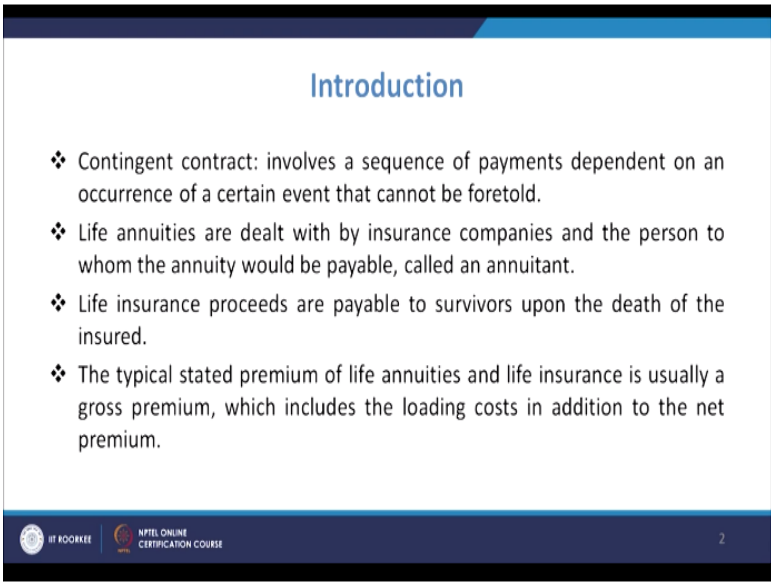
Financial Mathematics
Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology - Roorkee

Lecture - 56
Introduction to Insurance, Mortality Table

Welcome to the lecture on introduction to insurance and also in that we are going to discuss in this lecture about the mortality table. So basically all we know that insurance is very important you know part in today's you know world and we must know you know the related terminologies related to insurance. Before that we also discussed about the annuities you know in the earlier chapters this annuities different and in the case of insurance that annuity that is certainly different.

(Refer Slide Time: 01:14)



So the life annuities are normally related to the life insurance in that way. Now you have basically the existence of contingents, contingent contract which will be involving the sequence of payments dependent on an occurrence of a certain event that cannot be foretold. So basically we talk about the insurance in the sense that we ensure ourselves or any person goes for insurance you know.

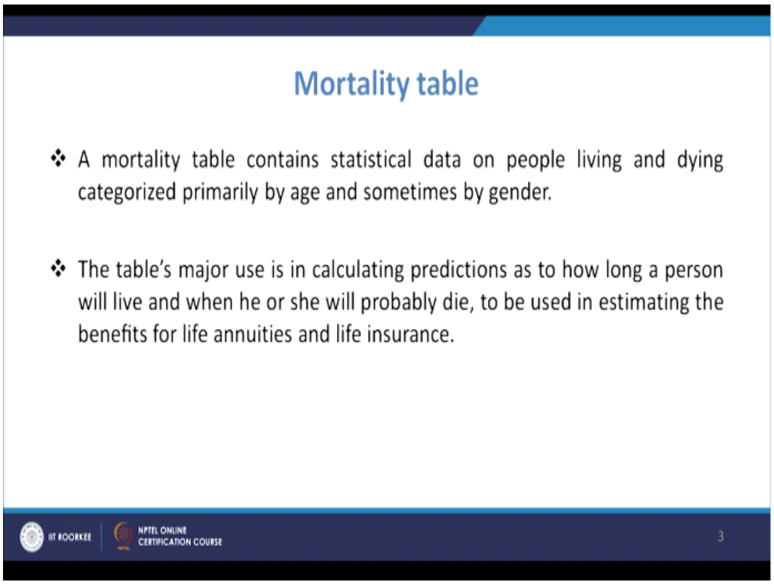
For that I mean keeping in mind that something you know something may happen you know people may you know meet with someone to an incident, there may be casualty, there may be death or so. So in that case what we do is normally we make a contract and what we do is under that we use to pay sequence of payments and because we expect that after that you know dependent of the person must be paid something.

So that is you know the contingent contract is developed and as we know that the life annuities they are dealt with by the insurance companies and the person to whom the annuity would be payable. So basically you have an annuitant I mean so the annuitant will be paying that annuity of the year he has to pay or it may be net you know single time also he can pay and then life insurance proceeds or basically they are payable to the survivors.

So the thing is that certainly you have the risk involved in life and there is I mean certainly there will be some probability that if suppose something happens with the person if he meets certain accident or in case of his death you know it will be so proceeds should be payable to the survivors. So, typical stated premium of life annuities and life insurance is usually a gross premium which includes the loading cost in addition to the net premium.

So certainly you have the loading cost also involved so in that. Now once we are going to discuss about it, so certainly we know that in that case we must know that what will be the chances of you know person meeting with that you know event that is foretold that we do not know or he is meeting with accident or death. So in those cases you should know something like the statistical you know must have a statistical table.

(Refer Slide Time: 04:14)



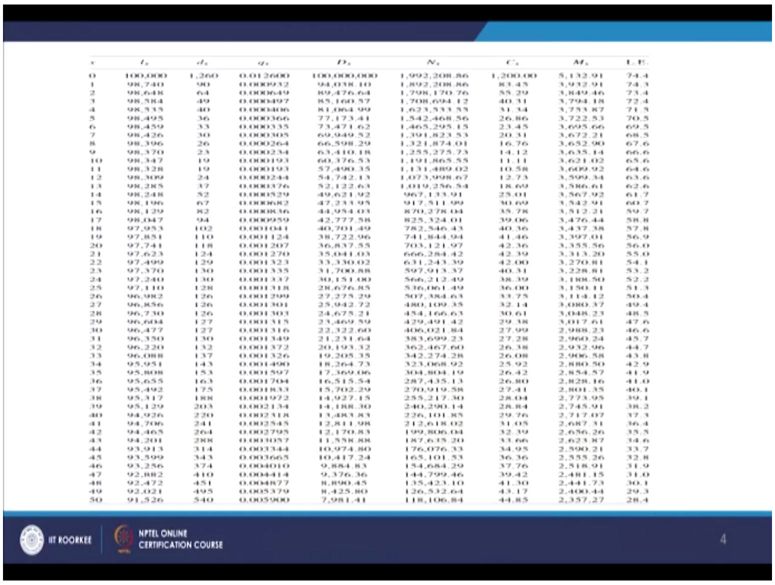
And that is basically predicted by the mortality table, so the mortality table basically it will be containing the statistical data on people living and dying categorized primarily by age and sometimes by gender also. So in that basically you will have the survey, it will be seen that and on that you know on a specific number of people of starting from a zero or so you can have the statistics that every year what has been, how many of the infants supposed have crossed and are raised to age 1.

Then, how many have gone to age 2 so like that you know there was a survey has been carried out and based on that on an average the mortality data will be you know stored and mortality table will be made and this table's measure use is in calculating the predictions. So it will be predicting as to how long a person will live and when he or she will probably die. So that is to be used in estimating the benefits for life annuities as well as the life insurance.

So this table will be you know referred to know all these values and then that those values will be used for calculating the value of the annuity and so. So when we talk about this table, now this table was prepared you know in 1868 so it was published in the American Experience Table of Mortality and it was this table is referred for I mean taking the data for the one lakh number of child.

And then it was seen that year wise what was the death, so how many are surviving and based on that you will have certainly some kind of you know terminologies which will be coming up. So if you refer to the table that table basically is the mortality table which is going like this.

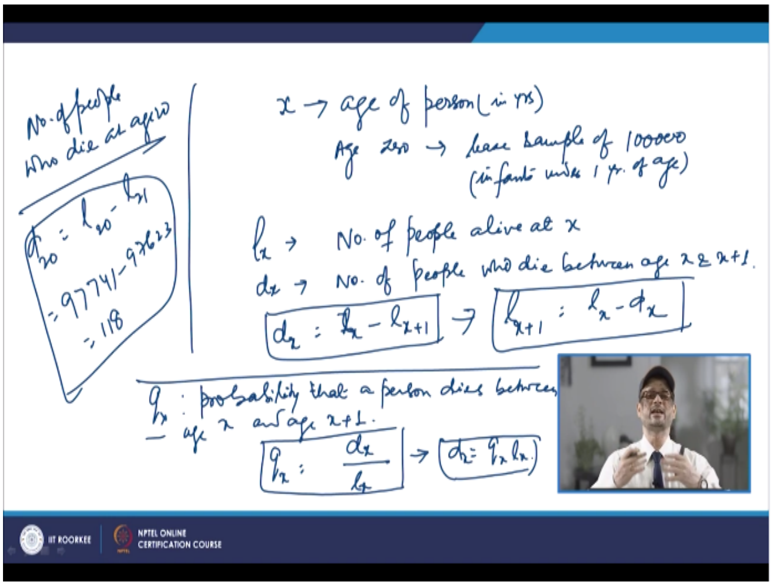
(Refer Slide Time: 06:52)



x	l_x	d_x	q_x	D_x	N_x	C_x	M_x	L.E.
0	100,000	1,260	0.012600	100,000,000	1,992,208.86	1,200.00	5,132.91	74.4
1	98,740	90	0.000932	94,038.10	1,892,208.86	83.45	3,932.91	74.3
2	98,648	64	0.000649	89,476.64	1,798,170.76	55.29	3,849.46	73.4
3	98,584	49	0.000497	85,160.57	1,708,694.12	40.31	3,794.18	72.4
4	98,535	40	0.000406	81,064.99	1,623,533.55	31.34	3,753.87	71.5
5	98,495	36	0.000366	77,173.41	1,542,468.56	26.86	3,722.53	70.5
6	98,459	33	0.000335	73,471.62	1,465,295.15	23.45	3,695.66	69.5
7	98,426	30	0.000305	69,949.52	1,391,823.53	20.31	3,672.21	68.5
8	98,396	26	0.000264	66,598.29	1,321,874.01	16.76	3,652.90	67.6
9	98,370	23	0.000234	63,410.18	1,255,275.73	14.12	3,635.14	66.6
10	98,347	19	0.000193	60,376.53	1,191,865.55	11.11	3,621.02	65.6
11	98,328	19	0.000193	57,490.35	1,131,489.02	10.58	3,609.92	64.6
12	98,309	24	0.000244	54,742.13	1,073,998.67	12.73	3,599.34	63.6
13	98,285	37	0.000376	52,122.63	1,019,256.54	18.69	3,586.61	62.6
14	98,248	52	0.000529	49,621.92	967,133.91	25.01	3,567.92	61.7
15	98,196	67	0.000682	47,233.95	917,511.99	30.69	3,542.91	60.7
16	98,129	82	0.000836	44,954.03	870,278.04	35.78	3,512.21	59.7
17	98,047	94	0.000959	42,777.58	825,324.01	39.06	3,476.44	58.8
18	97,953	102	0.001041	40,701.49	782,546.43	40.36	3,437.38	57.8
19	97,851	110	0.001124	38,722.96	741,844.94	41.46	3,397.01	56.9
20	97,741	118	0.001207	36,837.55	703,121.97	42.36	3,355.56	56.0
21	97,623	124	0.001270	35,041.03	666,284.42	42.39	3,313.20	55.0
22	97,499	129	0.001323	33,330.02	631,243.39	42.00	3,270.81	54.1
23	97,370	130	0.001335	31,700.88	597,913.37	40.31	3,228.81	53.2
24	97,240	130	0.001337	30,151.00	566,212.49	38.39	3,188.50	52.2
25	97,110	128	0.001318	28,676.85	536,061.49	36.00	3,150.11	51.3
26	96,982	126	0.001299	27,275.29	507,384.63	33.75	3,114.12	50.4
27	96,856	126	0.001301	25,942.72	480,109.35	32.14	3,080.37	49.4
28	96,730	126	0.001303	24,675.21	454,166.63	30.61	3,048.23	48.5
29	96,604	127	0.001315	23,469.59	429,491.42	29.38	3,017.61	47.6
30	96,477	127	0.001316	22,322.60	406,021.84	27.99	2,988.23	46.6
31	96,350	130	0.001349	21,231.64	383,699.23	27.28	2,960.24	45.7
32	96,220	132	0.001372	20,193.32	362,467.60	26.38	2,932.96	44.7
33	96,088	137	0.001326	19,205.35	342,274.28	26.08	2,906.58	43.8
34	95,951	143	0.001490	18,264.73	323,068.92	25.92	2,880.50	42.9
35	95,808	153	0.001597	17,369.06	304,804.19	26.42	2,854.57	41.9
36	95,655	163	0.001704	16,515.54	287,435.13	26.80	2,828.16	41.0
37	95,492	175	0.001833	15,702.29	270,919.58	27.41	2,801.35	40.1
38	95,317	188	0.001972	14,927.15	255,217.30	28.04	2,773.95	39.1
39	95,129	203	0.002134	14,188.30	240,290.14	28.84	2,745.91	38.2
40	94,926	220	0.002318	13,483.83	226,101.85	29.76	2,717.07	37.3
41	94,706	241	0.002545	12,811.98	212,618.02	31.05	2,687.31	36.4
42	94,465	264	0.002795	12,170.83	199,806.04	32.39	2,656.26	35.5
43	94,201	288	0.003057	11,558.88	187,635.20	33.66	2,623.87	34.6
44	93,913	314	0.003344	10,974.80	176,076.33	34.95	2,590.21	33.7
45	93,599	343	0.003665	10,417.24	165,101.53	36.36	2,555.26	32.8
46	93,256	374	0.004010	9,884.83	154,684.29	37.76	2,518.91	31.9
47	92,882	410	0.004414	9,376.36	144,799.46	39.42	2,481.15	31.0
48	92,472	451	0.004877	8,890.45	135,423.10	41.30	2,441.73	30.1
49	92,021	495	0.005379	8,425.80	126,532.64	43.17	2,400.44	29.3
50	91,526	540	0.005900	7,981.41	118,106.84	44.85	2,357.27	28.4

You have this as the year so this will be 0 and 1, 2, 3 or so and then you will have this lx that is number of people who are alive. So we have taken the sample size of 1 lakh and dx is the death which is given that year, so then you see that once it is 1260 so 987540 is remaining. So in the second again there is some death so based on that what is the probability of this death. Then other you know terminologies are also defined, so that we will see then.

(Refer Slide Time: 07:31)



So basically in that table what we do is we are basically taking x as the age of person in years. So we have taken age 0 so that is the base you know base samples at age 0 we have taken base sample of 1 lakh. So that way we are taking infants under one year of age. So from that number we are starting making the table, so under one year of age. Now lx any age so at any age x how many you know people are alive.

So number of people alive at age x, so they are basically represented by the term Lx. So further we take the dx. We have another column that is dx and dx will be number of people who die. So number of people who died between age x and x+1, so who die between age x and x+1. So that will be the dx, so basically if you look at the dx, dx will be basically defined as lx-lx+1 means the number of people who are alive of age x and number of people who are alive of age x+1.

So x+1 having I mean people of age x+1 will be smaller than the people of age x because there will be certain death in that period. So you know you calculate the dx like that. Now you can refer to the table and suppose you are told to find the number of people who die you know at age 20. So suppose you have to refer that value, so if you look at the age 20, now in that if you look at c, this is 118.

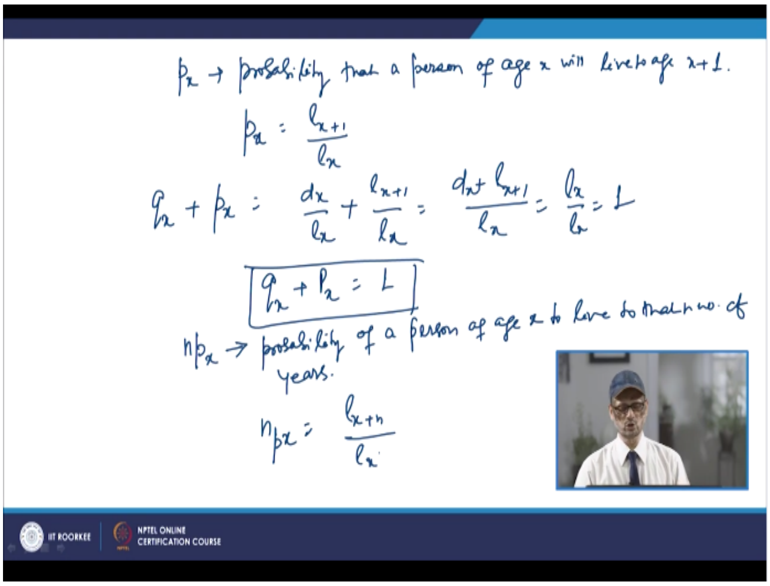
So this 118 it will be lx-lx+1, so that will be nothing but lx is 97741 and this is lx+1 that is l21 were of age 21 that is 97623. So 97641- so basically if you have to find you know the people number of people who die at so if you have to find the number of people die at age 20. So that you can find d20 and d20 should be equal to l20-l21 and we look at the table l20 is 97741 and 97623 so it will be 97 you know 741 and then it will be 97623.

So it will be 118 so this you can this way you can refer the mortality table to find and from here you can also find the lx+1 as lx-dx so we can if you are thinking of finding the number of people who are alive at age 21, so it will be l20 and if you know the number of person who are dying in age 20 so l20-d20 that you can find as the number of people who are alive at age 21 so lx+1 that way you can compute.

Now the second you know parameter which will be required to be known will be your probability that a person you know will die between age x and x+1. So if you are interested to find the probability that a person will die between age x and x+1 so that will be denoted by qx, so qx is the probability that a person dies between age x and age x+1. So that is your qx and it will be nothing but the qx will be basically the number of death at age x so that is dx/the number of people who are alive at age x, so it will be lx.

So qx will be divided by we found by dx/lx or you can find also from this expression dx will be basically qx*lx. So this is another expression which you know may be useful for finding that what is the probability that a person will die. Now similar to the probability that a person will die, you will have another term that is px and this is px will be defined as the probability that a person of age x will live to age x+1.

(Refer Slide Time: 13:43)



So px will be the probability that a person of age x will you know will leave to age x+1. So this will be basically you know px will be so px will be lx+1/lx means if because out of lx lx+1 you know persons are going to age x+1 so the probability that a person of age x is going to live will be living to age x+1, it will be px will be lx1/lx. So as you know that if you add this qx and if you add the px, so qx is the number of deaths during age x/lx.

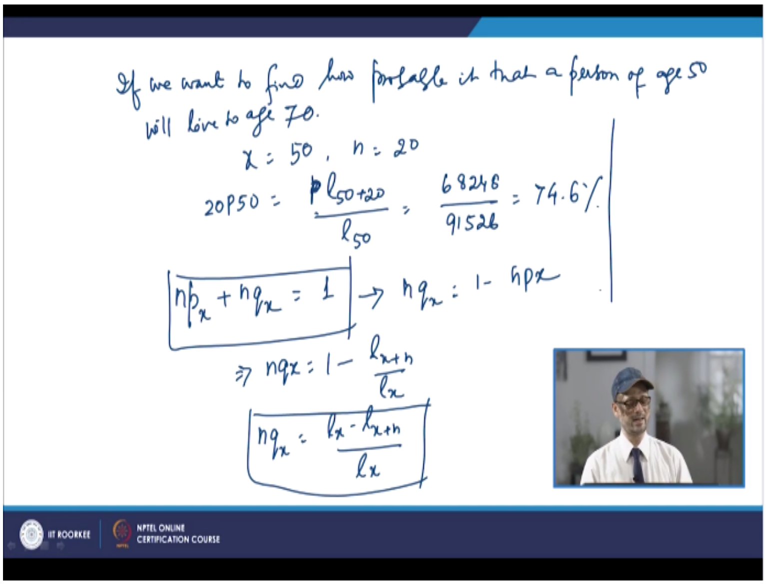
And the px is lx+1/lx so if you add them it will be dx+lx+1/lx so this becomes you know equal to so dx+lx1+1 is in fact lx. So this will be lx/lx that is 1. So what we find one of the expression is qx+px it becomes=1 so this is you know another expression which you can derive by from you know this table. Now we also see that many times we have to see that now from same thing you can further you know interpret these equations and you can find the values of qx as 1-px or px as 1-qx.

So this way the values can be found. Now so if the n is the number of years then the probability that a person will be living to the n number of years you know from now. So suppose today is he is of x age and if this person has to; so if you are thinking of the probability that this person will go to live to age x+n so that basically is denoted by the term that is npx. So npx is the probability of you know a person of age x to live to that n number of years.

So if you are talking about suppose someone is of 20 years and what is the probability that he will be leaving to age of 30, so that will be n will be 10 and x is 20. Now in that case, you can have the probability so that will be npx so npx will be you know lx+n so what is the number of people who are alive at age x+n and that will be basically divided by you know lx. So that will be giving you a term npx.

So whenever we have to find this you know probability we will use this term npx. So if suppose you have to find the probability that a person of 50 years' age what is the probability that he will be living to 70 years of age?

(Refer Slide Time: 18:11)



So in that case so suppose if we want to find how probable it is that a person of age 50 will live to age 70. So in that case, the person is of age 50, so we are taking x as 50 and we are to find the probability that this person will be living to age 70 so we are going for n as 70-50 that is 20. So we have to find the probability that this person of age 50 will be living to age 70. So for that this will be 20, so this will be 20p50.

So this is npx, now 20p50 will be p50 so this will be not p basically this will be number of persons at 50 years of age so that is plus 20 and number of people at 50 years of age. So this will be l70/l50.

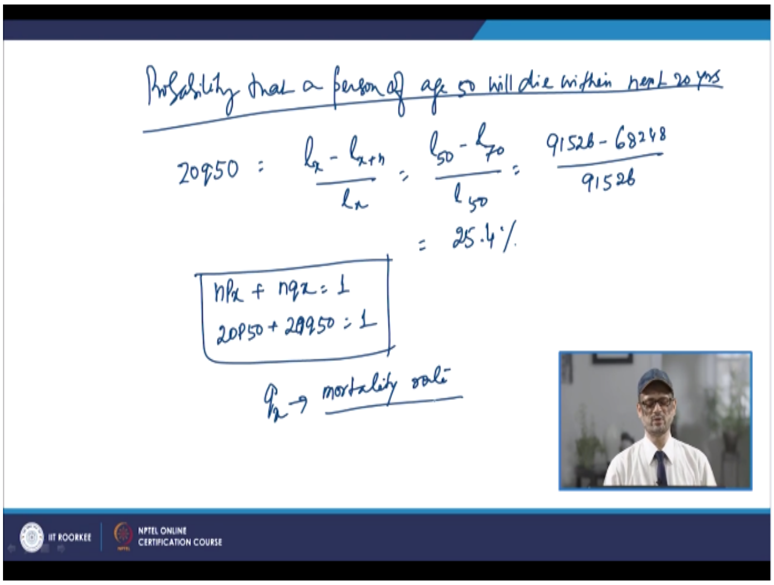
Now this data you can find from the mortality data, so you can refer to the number of people who are alive at 50 years of age and if you look at the table in the last row in the 50, the number of people who are living is 91526 and the number of people who are alive at is 70 will be 68248. So this will be 68248/91526. So this is coming out to be 74.6%. Now this 74.64 it is 74.6% probable that a person of age 50 will cross age 70.

So this way you are trying to find the value of the probability that a person of any age will live to how many years you know from there. So that way the probability can be found. Now npx and if you find the nqx, so what this you know if you add them npx+nqx that must be=1. So this is another you know formula which you get similar to npx you also have the nqx where nqx will be again you have to find the number of deaths in between.

So based on that you can find the nqx and if you add you know npx+nqx so it will be 1 which you can write as nqx=1-npx or you can also write nqx as 1-again npx will be l of x+n/lx, so you can write nqx=lx-lx+n/lx. So this is how you can calculate the value of the person. Now this will be talking about the probability that a person will die of age, a person of age x will not live you know till the age of x+n.

And for that the probability will be found you know by dividing this lx-lx+1/lx. So again if a person of age 50 is there and if you have to find the probability that this person will die in the another 20 years.

(Refer Slide Time: 22:33)



So probability that a person of age 50 will die this you know within next 20 years. So for that this will be nqx so that is 20q50. So for that this will be nqx so that is 20q50, so this 20q50 it will be so you can find this as lx-lx+n/lx. So it will be l50-l70/l50. So all these values you can again take from the mortality table l50 we know that it was 91526 and l70 was coming as 68248 and that will be divided by 91526, so it will be coming as 25.4%.

Now if you see that what is the probability that a person of age 50 will not die within next 20 years that is he will live for next 20 years was you know 74.6% and he will not live or he will die within the next 20 years is 25.4%. So you can say that if you know add both the probabilities that is npx and nqx, so that will be 1 that is your 20p50+20q50 so that is 20q50. It has to be 1.

So basically you must understand that what is you know meaning of these terms and times px so npx is not that n multiplied by px, it is basically there was a probability that a person of age x will go and live up to age x+n. Similarly, nqx is the probability that a person of age x will be dying in I mean before he reaches x+n. So that is what we must know that what are the meanings of all these terminologies.

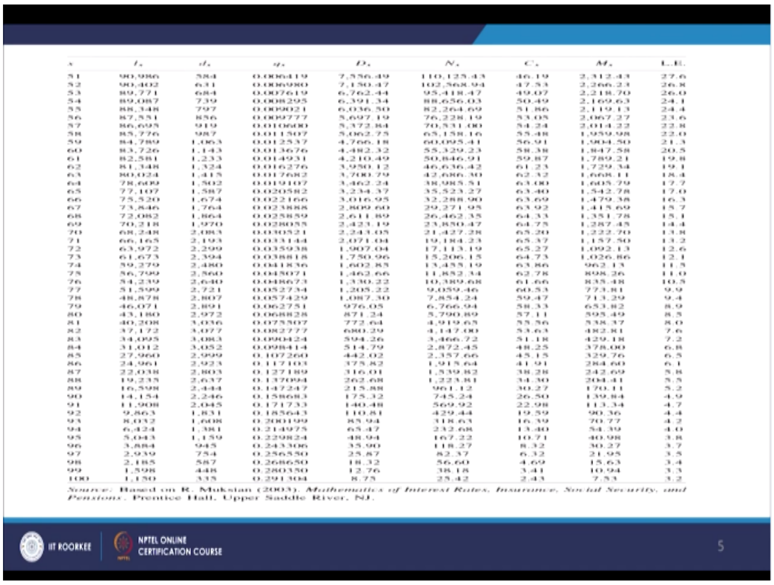
And they will be used basically for finding the terms you know as and when required in the case of you know terms calculated for finding these you know insurance annuities and other things. Now another thing is that what we define this term qx, basically this is known as the mortality rate. So this mortality rate you know based on that only we have the definition of this mortality table.

And what we intend to see also if you refer to the table. Now in this table as you see this table is for a certain rate of interest you know this is because we have other terminologies like dx, nx, cx, mx and then you have the life expectancy. So these terminologies like dx, nx, cx and mx they are the computation terms which will be used and basically they talk about you know that depending upon rate of interest that is chosen as 5%.

And as you know that this is based on you know source which is presented at the you know bottom of the table and these terminologies basically will be used for the calculation of the dx you know dx is here, nx is here, cx and mx and this is the life expectancy and then whole table will be used. So basically you can find you know once you have you can do for this analysis for any sample.

Suppose you can take any sample you just have the sample initially at 0 age and then once you have the data that how many dates are there in every year, so that will go into the next column and based on that you will have the value of these qx which will be you know calculated and if you look at these values now you see that it has started from you know from here, so this will be for 0 age and then in every year you have dx.

(Refer Slide Time: 27:43)



x	l_x	d_x	q_x	D_x	N_x	C_x	M_x	L.E.
51	90,986	584	0.006419	7,556.49	110,125.43	46.19	2,312.43	27.6
52	90,402	631	0.006980	7,150.47	102,568.94	47.53	2,266.23	26.8
53	89,771	684	0.007619	6,762.44	95,418.47	49.07	2,218.70	26.0
54	89,087	739	0.008295	6,391.34	88,656.03	50.49	2,169.63	24.1
55	88,348	797	0.009021	6,036.50	82,264.69	51.86	2,119.13	24.4
56	87,551	856	0.009777	5,697.19	76,228.19	53.05	2,067.27	23.6
57	86,695	919	0.010600	5,372.84	70,531.00	54.24	2,014.22	22.8
58	85,776	987	0.011507	5,062.75	65,158.16	55.48	1,959.98	22.0
59	84,789	1,063	0.012537	4,766.18	60,095.41	56.91	1,904.50	21.3
60	83,726	1,143	0.013676	4,482.32	55,329.23	58.38	1,847.58	20.5
61	82,581	1,233	0.014931	4,210.49	50,846.91	59.87	1,789.21	19.8
62	81,348	1,324	0.016276	3,950.12	46,636.42	61.23	1,729.34	19.1
63	80,024	1,415	0.017682	3,700.79	42,686.30	62.32	1,668.11	18.4
64	78,609	1,502	0.019107	3,462.24	38,985.51	63.00	1,605.79	17.7
65	77,107	1,587	0.020582	3,234.37	35,523.27	63.40	1,542.78	17.0
66	75,520	1,674	0.022166	3,016.95	32,288.90	63.69	1,479.38	16.3
67	73,846	1,764	0.023888	2,809.60	29,271.95	63.92	1,415.69	15.7
68	72,082	1,864	0.025859	2,611.89	26,462.35	64.33	1,351.78	15.1
69	70,218	1,970	0.028055	2,423.19	23,850.47	64.75	1,287.45	14.4
70	68,248	2,083	0.030521	2,243.05	21,427.28	65.20	1,222.70	13.8
71	66,165	2,193	0.033144	2,071.04	19,184.23	65.37	1,157.50	13.2
72	63,972	2,299	0.035938	1,907.04	17,113.19	65.27	1,092.13	12.6
73	61,673	2,394	0.038818	1,750.96	15,206.15	64.73	1,026.86	12.1
74	59,279	2,480	0.041836	1,602.85	13,455.19	63.86	962.13	11.5
75	56,799	2,560	0.045071	1,462.66	11,852.34	62.78	898.26	11.0
76	54,239	2,640	0.048673	1,330.22	10,389.68	61.66	835.48	10.5
77	51,599	2,721	0.052734	1,205.22	9,059.46	60.53	773.81	9.9
78	48,878	2,807	0.057429	1,087.30	7,854.24	59.47	713.29	9.4
79	46,071	2,891	0.062751	976.05	6,766.94	58.33	653.82	8.9
80	43,180	2,972	0.068828	871.24	5,790.89	57.11	595.49	8.5
81	40,208	3,036	0.075507	772.64	4,919.65	55.56	538.37	8.0
82	37,172	3,077	0.082777	680.29	4,147.00	53.63	482.81	7.6
83	34,095	3,083	0.090424	594.26	3,466.72	51.18	429.18	7.2
84	31,012	3,052	0.098414	514.79	2,872.45	48.25	378.00	6.8
85	27,960	2,999	0.107260	442.02	2,357.66	45.15	329.76	6.5
86	24,961	2,923	0.117103	375.82	1,915.64	41.91	284.60	6.1
87	22,038	2,803	0.127189	316.01	1,539.82	38.28	242.69	5.8
88	19,235	2,637	0.137094	262.68	1,223.81	34.30	204.41	5.5
89	16,598	2,444	0.147247	215.88	961.12	30.27	170.11	5.2
90	14,154	2,246	0.158683	175.32	745.24	26.50	139.84	4.9
91	11,908	2,045	0.171733	140.48	569.92	22.98	113.34	4.7
92	9,863	1,831	0.185643	110.81	429.44	19.59	90.36	4.4
93	8,032	1,608	0.200199	85.94	318.63	16.39	70.77	4.2
94	6,424	1,381	0.214975	65.47	232.68	13.40	54.39	4.0
95	5,043	1,159	0.229824	48.94	167.22	10.71	40.98	3.8
96	3,884	945	0.243306	35.90	118.27	8.32	30.27	3.7
97	2,939	754	0.256550	25.87	82.37	6.32	21.95	3.5
98	2,185	587	0.268650	18.32	56.60	4.69	15.63	3.4
99	1,598	448	0.280350	12.76	38.18	3.41	10.94	3.3
100	1,150	335	0.291304	8.75	25.42	2.43	7.53	3.2

Source: Based on R. Muksian (2003). *Mathematics of Interest Rates, Insurance, Social Security, and Pensions*. Prentice Hall, Upper Saddle River, NJ.

So dx will be going on subtracting and ultimately if you just look at the trend that in the later years this you know dx is increasing and at the age of say 100 what you see is that or before that basically it is increasing you know so it is basically initially it is there, so it is further then it is decreased in the age of 10, 11 or so. Then, further it is increasing and going on and then after that it is slowly increasing from the age of say 17 or 18 comparatively.

And then as you go further the increase is quite high as we cross the 70s of age that way it is increasing and it goes to maximum value of suppose say 3052 in the age of 84 and then further slowly it is decreasing because also that they are quite less number of people also. So that way you have this qx going on, if you find these qx probabilities, these probabilities are quite high.

If you look at these probabilities, probability is basically increasing as the age is increasing normally, the probability is basically increasing and it is coming close to the 0.9 or suppose say you know 0.3 times. So out of 1150 335 is likely to die. So basically that fraction you know of people who will not be alive so that part can be you know so that can be calculated and then how to calculate these computation terms.

So that we will discuss because that will be used for finding the annuity values, the single premium or how if the person has to pay in you know annually so how can he pay these premium values in you know annual basis or by differing for certain time. So there are many cases for those dx, nx, cx and mx will be how to calculate that, how to get these values, they will be you know where we will discuss in our coming lectures. Thank you very much.