Financial Mathematics
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Lecture - 55
Portfolio Diversification

Welcome to the lecture on portfolio diversification. So in the last lecture, we discussed about you know the assets X, Y or Z and W and we had seen that how you know if they are positively correlated and if they are negatively correlated then how you know risk or rate of return is affected and we have seen that normally you prefer to have the assets you know to be negatively correlated.

So that way you know in one case if there is you know the negative return then that can be compensated. So that was the situation in you know earlier lecture so in the previous lecture.

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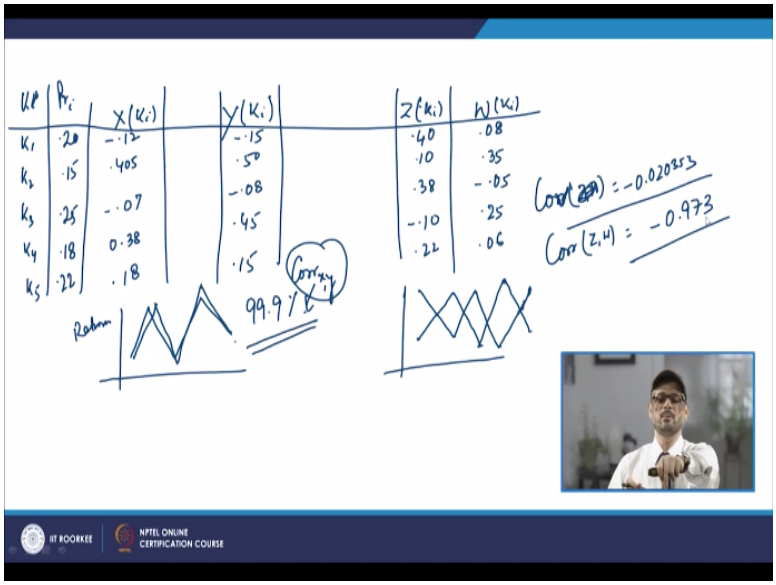

Now in the previous lecture if you recall what we have seen that you had two assets X and Y and similarly you had the asset Z and W and the probabilities were assigned and the probability values Pri was there, so that was for the Ki so for K1, K2, K3, K4 and K5 the probability values where you know 20% so it was and then 15%, 25%, 18% and 22%. So that was the probability values for the different returns you know that was yeah return values.

And if you look at these Ki values that is so this is you know K so Ki value for the x and y so for x you know Ki values was reported as something like -0.12 then 0.405 and -0.07, 0.38 and

0.18. Similarly, for Y you know these values were coming out to be something like you know -0.15 and similarly this was 0.50 and then you had -0.08, 0.45 and 0.15. So in the same line you had for the Z so that was the values of the return.

Similarly, we had seen for the asset you know Z and for Z the Ki values were you know taken as 0.40, then you had 0.10 and 0.38 and -0.10 and 0.22 and for the W asset it was actually 0.08 and 0.35 and then you had -0.05 and similarly you have 0.25 and then you had 0.06. Now all these cases we calculated you know the standard deviation and then ultimately we found you know the correlation.

Now we also discussed that how by looking at if you look at the graphs of these two values, if you look at the return values for these you know curves. So what you see that in this case, it is so wherever it has negative values and 0.15 or 0.12 then it becomes positive 0.40 so it also became 0.40. So basically if you plot the graph for you know return and you know the values, so in that case what you see is that the curves for A suppose it goes like this.

And for X in fact and for Y also it will be following the similar trend, so it will be starting from - of 0.15. So that way it will be same way and then it will have the similar trend, so it will go further, it will also go start and then further it will be dipping so then that was same. So what you see that they have whenever one is increasing another is also increasing, one is decreasing another is also decreasing.

Now in this case, if you look at their curves, what you see that in this case if it is 0.40, so in that case it starts from here from 0.40 it goes 0.10. Then, it goes to 0.38 then it goes to -0.1 and then it goes to 0.22, so this way it goes. Similarly, for this it will start with 0.08, so it will start from here and it will go in the next time to 0.35. So it will go here, then it will come down again to -0.05, it will come down to here.

Then, it will further go to 0.25, so like that what you see is that and then it will come down to 0.6. So what you see that in this case, this is basically the positive correlation and this is the negative correlation and that is why when we computed for X and Y, when we computed you know the correlation coefficient value, then this was coming out to be you know 99.9%. So this was you know correlation xy.

So that is what we calculated and this was coming out to be you know 99.9%. So quite extremely positively correlated and that is why when you know risk will be for this return will be high. So if it is very likely, so if it goes in the negative direction then that risk is basically you are multiplying, you are making it go higher and higher. So that way this is not preferred whereas this one is preferred where the correlation value when we calculated for the Z and W.

And when it was calculated basically in this case the correlation ZW when it was found, it was found to be - of 0.020353. So what you see that in this case it is negatively correlated and it was basically the covariance and when we calculated the correlation it was - of so it was the covariance in fact and correlation was found out to be - of 0.973. So that was the so if you look at this is quite positive and this is the negative value for these things.

And we had computed you know many parameters and we got the standard deviation, we got the covariance, so that was discussed. Now what does it show further when you combine the assets? So if you combine the assets, in that case if you take XY, so in that case the point is that how it is going to see that the risk or you know the value which you get the standard deviation what you are getting how does that affect when you combine the assets.

So when you are you know combining the assets, in that case you have to calculate these values, so this is 0.12 and this is 0.15. So when you are combining in that case, this will be 0.12+0.15/2. So we are taking the average of the two returns, so it will be - of 0.135. Similarly, this will be you know 0.905/2, so it will be you know 0.452 like that so it will be that and if you then compute so what will be happening in that case?

So basically we need to see that when you combine these assets, how it is going to affect you know risk.

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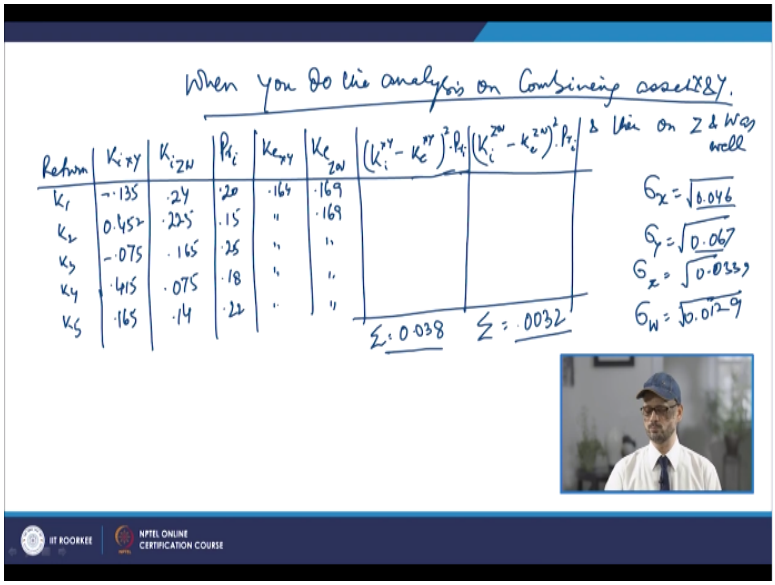

So if you combine, so the combination of assets for the combining, so when you combine you do the analysis on combining assets X and Y suppose and then further on Z and W and then on Z and W as well. So what will happen that when you take you know combination of the assets, so you return you know the return which you are getting like you have K1, K2, K3, K4 and K5.

Now for this these values basically when you combine so that will Ki you know XY so for Ki XY it will be this -0.12+ -0.15/2. So this way it will be written so it will be - of 0.135. Similarly, 0.45 to next data is basically - of if you look at this value -0.07 and -0.08 so it will be -0.075. So – of 0.075, so this way the values can be written as 0.415 and also 0.165. So similarly you can write Ki ZW.

And Ki ZW if you compute what you see that in this case it will be 0.40+ you know this 0.08/2 so it will be 0.48/2 so 0.24, so 0.24 then it will be 0.225, then this will be basically 0.33/2 so 0.165 like that. So it will be going you know like that, so it will be 38 and -0.05, so as we discussed that it will be 0.24 so it will go like this will be 0.24. Then, it will be 0.225, then it will be 0.165, 0.075 and 0.14.

So this is you know once you combine the assets and then you know that the Pri was already defined, it was 20% then you have 15% 0.25, 0.18 and 0.22. Now the thing is that based on that you will be finding the Ke XY and so that will be the addition of them. So addition of them will be you know it will be like 0.164. Similarly, for you know for ZW it will be coming as 0.169.

So it will be Ke XY will be 0.164 and that will be for all and Ke ZW it will be there, so it will be there for all. So that will be 0.169. So that will be you know Ke XY will be the summation of all this by 5. Similarly, for the Ki ZW it will be 0.169. Now based on that based on all these values you can find you know standard deviation and you can find the correlation and covariance values.

And what you see that in such cases so if you find, if you try to find you know suppose Ki XY-you know Ke XY and for that it is multiplied by so it will be taken square*Pr for finding you know standard deviation summation and its root will be the standard deviation and similarly you will have Ki ZW-Ke ZW and that will be square*Pri. So if you calculate these values so that will be basically you can calculate.

So this minus this that will be square so this way you can Ki XY that will be 0.135 and this is 0.24 so it will be difference will be something like 0.105 that will be square so you know 0.01 you know 0.10*0.10 so 0.011025 something like and then that will be multiplied by the Pri so 0.20. So that way you will have these values and this value you know if you add so you have to sum them.

And if you sum these values, this will be coming as 0.038 and similarly if you sum this value this is coming at 0.0032. Now if you look at the standard deviation, if you see the standard deviation for the earlier case, now in this case if you find the standard deviation for Xi.

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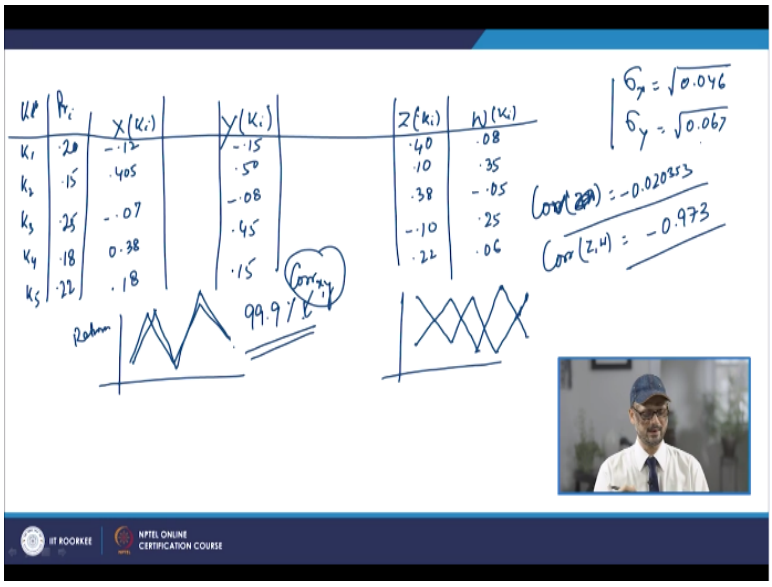

Then, for the standard deviation of Xi is coming as you know sigma X it is coming as under root of you know 0.046 and sigma Y is coming as under root of you know 0.067. So you can write here. So sigma X if you look at it will be under root of 0.046 and sigma Y will be under root of 0.067. Now if you see that if you are combining then this standard deviation which you are getting this is going to be lesser than the either of the two.

So when you combine the asset, it is going to be quite smaller. Now the situation is even better if you look at but what you see that the asset X and Y they are positively correlated and even in those cases when you are combining the assets, in those cases the standard deviation which basically will be talking about the risk and all that. So that is basically decreasing with respect to you know loan assets like X or Y.

Similarly, will be the case of the Z and W, sigma Z which we are computing now sigma Z was basically the under root of 0.0339 and sigma W was computed out to be under root of 0.0129. So what was you know seen that if you look at this value and this standard deviation will be the square root of this. So in fact this is<0.0339 or 0.0129 and this value is even a smaller then this.

So what we see, now this value or this value and this standard deviation will be square root of this so that should not be misunderstood. So what you see that even for them it is quite even you know smaller, so this value becomes quite small, so the thing is that first of all the risk will be primarily depend upon you know correlation coefficient values if they are negatively correlated assets.

In that case, you know you have you know the chances of less risk and then if you combine the assets then you have always the chance to reduce the risk that is what is being observed. Now what is happening that in that case you can see that there are certain you know way, so by which you can combine these two assets. Now what we see that when you are combining the assets the risk is certainly you know one is you know that return.

That return, this Ki and all that so how that return is going to be changed and how the risk is going to be changed that has to be computed. So this can be understood basically by a graph because from A to B when you are combining the assets then you can see by looking at the graph you can explain that if you what we say that you have asset A so that has the expected

return that is KA and similarly you have so that has some risk level of A so that is sigma A and similarly for B if we get the risk level of or standard deviation is sigma B.

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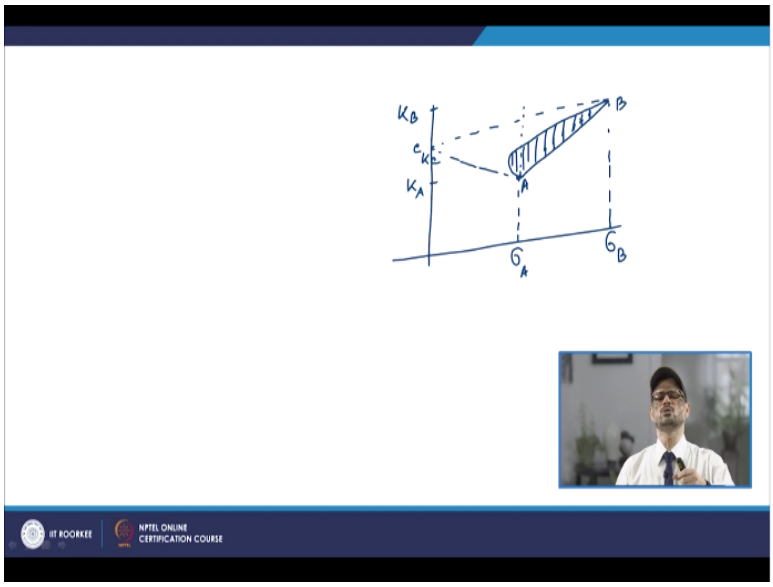

Then, there are 3 combinations you know two extreme combinations and one you know one more combination that can be you know originated or that can be inferred and that can be understood like this that one extreme combination is that you know it will go through this straight line, so it basically will be increasing you know the standard deviation and it is basically the case of positive correlation and your return will vary in between.

And the second one is that when it is negatively correlated then in that case it will move through this line. So that will be your so if this point is A and this is you know B then this point will be C and this return will be KC. So the second will be this and the third you know the in between you know when you are in normal case when it is positively or negatively you know correlated, so it is most likely to occur you know in such cases.

So that is common combination and in that case the variation is going like this. So basically this is the you know third case where how these you know return and risk are going to be you know correlated so that is seen. So this is how there are 3 basic you know combinations, 3 basic you know ways to combine the two assets and this will show that how you can expect or you can calculate the values of the Ke or you know KC, how that will be?

So it is expected return and what will be the respective you know standard deviation, so these are the 3 ways by which you can represent them.

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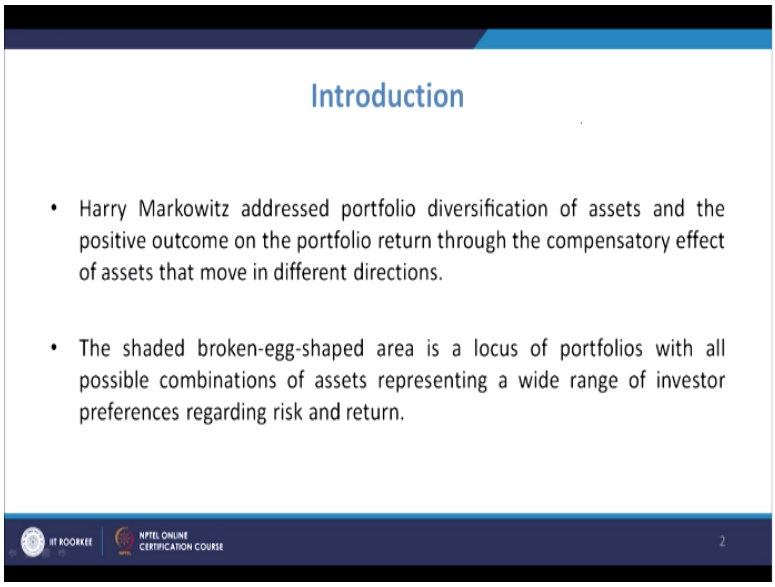

Now based on that so there has been further the modification given by you know the Markowitz and that is known as the Markowitz's two asset portfolio and he has also talked about the risk and return and there also he has shown that how you know this risk and return portfolio you know construction or these graphs you know are seen. So that we will see in the case of the Markowitz's two asset portfolio.

So Markowitz's two asset portfolio basically portfolio we were talking about the risk and return, the portfolio risk and return basically in which the diversification of assets will be in certain proportion. So in the earlier case what we say that we are basically taking the average and then see you know effect you know on the return and you know the risk but what you know Markowitz has suggested that when you are basically giving the unequal percentage of you know investment.

In those cases, how that return and risk can be you know calculated, so for that suppose you know you can understand it with one example.

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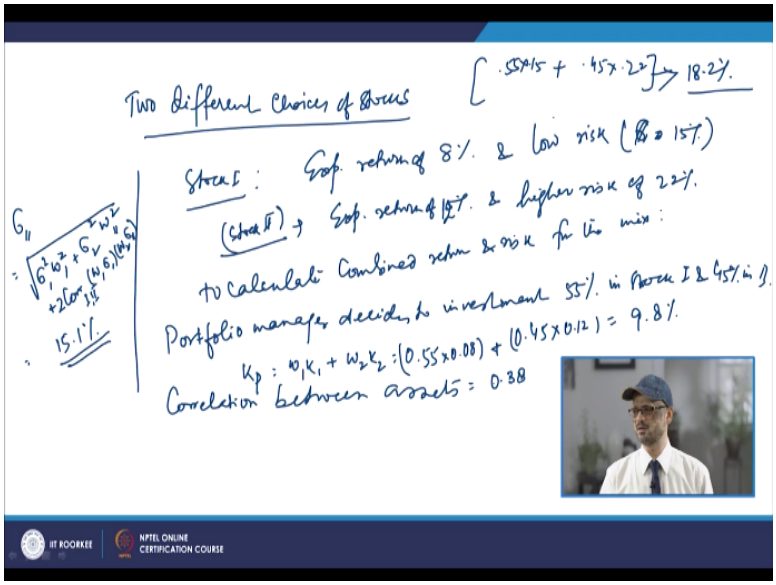

Suppose that you have two different choices of stocks, so you have two different choices of stocks, now in that case you know stock 1, so stock 1 is you know giving expected return of 8% and there is no risk. So it is giving you know low return so it is giving you the low risk and low risk is basically by standard deviation of 15%. So that will be not a standard deviation, it will be represented by standard deviation, so it is 15% of risk you can say.

Similarly, you have the stock 2 and stock 2 it is giving you the expected return of 15% and you have you know so that is no this is 12% stock 2 and your risk level is so you have higher risk of you know 22%. So suppose you have this 21 and you want to you know further predict. So that you want to predict you know to calculate combined return. So you have to calculate the combined return and risk for the mix.

So now we know that when you are going for the return of lesser values, load is catalyst and you are going for higher risk, higher return then you have higher risk catalyst. Now what is also you know input is that you know the portfolio manager is going to you know invest 55% to stock 1 so portfolio manager decides to invest 55% in stock 1 and the rest is 45% and 45% in 2.

So that way you know because risk is less so he has thought of putting 55% in 1 and 45% in 2. Now if you look at the portfolio rate of return so that will be the weighted average of you know the 2 return. So that will be weight 1*K1+weight 2*K2. So weight is 55% so it will be 0.55 and return is 8% so it will be you know 0.08. Similarly, you have return is you know the weight is 0.45 and the return is 12%.

So if you look at that this value comes out to be 9.8%. So this is you know return for the combined mix of this. Now we have to find you know portfolio risk, so that will be calculated by you know by the combined source, we have to find the determined by the standard deviation of the combined assets and it is found that the correlation between the two assets. So it is given that the correlation between assets is.

So that is given as 0.38, so on that you know basis you can calculate the sigma 1 2 you know that will be talking about the risk level. So you have to find that sigma 1 2 and for sigma 1 2 there has been given one you know correlation you know that is formula and that will be found by sigma 1 square W1 square+sigma 2 square W2 square and then two times correlation you know 1 and 2, so that is sigma 1 and sigma 2.

And then you will be multiplying with W1 sigma 1*W2 sigma 2. So that way you are going to calculate the risk which will be there for the combined level and if you calculate these values in this computation and if you put the values like sigma 1 is your 0.15 and your W1 is 0.55 then sigma 2 is 0.12 and W2 is 0.45 and then you have correlation is two times you know 0.38 that is given.

And then W1 sigma 1 so W1 and sigma 1 we know, W2 and sigma 2 we know, so if you put all these values, this is coming out to be 15.1%. So what you see that it will be less than the weighted average of the risk basically. If you find the weighted average of the risk, now this is the risk if you are combining the assets, if you find if you try to find the weighted average of the risk, it will be 0.55 times you know 0.15+0.45 times 0.22 and that will be 0.55 times 0.15+0.45 times 0.22.

So this is the weighted average of the risk and this is coming out to be about 18.2% and here we are getting 15.1%. So what we see that when you are combining the assets, you are getting a return of about 9.8% you know and the reasonable level of risk is coming out to be 15.1%. So that is how this is you know this is the correlation which is to be used for finding that risk and the rate of return.

Now if we take the different you know values, if you take you know if you take the large number of combinations, then we are going to have similar kind of graph which we have got

earlier here. So we are going to have the similar kind of graph and this area which is shaded this is basically known as the broken egg type of you know shape and this talks about you know the different situations basically.

And you will have the different positions where some of the cases will be talking about the very high risk you know and there are you know certain regions are basically denoted by the portfolio which is you know Markowitz you know efficient portfolio curve, so that we can see from the graph. So next we are going to discuss about the term that is known as the risk free rate of return.

Many at times you know when people are investing in stocks, so they have you know different levels of returns and also risk you know associated with that and also you have the situations like you have the government you know or sometimes the certain municipality or so, they are giving certain you know minimum you know risk that will be giving you the risk free return of you know 5% that is there is no risk.

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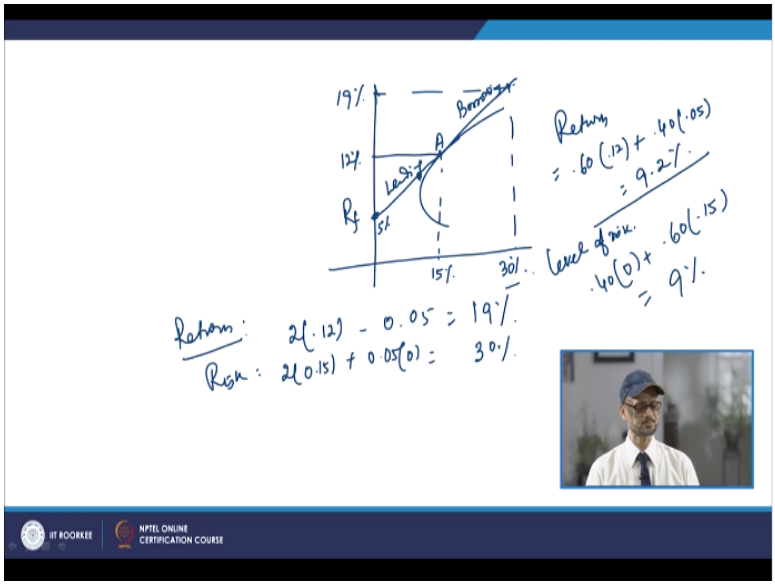

So basically that will be coming to a situation like you have you know so what happens that you know there is free rate of return. So you will have a risk-free you know value that is that is known as risk-free and some value is basically given and that basically will be applicable when you are going to have the lending or borrowing in certain cases.

So that this case will be talking about when you are going for the lending or we are going about the borrowing cases. So when we analyze this risk free you know assets suppose a

person wants to split his investment you know into asset A and suppose and also one risk free that is by the municipality. So suppose the A is about you know 12% and so A will be here that is it is giving you the 12% return.

And you know he is going to talk about he is going to invest also in the you know risk-free you know, so this 12% return, for that he has a risk level, certainly it will be risk level of suppose 15% and then you have basically the risk-free that is giving him the 5% and suppose you know you have, so he puts 60% you know in the case of you know A and 40% for B. So in that case for 40% you know B means that is so this is for A.

So if that be the case, he will be getting you know return, so that return will be 60% in A, so it will be 60 and return is 12% and 40% in the risk-free state that is so it will be 0.40 and into it is only 5%. So that will be coming as 9.2% and if you look at the risk level, so level of risk will be basically 0.40*0+0.60*0.15. So it will be basically 9%, so that is what will be the case in the case of the risk-free values.

Now if so he will be you know at somewhere at this point B, so that way this is normally when he is investing in that and if suppose he is basically going for you know if he is completely if you know he is you know if his whole amount whatever he has and he has borrowed the same amount and then he is investing that in asset A alone, in that case if you look at the return.

If he is investing the twice the amount he has, he has certain amount, that amount and that some amount he has borrowed also. So in that case, his return will be 2*0.12-0.05, so it will be you know 19% and then if you look at his risk, so it will be two times 0.15+0.05*0. So it will be 30%. So basically he is coming to this, this is your 19% of his return is getting but at a very high risk of 30%.

So it depends upon that how much percentage he is putting where and then how this risk-free rate of return is being calculated.

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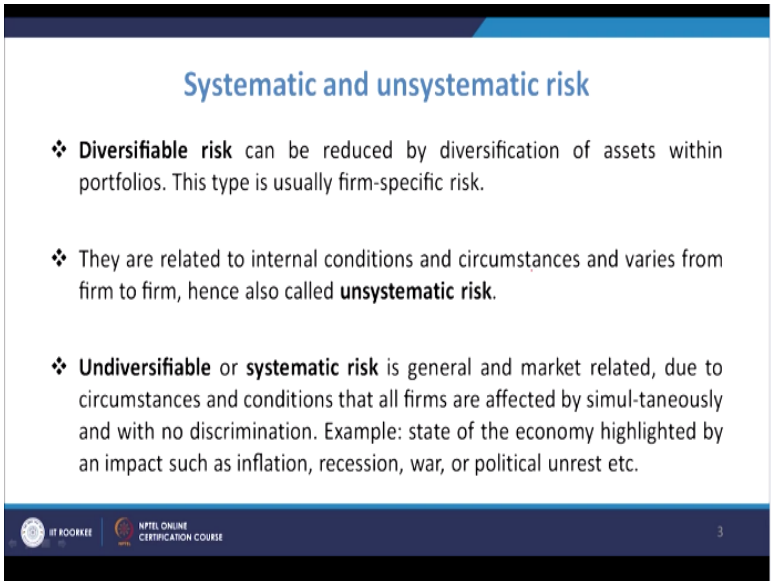

Now we are going to have also some knowledge about the type of risk you have you know as we see the diversifiable risk can be reduced by the diversification of assets within the portfolios and they are also known as the firm-specific risk. So they are related to the internal conditions and circumstances and varies from firm to firm, so they are also known as the unsystematic risk.

You know because you have the control you know form a specific it is you can reduce it by you know that how by you know by diversification or by putting what percentage in where in portfolio you can reduce these risks, so these are known as the diversifiable risk whereas there is another type of risk which is known as the undiversifiable risk and it is in general and is general and market related.

So this will be because of the circumstances and conditions like you have many a times you have the war situations or inflation or political crisis or so. So that is basically the case of you know undiversifiable risk or systematic risk. So when we try to you know present these risks, so what we see that you have if you try to present these risks so that can be you know one will be your so this will be how is the total risk will go.

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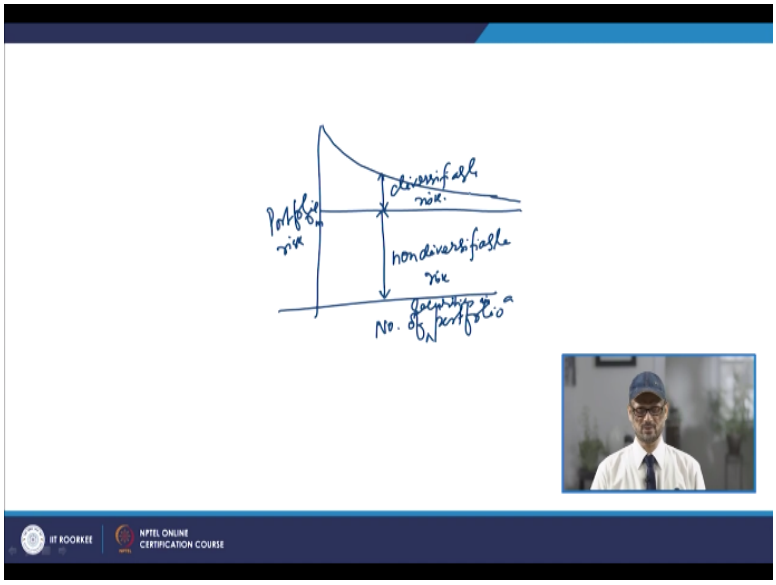

And one part will be you know constant so that is so this part which is you know which is the non-diversifiable risk. This is known as non-diversifiable risk and this is so that is not in our control because that is because of the political crisis or so and this part is known as the diversifiable risk. So this is about the different type of risk which we see. We cannot do much here but for this we have you know we have the way how to see that.

And that will be total risk, so you have you know number of portfolios here so if you increase the number of portfolio you know number of you know securities in a portfolio, number of securities in a portfolio and this side you have portfolio risk. So what you mean to say that this is your sigma M. So if you increase that number of securities in the portfolio, in that case this diversifiable risk is going to decrease.

However, this is going to be constant and total risk you can minimize. So that is what is about this risk you know situations in the case of diversification of portfolio. Thank you very much.