Financial Mathematics
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Lecture – 54
Risk Premium, Portfolio Return and Risk

Welcome to the lecture on risk premium and portfolio return and risk, so in this lecture we are going to have the discussion about in risk premium, risk aversion basically also that many a times the investors wish is that to averse the risk so, risk aversion refers to avoiding risky situations, most investors are risk averse that is and they chose the less risky investments so basically, many a times, almost of the time when you asked the investor, you would try to see, will try to tell you that he will go for the less risky investment.

So, in the less risky investment, the return will be less, investment amount will be more you know, return I mean will be less whereas, in the more risky you know situations; the situations will be opposite, so the tendency comes with the understanding that the returns expected from less risky investment would not be high.

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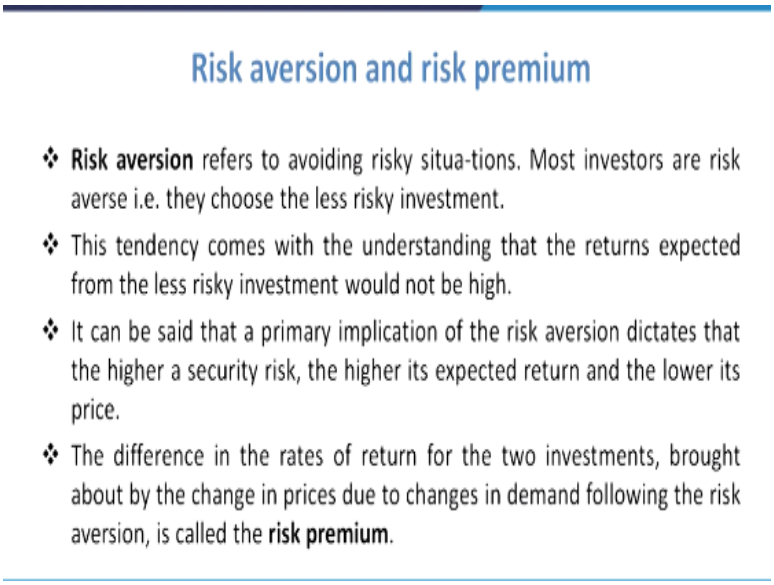

Certainly that is the one but then people do not want to take risk, the tendency of risk aversion, so that is why they go for the less risky you know options, it can be said that a primary implication of the risk aversion dictates that higher; the higher security risk, the higher its

expected return and low its price, so that is normally the you know what can be said and many a times the difference in the rates of return for the two investments because of the change in prices, due to changes in demand that is known as the risk premium.

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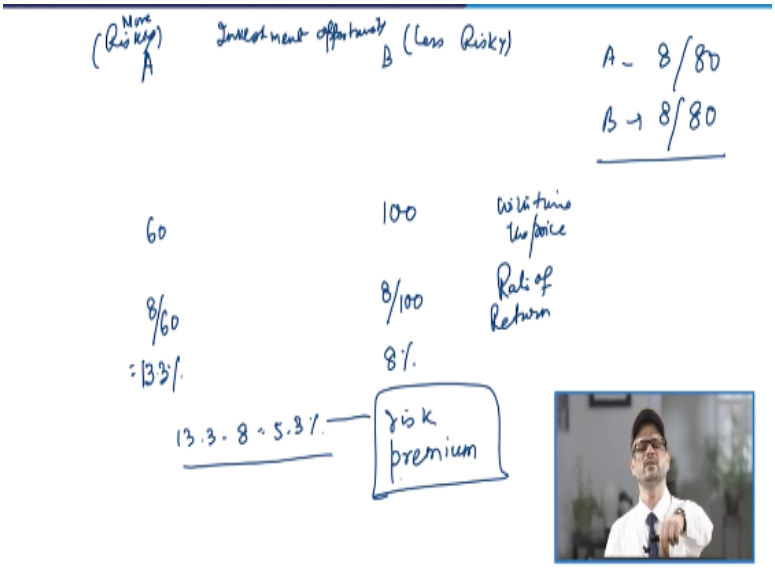

So, we will talking about this risk premium in fact we can try to understand suppose, you know there are 2 investment opportunities and let us say, one is A and one is B there are; these are the investment opportunities, so you know and here A is said to be riskier, A is you know riskier than B, so more risky and this is less risky, now in that case you know, so now basically what happens that suppose we started with A and B and they are giving you the same return.

So, suppose you know on the share price of 80 rupees. each one of A and B is giving you the you know return of 8, so that is your 10%, so suppose, you know it is; it started like this that A is also giving you know 8 for 80 rupees and B also is giving 8 for 80 rupees, now the thing is that with time, if A is said to be more risky in that case B, people try to prefer you know invest in more, invest in you know in more numbers in B.

So, people will queue for B and more number of investments will be there, in that case there will be demand for B and B share price will go up so, with time you know with time the price of A and B will be changing and B will be more than 80, so it will be going as suppose, 100 and A,

since its demand has gone down, so its price basically is coming down to you know 60, now if you look at you know in that case, the return is still is the same.

This is giving the return of 8 at a price of 100 and this is giving a return of 8 at a price of 60, so if take the you know rate of return analysis, what you see that you have rate of return will be you know less for B then that for A, so our rate of return will be 8 for 100, and this will be 8 for 60, so basically this will be about 13.3% and if you look at its you know, rate of return it will be only 8%.

So, what you see that this rate of return has you know there is rate of return that is you know decreasing in the case of those you know in such cases where you have, this is the less risky you know, option you have the rate of return as the smaller one than the you know that the options which the more risk, so the difference; this is difference of you know 13.3 - 8 that is you know 5.3%.

Now, you know this is because of the change in prices of these 2 because of the risk aversion you know notion, there was change in price between them and because of that this is the change in the rate of return that is 5.3%, this is known as the risk premium, so because of the risk aversion, people try to go, you know run after those you know opportunities which are assumed to be less riskier.

In that case, the you know higher rate; the margin of rate of return which you get in such cases you know that will be known as the risk premiums, so that is the you know case of risk premium.

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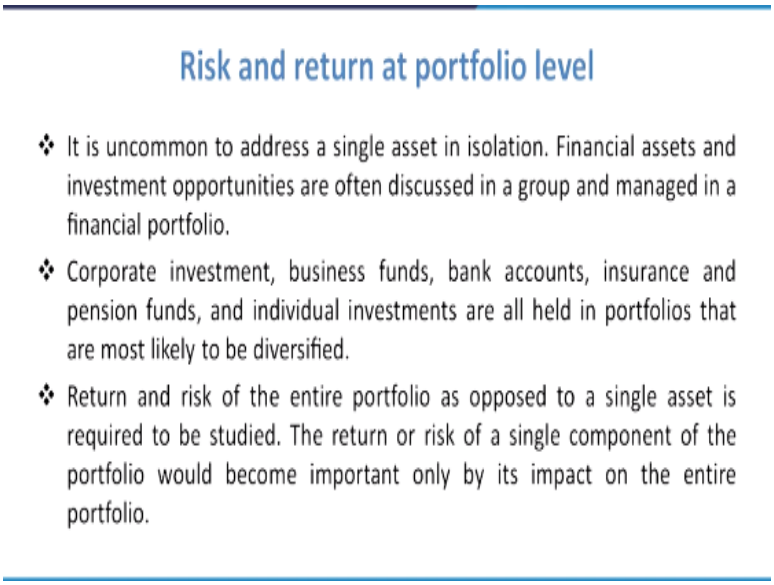

Now, we will talk about the return and risk at the portfolio level, now what happens that many a times we have talked about the single situations where people purchase and they are investing but many a times you know we are not going to address single asset in isolation, these financial assets and investment opportunities normally they are often discussed in a group and managed in a financial portfolio.

So, you know the all this corporate investment, businesses, funds, bank accounts and then insurance and pension funds and they are individual investments, they are all held in portfolio that are most likely to be diversify, so basically you have you know there are many you know things grouped together and then they are managed. Now, you will the return or risk you know or the entire portfolio, it is not because of the single you know entity, single asset.

So, it has you know single component will have certain bearing on that it will be doing that so, this will be becoming important because it will have the impact but then you know you cannot say that by one you can tell that either it is you know you are losing or you are gaining or source it will have its impact because they will be having their individual impact and how that impact is accessed that we can understand.

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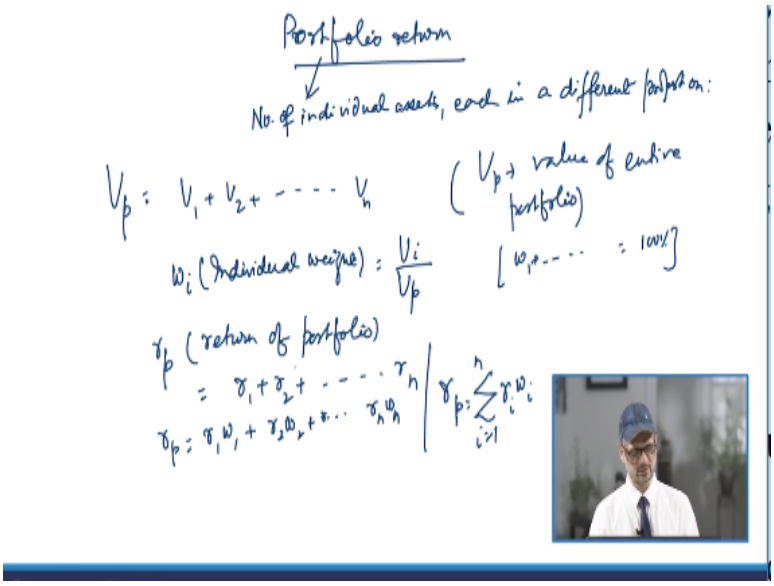

So, if you can look at that situation now, what happens that in most of the cases when we talk about the portfolio return, so when we talk about you know, the portfolio return so basically, the portfolio means you have number of individual assets, so portfolio means you have a number of individual assets and each in a different proportion, so each in a different proportion, now when we try to find the market value of the entire portfolio that is Vp, so that is market value of the portfolio or entire portfolio.

This will be the summation of the values of the individual assets, so you have many assets combined together, it will make a portfolio and the value will be the submission of all these you know, individual values, so it will be V1 + V2, so this will be going up to suppose, you have n assets, so you will have Vp will be value of the portfolio will be V1; so that it will be entire portfolio, so Vp is the value of entire portfolio.

So, this will be; V1 + V2, so that will be +Vn, now each asset which is V1 or V2 or so, it has its own proportion, so this proportion you know or it will be talked as its weight, so individual weight, so this weight, wi that is your you know individual weight, now this will be basically its proportion divided by the total value, so individual weight that will be its ratio of the you know ith you know assets value and divided by the value of the entire portfolio that is Vp.

So that will be your Vp and you have i varying from 1 to n, so you have n different weights defined now, what happens that if you sum all these weights so, you know that will be coming to you know 100%, so that will be coming like that now, what happens that you can find the rate of return in that case and all these returns on all these individual you know assets they are going to contribute to the return; overall return basically.

So, the return will be you know; return of the portfolio rp that is return of portfolio, so it will be summation of the you know, individual returns that will be r1 + r2 + rn and the rp as you know rp will be basically, the r1w1 + you know r2w2 + you know this weight will go and it will be rnwn, so what happens that if you try to see that altogether you will have you know, altogether you can define this rp as summation of i = 1 to n, then you have ri and wi.

So, this summation altogether, it will be talking about the return of portfolio and you have summation is ri wi, so it is the if you talk about the portfolio return, it is the summation of weighted you know, individual returns so that is you ri wi of its all you know component assets and the weights proportion we can find as you know that depending up on the value of the assets, so this way you know portfolio return you know is calculated.

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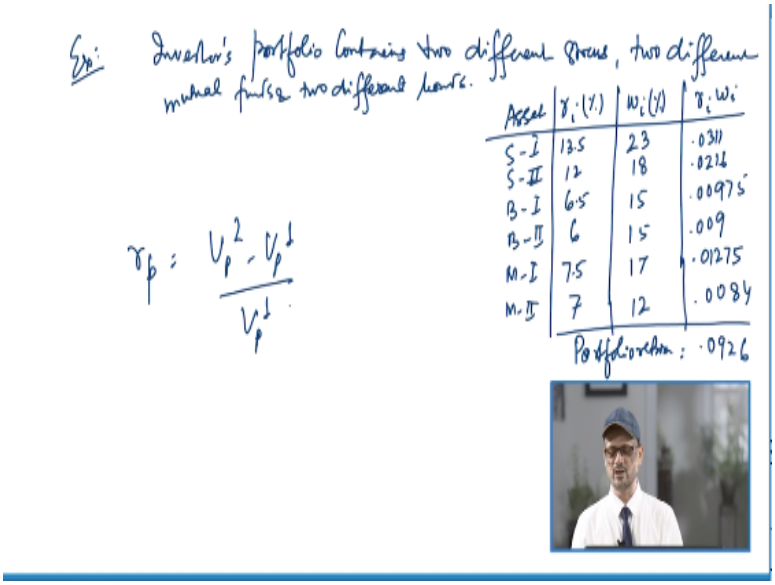

Now, you can have one example like we can see that suppose one investor, so if suppose we see that one investor's portfolio is there and it contains 2 different stocks, so investors portfolio

contains two different stocks, so you know 2 different stocks are there, then it has you know as 2 different mutual fund is there and also you know, it has 2 different bonds are there, so it has 6 you know assets.

And among them, you have to calculate the portfolio return and its values are given suppose, you have asset and for that you have 2 different stocks, so you have a S1 and S2 similarly, you have 2 different you know bonds, so this will have B1 and B2 and similarly, you have 2 different mutual funds that is M1 and M2 now, in that case the return which is you know given, so for that basically, you have the data given and its weight suppose is given as you know r, so that is your ri.

And similarly, you will have values of wi that is weight percent and then you will find the ri and wi and then you will find the summation of ri, wi and that will be giving you the value of you know the portfolio return, so you will be given their individual rates and suppose their individual rates are given as 23, 18, 15, 15 and 17 and then 12, so altogether it should be 100, so this is 41+15, 56, 71, 98 and 100.

And if suppose, their individual returns are computed out to be you know 13.5, so this is in the percent similarly, this is also in percent, so 31.5, 12, 6.5 then 6 and 7.5 and 7, so this is suppose their individual you know returns of the asset in that case, the r1, wi if you calculate if you; you know if you multiply, so it will be in percentage, so if you take in terms of decimal, it will be.0311 similarly, it will be .0216, this will be .00975, this will be .009, this will be.01275 and this will be .0084.

So, r1, wi is calculated and if you have to calculate the portfolio return, then portfolio return will be summation of the ri, wi and if you sum them, it is coming as .0926, so portfolio return you can say that it will be 9.26%, so that is why and the way you calculate these portfolio return, now many a times, you are; you know you have to find the you know change in value of these the original value to find the estimated rate of return when the change of value is from one year to another year.

So, in that case if you find the know you know portfolio rate of return, so that will be calculated by rp will be Vp2 – Vp1 divided by Vp1, so this will be your you know market value of the entire portfolio at the start of the year and suppose the market value at the end of the year is you know Vp2, so in that case what will be you know, the rate of return, so that can be calculated based on that.

So, suppose you calculated this, you had the value calculated for certain situations suppose, you have a certain points and then that increase to so and the value was increased to some value, so you can have this used this formula and you can get these you know rate of return you know, for the portfolio how it has you know grown-up or so, so that way you know from the year end, first year you know in any year when started and when it ended, you know that rate of return can also be you know calculated.

Now, what we so; what we further try to understand is about the portfolio risk, now what is these portfolio risks?

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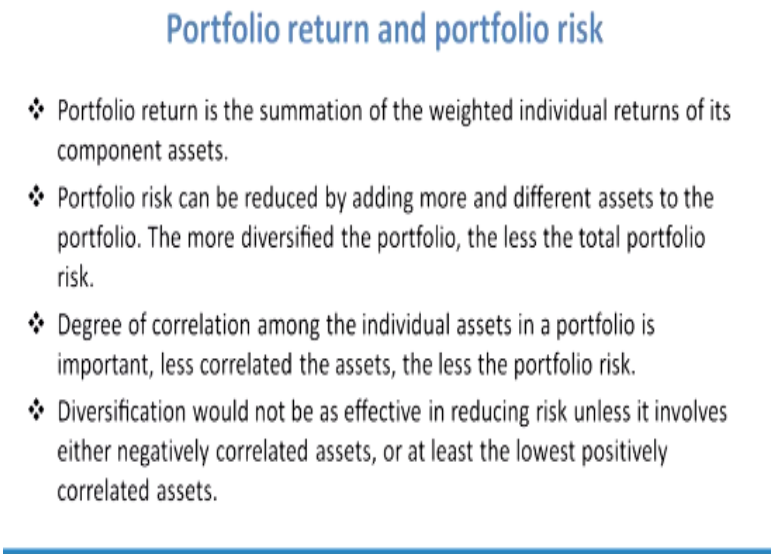

So, what we see that portfolio return and portfolio risk, what we have seen that portfolio return is a summation of the weighted individual returns of its component assets that is what we have understood in that case and now, we try to have the definition about the portfolio risk now, if you talk to portfolio risk, you know it is; it can be reduced by adding more and different assets, now

when we talk about the risk, then what will happens that if suppose, you have you know different components you have you know if there is a diversification of the portfolio.

Now, what happens that when they are correlated, in that case the correlation among them becomes important, so for that you find the correlation coefficient, how that return you know rate of return for the all these components are coming up suppose, if one is increasing, another is also increasing that is basically, positive correlation similarly, if one is increasing, another is decreasing in that case it is the negative correlation.

Now, if it is very, very positively correlated that is not a very, very you know the favourable case because that may lead to suppose, there is a loss in that case, it will be an accumulation of loss, so what we see that it is not be very, very highly you know positively correlated, so in the case of the you know quantisation of the portfolio risk, we need to study also about the correlation among the variables.

So, we will see that how that can be studied, so as we discussed that the degree of correlation among the individual assets in a portfolio is important, less correlated the assets, the less will be the portfolio risk so basically, when we are talking about the different assets in the portfolio and there if their correlation coefficient is -1, it is they are completely you know negatively correlated.

So, in that case they basically reduce the risk but if they are very, very highly correlated positively correlated in that case, there will be chances of this risk, quite high risky you know situation may come into and we will see that how there is such situations are basically you know understood that when there are chances of less risk or high risk, so how you know you see those component.

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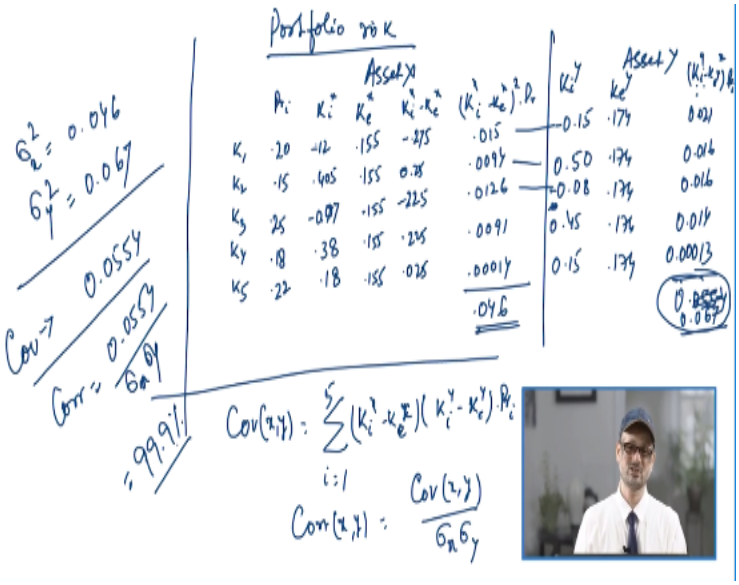

So, if you look at the you know case of portfolio risk that can be understood by looking at one example, now that can be understood by you know looking at the 2 pair of assets you have x and y, so if you know for asset x and y, you have the values given and based on that you can say, you can calculated the correlation among them and then you can say that whether you know there is a chance of less risk or high risk.

So, suppose you have you know asset x and you have asset y, now for the asset x is suppose the values are like you have you know, so for K1, K2, K3, K4 and K5, now in this case you know their probabilities are assigned, so probabilities are like the 20%, 15% then 25%, 18% and 22%, so that is your pri then, for that you know you will find you know the value of Ke and also before that you have the values of Ki.

So, Ki values which we are getting is -1.; you know, -.12 similarly, it is.405, then it is - of 0.7; 07 and similarly you have another value as .38 and you have .18, so if you add, it is 56 and -7 is 49 and then so based on that you will be getting the; so if you add them and based on that you will be getting that Ke, so expected value, so that will be Kex, so, Kix will be these values that is the you know return which you get.

And what you get is; if you add them, so 56 and 49+, so that way you know 99 and then, so that way you are further you are subtracting these and divided by 5, so it is coming out to be .155, so

this will be you know, then you have all these values will be coming as .155, now what you calculate is the Ki – Ke, so that will be your these values, so that once you calculate, it will be minus of .275, then it will be minus of; so this will be .25, then it will be a minus of .225, this will be minus, so this will be .225 and this will be .025.

Now, the purpose is that you have to find the correlation and for that you want to find the covariance and once you find the covariance then the correlation can be found by covariance divided by you know some of the standard deviations, so covariance of the xy you have to find, now for that you have to have the you know the formula now, for that we will; for this also you will find the Ki – Ke x, so this is the way to find the dispersion basically, or standard deviation formula.

So, this will be square * Pr, so if you calculate that it will be .015, this will be .0094, this is . 0126, this is .0091 and this is .00014, so this summation comes out to be .046, now this is for asset x and then that the main aim is to find correlation and for finding the correlation actually, the formula which will be used will; so first of all you will find the covariance xy and the covariance xy basically, will be found by summation of i = 1 to 5, this will be Kix – Kiy.

And then it will be further Kiy -; you know, so this will be Kix -Kex and then Kiy -Key and then it will be multiplied with pri, so this way you are going to have the covariance and then you will find the correlation between xy and correlation between xy will be covariance xy divided by sigma x sigma y, so we have to first out the sigma x and sigma y, so once you have the returns given then this is how you find the you know sigma x square and sigma y square.

So, sigma x square will be Ki = Kex square pr, its summation, so you are getting the sigma x square as this summation that is your 0.046, now so you will similarly find the you know sigma y square and sigma y square for that you need the data, now this is for the asset x now, you need to find for asset y, now for asset y, again you had the similar assets; K1, K2, K3, K4, K5 and in that you know you have the pri that is you know for asset y will be you will have the pri as point you know, so that is Kiy is given for here.

And Kiy is given as minus of 0.15, then you have the values like 0.50 and then you have minus of 0.08 so this is -0.08, this is second; this is first, this is second, this is third, then you have next value is 0.45 and then you have 0.15 now, if you look at the; these returns, you can see a trend now, this Kix is -.12 then it is -.15 it has increased to .40, it has also increased to .50, then has the decreased value of -0.07, it is -0.08, this is.38, .45, .18, .15.

Whenever it is increasing, this is also increasing whenever this is decreasing, this is also decreasing, now by looking roughly you can have a sense that how they are correlated, if one is you know, increasing another is also increasing, one is decreasing, another is also decreasing, so there is a high correlation among them but then you have to find these values for Key, again you will have getting you know the so; for this is Kiy, then you will be getting that Key and further you are going to get the Ki – Ke square and all that dot pr.

So, ultimately for this if you are getting this value is coming out to be 0.0554, so that can be calculated because you will have Key as the; if you have to sum it, so it will be no 60 + 50; 110 -23, so it will be 87 that will be divided by 5, so it will be basically.174 will be the Key, so .174 then you can further you know you can calculate all these values like Ki - Ke then Ki – Ke square * pr, so pr is all you know known, so that way you can have these values, you will be having the values like point; so you will have Ki - ; Kiy – Key and that you know * pri.

So, you know that value can be calculated and then what you do is; you calculate these covariance, so basically when you calculate the covariance, this value, this 0. you know, this sum is basically coming out to be 0.067, so this will be Ki – Ke square * pri, so these values will be coming as 0.021, 0.016, 0.016, 0.014 and 0.00013, so you sum them 0.067 it is coming and this is 0.046 is coming.

So, sigma x square will be 0.046 and sigma y square will be coming as 0.067, now based on that you can find the covariance, now covariance will be you know Kiy – Kix – Kex * Kiy – Key * pr, so it summation pri, so that will be giving you the covariance and that will be coming as 0.0554 and if you find the correlation; the correlation will be 0.0554 divided by sigma x square * you know sigma y square under root, so it is sigma x * sigma y.

So, if you find these values, it is coming out to be 99.9% correlated, so the thing is that such situations are giving you risk because in one case when the return is giving more, this is also given return more but when this is giving return less, it is also giving return less, they are basically these assets are very positively correlated, so we try to have a situation where the they are they should be negatively correlated not positively correlated.

So that if there is a risk in; if there is loss in one case then that can be compensated by the gain in the other case, so that may be situation you know, that situation can be understood because that can have another example that you know we can see in our coming lectures where we will see that the return which we are getting for the x and for y, they are different when this is increasing, then this is not increasing, this is basically decreasing.

So, suppose, -.1 to .140, it is going to .40, it is coming from .15 to suppose, .05, so like that when it is there, when it is increasing, this is decreasing that will have or when this is decreasing, this is increasing, so that will give you a negative type of correlation and that will be giving you the minimum type of risk, so such you know correlation; coefficient value when you calculate that basically you know gives you the value of the you know portfolio risk.

And that analysis is required to ensure that you have minimum of the risk. when you invest you know for the manager of the you know, portfolio is managing all that you know, investment, you know assets, so these point needs to be understood and this points need to be you know kept in mind. Thank you very much.