die at age x.

So what you do is that you know you multiply that the number of people basically who will die at age x suppose and that will be multiplied with the present value of $1.00 at the time x+1. So what you do is you calculate the Cx as the number of people who are dying at age x and then it will be multiplied with the vx+1. So basically this is the present value of $1.00 so you are going to give you know after that you are going to give the you know the payment.

So you have to multiply with the factor so it is Cx will be computed as dx* v raise to the power x+1. So you can find so if you look at the tables this Cx table it will be talking about these values so your Dx that is number of deaths and then that will be multiplied with these values if you look at the Dx it will be here and you can for you know 0 it is certainly so it is coming as 1200 that way.

Then what happens for the; this way you are going to compute you know these value 1260 multiplied by 1/1.05 so that gives you 1200 but after that your other values are slowly changing. So basically this value can be computed at any points.

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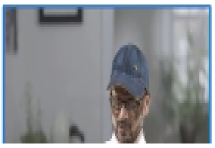
M_x: Summation of all C_x : It represents present value of \$1.00 payment for all people who eventually die, but are still alive at age x.

$$M_x = C_x + C_{x+1} + \cdots C_{x+100}$$

$$= \sum_{k=x}^{x+100} C_k$$



Then one of the other term which is used also that is Mx and this Mx is basically the summation of all the Cx. So as we see that the Cx present value of the for \$1.00 of a payment of people who die at age x. So in that case you are talking about you know the person who are dying and then you are multiplying that with that factor. Now you are summing them all you know from there up to age 100.

So that way you will be finding the value of Nx. So if you can write it represents present value of \$1.00 you know payment for all people who eventually die but are still alive at age x. So what you do is just like Nx here also you are going from age x to age 0 in fact. So what you do is Mx will be Cx+Cx+1 so like that it will go up to you know 100. So basically you are finding k=x x+100 and then you are finding the Ck.

So that way you find these Mx terms. So all this so if you look at so what we mean to say that if you have these table you are knowing these computation terms like Dx, Nx, Cx and Mx and we have to refer these values when we discuss about you know the endowment or the life annuity values. So now we will come to the you know the value of the or the diffusion of the endowment.

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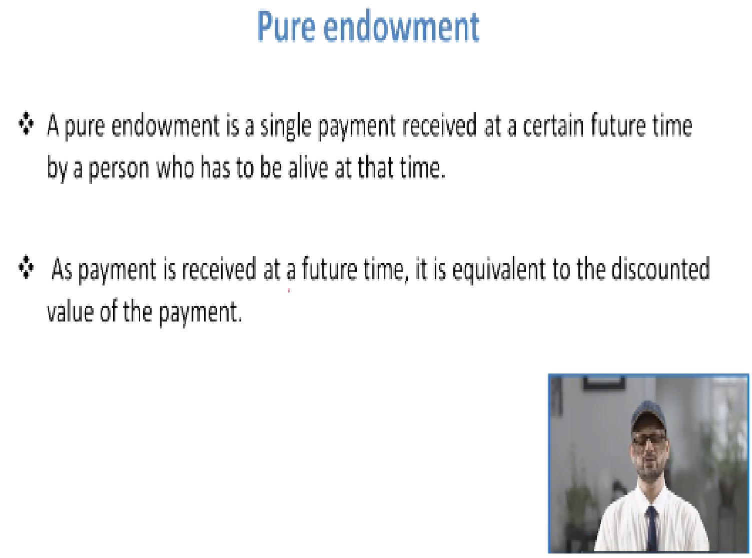

Now what is endowment so endowment a pure endowment is single payment received at a certain future time by a person who has to be alive at that time and I mean since the payment is received at a future time so certainly you will have to multiply with the factor to find the its present value. So here it is equivalent to the discounted value of the payment so that way we find this you know in pure endowment cases. Now suppose if nEx is the contribution so far.

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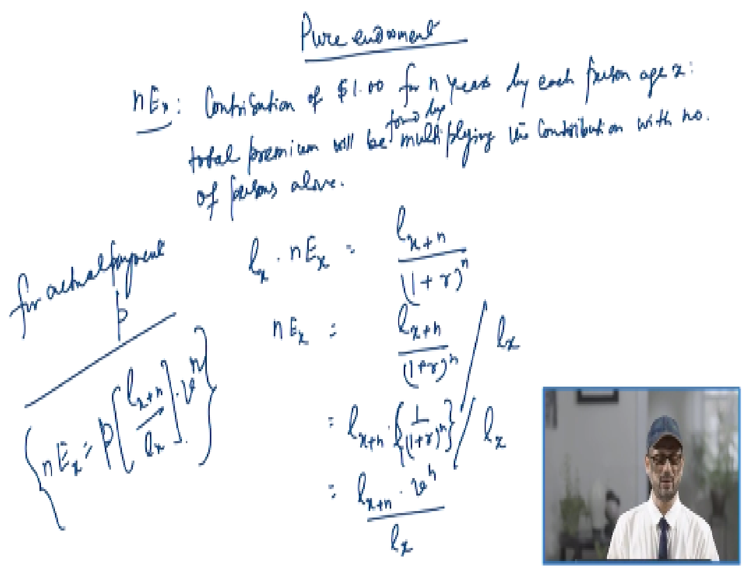

Suppose for the pure endowment cases so for pure endowment suppose if the nEx if it is the contribution of $1.00 for n years by each person age x. So in that case the total premium you know will be multiplying this contribution by the number of people. So you know so if you look at the total premium will be multiplied will be found by multiplying the you know these contributions the contribution with number of persons which are alive.

So multiplying the contribution with number of persons alive. So in that case your number of alive persons alive at any age we know that is the table under the column lx so you are to find you know so what you do is that if you (()) (18:16) introduce that if you are discounting you know into age x in that case and if you are receiving at x+ n. So in that what you do is you get lx* nEx.

So because you are going to for n year so lx* nEx so it will be basically lx+ n and divided by with the factor 1+r raise to the power n. So this way you will be finding these values after discounting. So if you try to find the value of you know nEx. So nEx will be basically lx+n/1+r raise to the power n and then it will be/ lx. Now this can be further written so this 1+r raise to the power n.

So you can write that lx+ n and * 1/1+r raise to the power n and then it will be/lx. So this factor as you know 1/1+r raise to the power it is v raise to the power n that is the factor. So you can write you know lx+n * vn/ lx. So this is for a hypothetical value of $1.00. Now if the actual payment you know if you have to calculation for the actual payment p. In that case you have to multiply this with the value p.

So in that case so if you try to calculate for the you know other actual payment p. So for actual payment p you have to multiply that with that payment so it will be nEx it will be p times. So you have to multiply that with p and then you have lx+n/lx and then you are multiplying that with vn. So this is how your nEx which is the contribution of you know for the n years by each person of age x. This can be computed. This value can further by simplified.

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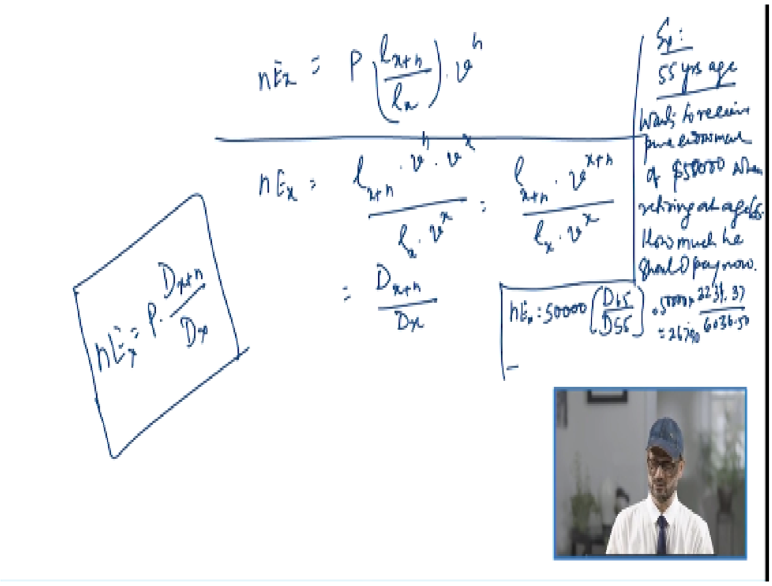

So if you look at the expression that is nEx so you can so write it as you know P lx+n/lx* v raise to the power n. Now that can further by simplified and nEx if you write you can write it as lx+ n *vn* vx/lx *vx. So we are not taking you know at this time as P or you can further simplify this as these values. So this will be like lx+ n and it will be vx+ n and then in the bottom it will be lx*vx.

So you can write now this is basically lx*vx it becomes you know if you recall that becomes=Dx. So you can write it as Dx+ n/Dx. So for the payment P you can write that you know nEx for the actual payment P will be P* Dx+n/Dx. So at any point of time if you have to calculate this nEx it will be P times you have to refer to the table D x+n/ Dx and you can find the value of nEx.

So if you have you know you can you know you can refer to you can solve the problems based on such data if you have if you are asked suppose a person suppose one example is there that for a 55 years' age old person now he wants to you know receive a pure endowment so wants to receive pure endowment of $50,000 and he is so this is 50,000. Now he is 55 years' age and he is you know after retiring at so when retiring at age 65.

So in that case how to find you know how much he has to pay now. So how much he should pay now. So in that case what is there that you are putting some amount and then he will be getting may be after 10 years so your n is 10 x is 55 so x+n is 65 and in that case if you have to calculate you will calculate nEx as P times so it will be P is 50,000 so that is what he wants to receive.

And then it will be D of so you have to calculate this D of 65/D of 55. So directly you can use this table and if you look at the table so D is 65 so D 65 will be here so D 65 is coming out to be 3234.37 so that is what it is 3234.37 and D 55 will be here. So this will be 6036.50 so if you take 50,000*3234.37/6036/50 so that will you this value so it will be 50,000/234.37/6036.50 so it is coming out as 26,790.

So like that you can calculate you know these values. Now the next thing which we wish to get introduced will be life annuities. Now this we are going to discuss something about the single life annuities

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Life annuities

- Single-life annuities are for a single living person and they cease when the person dies. There are mainly of two types: whole life annuities and temporary life annuities.

- Whole life annuities are paid to the annuitant as long as he or she lives. It is under three categories: ordinary, due, and deferred.

- Temporary annuities pay only for a certain contracted period of time, given that the annuitant is alive throughout that period. Temporary life annuities are classified into ordinary, due, and deferred.

And single annuities are for a single living person and cease when the person dies. So they are normally of 2 types one is whole life annuities and another is temporary life annuities. So the whole life annuities as we know the name indicates it is paid to the annuitant as long as he or she lives. It is under 3 categories ordinary, due and deferred. Similarly, you have temporary annuities as contrary to the whole life annuity where it is for the whole life it will be for a certain contracted period of time given that he is alive through that period.

And again that also is classified into ordinary and the due and the deferred. So we will have some introduction about these and we also try to you know find the value of the endowments in such cases. So coming to the you know whole life annuities if you come to the whole life annuities so as we know that in whole life annuities you are going to so the person is going to pay that annuitant is going to be paid as long as he is living.

And in that we are having you know the different types of this whole life annuities and among them you have you know ordinary whole life annuity.

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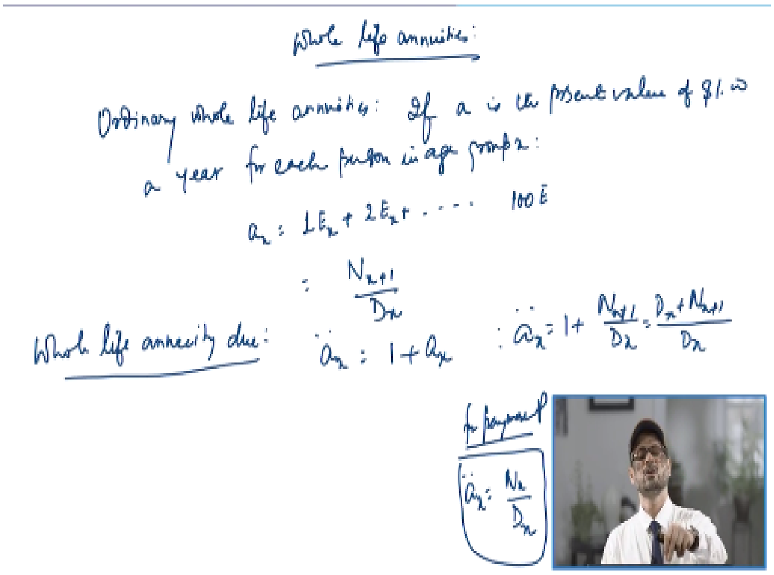

So if you go to the whole life annuities. So here you have ordinary whole life annuities so here you have ordinary whole life annuities. So in this case basically receipt of payment will be at the end of every year and there will be series of individual you know pure endowment. So in this case if suppose if a is the present value of $1.00 each year for each person for one year for each person in age group x.

So the collective endowment in that case will be you know 1 Ex+ 2 Ex+ something like so it will go up to 100 E. So that way you can find these collective endowment values and in this case if you do the simplification and if you do the computation so you can write further Ex as so it will be Dx/Dx+1/Dx so it will be further Dx+2/Dx like that it will go. So basically what is happening if you calculate this as Nx+1/you know Dx.

Because in the numerator you have Dx+ Dx+1+ Dx+2 up to 100 so its sum is Nx as we have found out and that will be Nx+1/Dx. So basically by using this formula you can find the endowment values in such cases. So this is the case of ordinary whole life annuities. Now if you have there is example of there is another type of this that is whole life annuity due. Now what this annuity due means that you know in this case the first payment will be made at the time of purchase itself.

So that is why it is known as the annuity due. So you are making one payment while purchasing itself. So in this case what happens that you are making so that is why we define it as 1+ ax so because we are making one payment at the time of purchase itself and this is for the whole life annuity due and this is for the ordinary whole life annuity and in that case you are adding 1.

So basically what you do is in this case if you further simplify you will have a double dot x it will be 1+ Nx+1/Dx so basically it will be Dx+ Nx+1/Dx. So basically you are adding all of them and you can further find. So if you do the simplification basically for payment P so if you have you know for payment P you can calculate the a double dot x it is as Nx/Dx. So basically you are showing a summing.

So you are once you are adding as you know that Nx is the summation of Dx so everytime. So once you sum this, this will be Nx. So basically in this case it will be so while in that case it is Nx+1/Dx it will be here in the case Nx/Dx. So that is how you calculate these whole life annuities due. Similarly, you can have the annuity deferred to certain you know certain time certain time period N.

So that is the case of you know annuity you know deferred whole life annuity so in that case you are deferring the payment.

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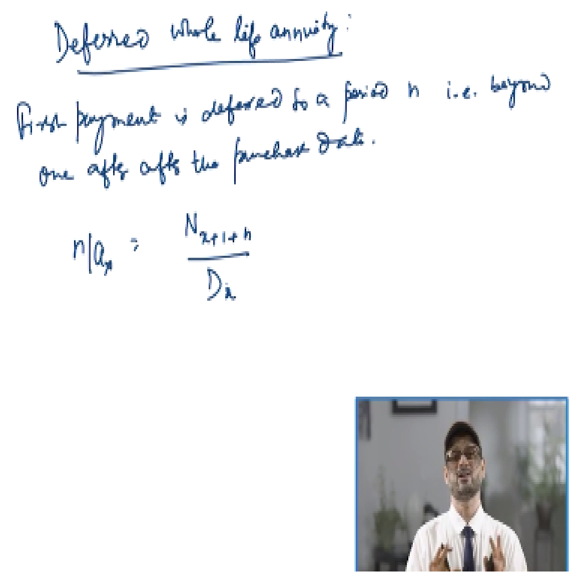

So this is deferred whole life annuity. So in this case basically you are so first payment is deferred to by the period N. So first payment is deferred to a period N that is beyond 1 year after the purchase date. So in this case you know purchase is occurring at age x. Now in this case your first payment will be deferred by certain time so it will be x+n it will be going like you know so in that case so x+1+n it will be otherwise it is after the first year.

So then also it is n years of deferment. So it will be x+1+n. So in this case we define this value as n ax that is deferred by period n and in that case it becomes N x+1+n/Dx. So in normal ordinary whole life it is N x+1/Dx. Then you have the annuity due, you have Nx/Dx and if you are doing the deferring you know to a period of N then it becomes Nx+1+n/Dx. So that is how this values are computed.

So this is about the whole life annuity then we have temporary life annuities. So as you in this case it is for as the whole life annuity it is the payment is made for the whole life. Now in this case it is not for the whole life.

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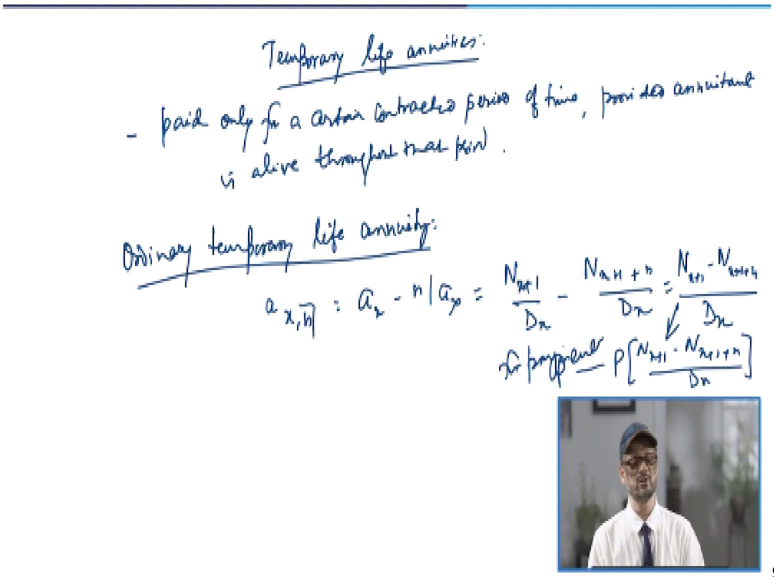

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So this is temporary life annuities. So it is only paid for a certain you know contracted period of time. So paid only for a certain contracted period of time and there also you have a condition provided annuitant is alive throughout that period. So in this case also when the person will die the payment will cease. Now for the ordinary temporary life annuity now in this case as we know basically in this case you know if the term is annuity is n.

And the present value of the temporary life annuity is a you know x n and which is shown by this so this is for the temporary life annuity ordinary temporary life annuity. So it will be basically the difference between the whole life and the deferred whole life annuity. So it will be ax- n ax. So as we know that ax is coming out to be n x+1/Dx and then for the deferred annuity you have n x+1+ n/Dx.

So basically it becomes=Nx+1- Nx+1+n/Dx. So this is how you are computing for the ordinary temporary life annuity and if it is for payment P then you are going to certainly multiply you know P here so it will be P* Nx+1- Nx+1+n/Dx. So this will be for payment P. So similarly you have also the cases like I mean different types of temporary life annuities also.

And you have the deferred also temporary life annuities which has the different expression and you can study and have you know the idea about these annuities like if you find the for (()) (37:16) of annuities it will be Nx-nx+1/Dx+1. Similarly, if you have the you know deferred temporary life annuity it will be Nx+k so (()) (37:31) Nx+k- Nx+k+n/Dx. So this way you calculate this (()) (37:40) type of temporary life annuity and the deferred life annuity.

And you can calculate all this values by referred to the tables. So this way you can you know have the idea and do more practice to have you know the control over these things. Thank you very much.