Financial Mathematics

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Lecture - 53
Decision Under Uncertainty

So welcome to the lecture on, Decision under Uncertainty. So we discussed about the situations when you have the risk and probabilities related to the alternatives are or the options are higher, provided and those cases we have got certain methods, we have studied certain methods that how to you know, use those probability values and come to the conclusion to have you know, the knowledge about the best alternative which should be selected or you know, related cases like based on the expected value or the most probable values or the based on variance.

So all these things we have discussed. However, many a times these probability values are not given and there are many situations like it may be impossible to assign the probabilities to several features identified for the given decision situation. So in our one of the you know, example which we discussed, we saw that for the 3 futures you had the probability given, you know that in that case the formula could have got C1 or C2 or C1 and C2 and for that the contract you know, the probability was 0.23 and 0.25. So like that.

But many a times no meaningful data are available from which probabilities we will develop.

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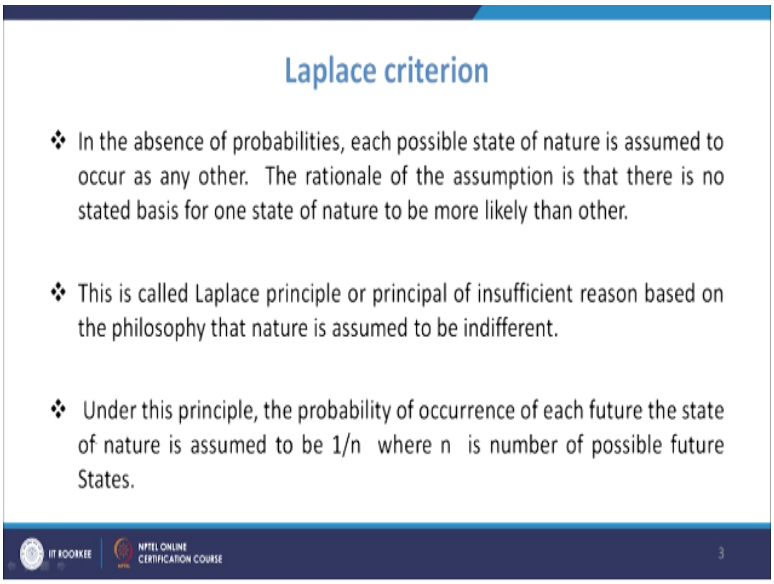

So these are very much situations which are practically relevant that the meaningful data are not available, and so the probabilities cannot be found out. Now in many cases the decision maker is unwilling to assign subjective probability because he feels that whatever future outcome it will be shown because of the respective probabilities to the different components that will not be welcome I mean for him, he will not feel good that way, so it will be unpleasant you know, outcome for him.

So he does not in fact want to you know, assign these subjective probabilities in those cases. So the probability is not known, I mean in that sense. Now when these probabilities are not available for the assignment to you know future events, so when there are even probabilities are even you know, available, then that is decision under risk and you use those probability values. But when this probability values are not known in that case we have to still come to certain decision.

And in that case the situation you know, is classified as the decision making under uncertainty. So that is how you will have few ways by which under these cases of uncertainty when the probabilities values are not given, you can come take certain decisions and that is known as the decision making under uncertainty.

So there are few rules and these rules are like you have Laplace criterion, you have Maximin and Maximax criterion, you have, the regret you know, criterion like that, you have Hurwicz

criterion, so that we will discuss in this case, in this lecture and we will know that what is these criteria means. So coming to the first criteria that is your Laplace criteria and this criteria you know, it tells that in the absence of probabilities, each possible state of nature is assumed to occur as any other.

So, whatever alternatives we have, whatever options we have or whatever features you have, you will feel that they all have the equal you know, possibilities to occur and that why they have equal probabilities. The rationale of the assumption is that there is no stated basis for one state of nature to be more likely than other. So they are basically derived from some of the features of nature, because nature does not favor anyone.

It is equal for everyone. So that way it assumed that there will be equal probability, you know for everything, for every alternative or for every choice. So this is called the Laplace principle or the principal of insufficient reason based on the philosophy that nature is assumed to be indifferent. So, that is the Laplace principal which tells that nature is assumed to be; or this is based on assumption that to be indifferent.

Now in this case when you have n number of possible states you know, if you have possible future states equal to n, in that case every future state has the probability of 1/n, so that all together 1/n you know *n that becomes 1. So on that basis basically we select you know, we take those you know features, we judge according to that. So you can say that now we try to basically, we will try to analyze based on the; this Laplace criterion.

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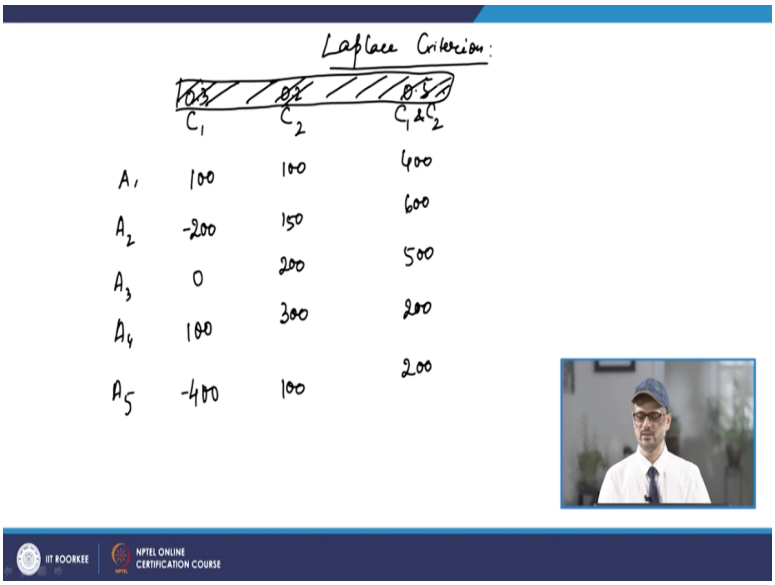

And in that we are going to further discuss about situations which we discussed for the decision under risk in which the contractor maybe awarded either contract C1 or C2 or C1 and C2, so we had that value; so we had C1 or C2 or C1 and C2 both and they had the probabilities of C1 have their, getting awarded C1 is 0.3, for C2 0.2 and this was basically for C1 and C2 it was 0.5, so that was the probability values and also if the work, any contractor, he got then there are five ways to do as alternatives, so that will be alternative A1, A2, A3 and A4 and A5.

So now we knew about these problems and we know that for them we had the profit matrix was there, so is the profit in 1000s of rupees will be varying if the C1 is contract is awarded so with alternate you know A1 will be 100 and then this with A2 it will be -200 so that will be loss then you have 0, 100 and -400 so that way you had; this was the, the values in that problem.

Similarly, for C2 it was 100, 150 then 200, 300 and 100. So this were the 1000s, you know, so that much 1000s of rupees was the profit. And for C1 and C2 if awarded this will be 400, 600, 500, 200 and 200. Now, this was, you know in the earlier case when you had these probabilities awarded but now when we talk about the cases under uncertainty then this value is not given, so we do not know this value and we have to use that different criterion, you know and then we have to come and choose the best alternative you know, which should be taken.

So using the Laplace criterion what we do is that we have to find the profit, average profit for all A1, A2, A3, A4 and A5 and we assume that their probabilities for C1, C2 were C1 and C2 were all same. So if you take the alternative so you come to the you know.

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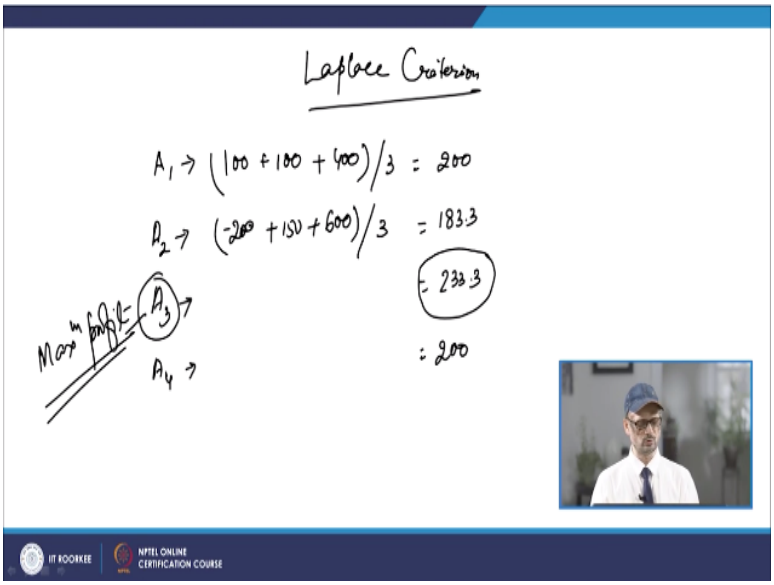

If you take the Laplace criteria so in that case Laplace, so in that case if you take for A1 then you have to take 100+100+400 and each one have the equal probability, so it will be 100+100+400 and divided by 3, so it will be 200. Similarly, you can calculate for A2, for A2 it will be you know, -200+150+600 so it will be 750-250 550, so – of 550; so this will be + of 500, so for A2 it will be -200 and then you have +150+600.

So this one; and divided by 3, so it will be 550/3 so it will be 183.3. Similarly, for A3 if you look at; for A3 it will be 0+200+500, 700/3 will be 233.3, so it will be 233.3. Similarly, A4, for A4 you know, we will be getting the 100, 300, and 200 * total 5+1 600/3 so it will be 200. And anyway, we had struck of the A5 because of the dominance criteria, so that is anyway not coming to picture.

So the thing is that if you use that Laplace criterion in that case we are going to give the equal probability to all the features and then if you choose among the alternatives then by deciding about them you see that this alternative A3, A3 is the, is proved to be the best because it gives you the maximum profit. So if you choose Alternative A3 then this is giving you the maximum

profit. And with Laplace criteria you have to say that the A3 alternative should be chosen. Now we will move to the next criteria.

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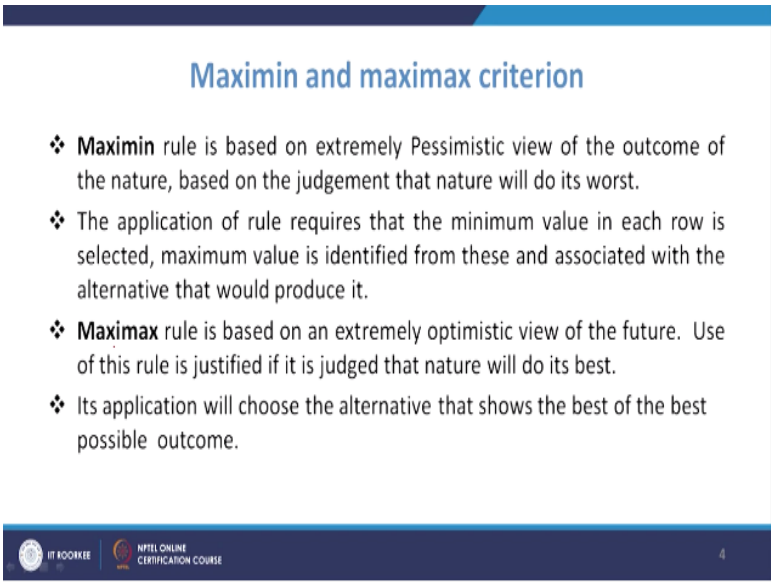

The next criteria will be the Maximin and Maximax criteria. So Maximum rule is based on extremely pessimistic view of the outcome of the nature, based on the judgment that nature will do its worst. The basically there are two criterion one is Maximin and Maximax. Maximin is based on extremely pessimistic and Maximax is based on extremely optimistic view. So one assumes that nature will do its worst and another assumes that nature will do its best.

So in that case what we do is that in the case of Maximin criteria where it is expected that nature will do its worst, in that; the application rules requires that the minimum value in each row is selected so for every alternative you are going to choose the minimum value and then among those minimum value the maximum value is identified from those minimum values and associated with the alternative that would produce it.

So that way the maximum of those minimum values you know, in that case will be choose, that alternative will be chosen. Similarly, the Maximax rule is based on extremely optimistic view and it will be justified, if it is judge that, this only justifies on the judgment that nature does its best. So it has, you know it will choose the alternative that shows the best possible outcome and

further to what you do is you are choosing the; in that case the maximum value then among them the maximum, so that is your Maximax rule.

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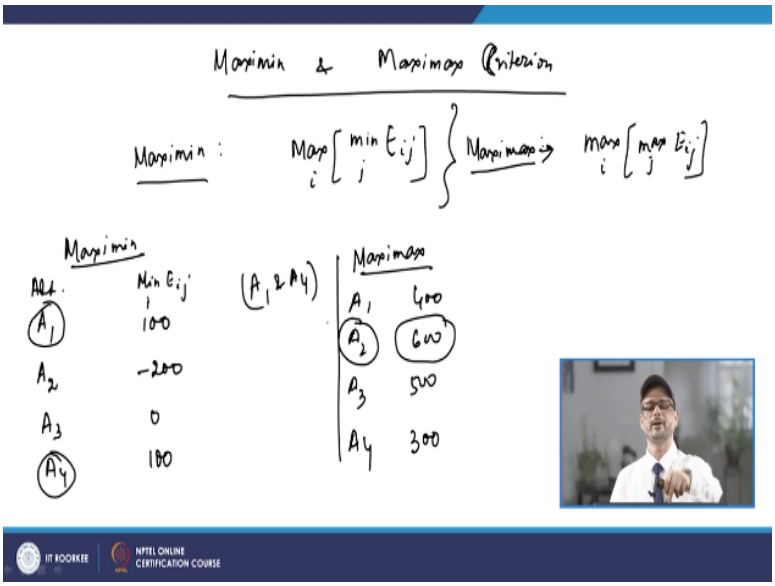

So if you try to analyze the Maximin and Maximax rule, so this is criteria basically you can say. Now if you take the, if you try to interoperate you know the Maximin or Minimax criteria you can have the definition for suppose, if the case of the Maximin what you do is, you take that, you do the computation, and in computation and you take the maximum value for among the, for the j state of nature.

So first of all you will get the minimum value of E, i, j for the j state of nature and on then you will be finding the maximum value. So that way, this is for the ith alternative and this is the jth state of nature. So for that you will be getting first the minimum value and then you are getting maximum out if it. Similarly, if you have to find for the Maximax, so for Maximax, for Maximax it will be done like this but you will have maximum value and that too for the Maximum of for you know E, i, j, so this way you try to have the Maximin and Minimax values.

Now if you try to see the you know, values in the you know, in this matrix what you see that for alternative A1 the, if you take the Maximin value, then using Maximin the minimum is 100, similarly, for A2 the minimum is -200, for A3 it is 0 and for A4 it is you know 100, so that is your Minimax, so I mean minimum values.

So you using the you know, the Maximin you know; so for Maximin if you see for A1 you know, if it is alternative and this is the minimum of E, i, j, so in that case what we will be getting, you will be getting for A1 it is 100, for A2 what we have just seen. For A1 it is 100 then -200, 0 and 100, so A1 100, A2 -200 and A3 is 0 and A4 is 100 And now then, you have to get the maximum of the values and the maximum value is A1 as well as A4.

So A1 and A4 you know, are basically the alternative, they are basically taken or preferred using the Maximin conditions. Now if you take the Minimax condition, now in the case of Minimax what you have; not Minimax, it is basically Maximax. Now in the case of Maximax if you take A1, A2, A3 and A4, now in the case of Maximax you know that it will be 400, 600, 500 and 300. So it will be 400, 600, 500 and 300 and in that case you get the A2 as the maximum value of the maximum.

So it is more optimistic and you know, we say that, this among the optimistic it is too optimistic and we are taking the maximum of that you know, outcome that is 600. So A2 will be taken as the criteria you know, that will be the decision which will be taken under this criteria, so the profit of 600,000 you know that is. So basically, it is fine but what you see that in that case you are likely to have even the loss of -200.

So that is certainly the risk is there but if you; it is taking that it has to get maximum profit so that 600 is, so alternative A2 is chosen. So that is your Maximax criteria. Then, comes the Hurwicz criteria. Now what is Hurwicz criteria?

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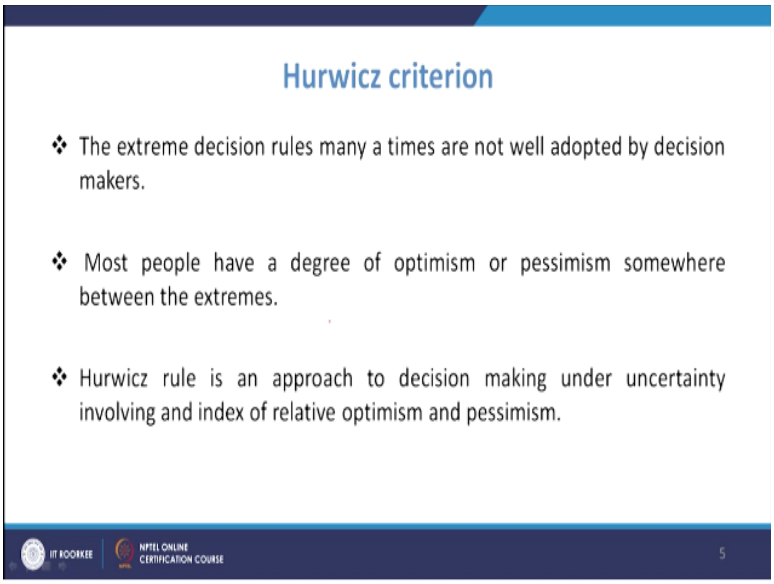

So that is, so this is Maximin. Now what we see that in the cases of Maximin or Maximax you have been taking the extreme decisions and then you are choosing the alternative. Now, many a times these extreme decision rules are not well adopted by the decision makers. So what they, you know, they adopt a degree of optimism or pessimism somewhere between the extremes if you are completely pessimistic then it is degree of optimism is 0.

And if you are very, very optimistic then the degree of optimism is 1. But then, there has to be you know, something in between. So the Hurwicz criteria gives your one parameter that is degree of optimism or so or degree of pessimism you can say. Now in this case we call it that as index of optimism that is alpha and that basically talks, so when it is 0 it will be the you know extremely, extremely it was a side of pessimism and if it is 1 it will be it was the side of optimism.

So Hurwicz rules is an approach to decision making by making you know, this decision making; now in this case we make the decision under uncertainty by using this index of relative optimism and pessimism and that alpha is this value is used; how to use that, so that can be understood. Now let us say that we got this, we have got this A1, A2, A3 and A4 and we know that by the; you know, by pessimistic way you are getting certain values and by optimistic way; by extreme optimism you are getting some outcome and by extreme pessimism you are getting certain outcome.

So like for suppose, if you are extremely pessimist in that, and your idea is pessimism in that case A1, A2 and A4 will be like what we see have seen the minimum of the values that 100, -200, 0 and 100. And if you are completely optimism in that case you are going for this Maximax criteria that is your 400, 600, 500 and 300. Now if you take that, you know degree of optimism or index of optimism.

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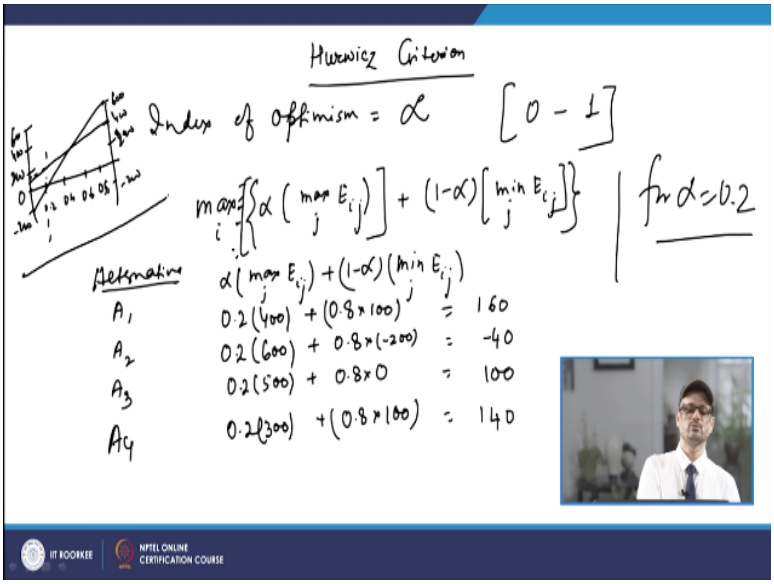

That is, alpha, so it is called as Hurwicz criterion. So in that case you take index of optimism is chosen and this is taken as alpha and its value is between 0 to 1. So with 0 it will be completely extremely you know, pessimism is there extreme pessimism and towards the right, you are completely optimum you have extreme optimism.

So using this what we do is we are getting you know using the alpha, we are having the criterion that we will be doing maximization of in alpha into you know, maximize j and then E, i, j and then +1-alpha*minimum of E, i, j. So that will be there, and it will be all you know, so this will be from here it will be going like this. So alpha into you know, Maximin E, i, j, Maximum of E, i, j +1-alpha*minimum of E, i, j and then you will be doing the maximum of that value that is your Hurwicz criterion.

So if you take the different alternatives in that case how it comes you know, for other different alternatives, if you take alternatives like you know, A1, A2, A3 and A4. Now for that what do we

know, that if alpha is so, for alpha suppose we take as 0.2 so in that case what we do is we are getting first the value of alpha*maximum of E, i, j and then +1-alpha*minimum of E, i, j. So that is what we are calculating.

So we will be calculating alpha as 0.2 and this is the index of optimism so we are taking the maximum value. So in the first alternative the maximum value was 400 and 1-alpha will become 0.8 and the minimum value which was there was 100, so it will be 100. So this value will be 80 and this will be also 80, so it will be 160. Similarly, if you take for A2 it was minimum was -200 and maximum was 600, so we will be getting 0.2*600 and +0.8*-200.

So it will be 120 and it will be basically you know, 160. So 120-160 so it will be -40. Then you are getting the A3; for A3 the minimum is 0 and maximum is 500. So it will be 0.2*500+0.8*0 so it will be 100 and then for A4 the minimum was 100 and maximum was 300, so it will be 0.2*300+0.8*100. So it becomes equal to 100*0.8 so 80 and +40 so it will be becoming 140. So what you see that the value of this expected values which we calculate it is going to be maximum for A1 and using the Hurwicz criterion we can go for the value 160 that is alternative A1.

If you are taking the index of optimism as 0.2, if you take the index of optimism the different values in that case this decision may be changing and even that can be you know, seen by the graph so you can even plot a graph and you can see that suppose on one side you will have the values in this side and on the x-axis so you can have the plot and if suppose this is 0 and you have positive here 200, 400, 600 and you have -200.

Similarly, you have -200 and; 200 400, 600. And this is 0.2, 0.4, 0.6, 0.8 and 1. So this is index of optimism. And for A1 to A4 you can have the lines. So suppose for A1 it is minimum value is on this side it will be minimum is 100 and this side is 400, so 100 to 400. Similarly, for the second one it is -200 to 600 so it will be -200 to 600 so like that. So basically, this line will talk about these which criteria to be taken.

So this will be talking about those values and it will; you can calculate from here, you will say that is negative 40, and in this side it will be coming as 160 something like here. So this way this

line also can be tell you, that where this points are lying and based on that, if you take the degree of optimism quite higher side so you will see that this will be coming in that case. So if you take the higher value of this, suppose if you take 0.8 in that case it will be 480 and this will be 2 so it will be quite high so that is what it is taken using this graph.

So this graph also helps you understand that how the degree of optimism can be you know effective in telling you which alternative should be chosen. The next criteria is the Minimax regret criterion. Now what is regret?

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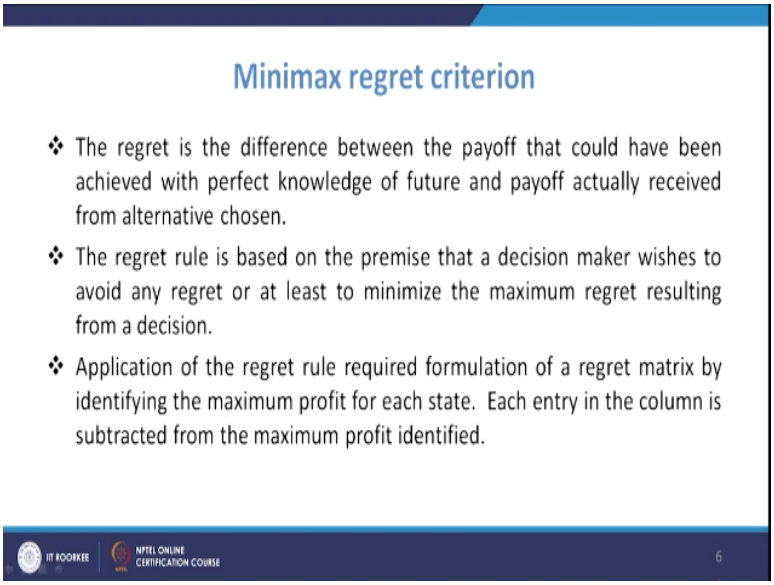

Basically, the regret is the difference between the payoff that could have been achieved with perfect knowledge of future and payoff actually received from alternative chosen. So basically, you know, every time there may be a sense of regret if somebody has got some alternative some outcome. He may always have the regret that he could have got the best one. So there is always a regret and based on that there is a rule which tells that decision maker; a decision maker wishes to avoid any regret or at least to minimize the maximum regret resulting from a decision.

So that is what he feels to have and application of the regret rule requires the formulation of a regret matrix by identifying the maximum profit each state. So basically, you are going to have a regret matrix and for that you are identifying the maximum profit for each state and then each entry in the column is subtracted from the maximum profit you know, that is obtained. So that

way you are going to have you know these values. So you are obtaining one regret matrix and if you take the, you know, if you choose; if you look at the matrix which we have seen here.

Now what we see that for you know, for any matrix if you look at here, now in this case this; the every future you have to find this regret matrix and what you have to do is that you have to see that what is the maximum profit and that has to be you now subtracted basically, so from that you have to see the you know, the amount of regret for that. So in this case, you know the maximum profit is 100.

So you have to subtract you know; so for this the regret will be 0 whereas for A2 the regret will be 300. For A3, regret will be 100 and for A4 it will be 0. So that way we you know, calculate the regret matrix. So for A1 you know, for alternative A1 and for C1 future the regret values will be 0, 300, 100 and 0. Similarly for C2 for future 2 you have maximum profit is 300. So for 300 the regret is 0 whereas for others the regret is 200, 150 and 100. Similarly, for the C1 and C2 you will have the regret of 0 if you get A2 otherwise you have regret of 200, 100 and 400 for A4.

And then you are going to have the maximum value these regrets you know, for the alternative, so that way and then among them you are going to have the minimization. So what you do is, in this case you first of all you calculate.

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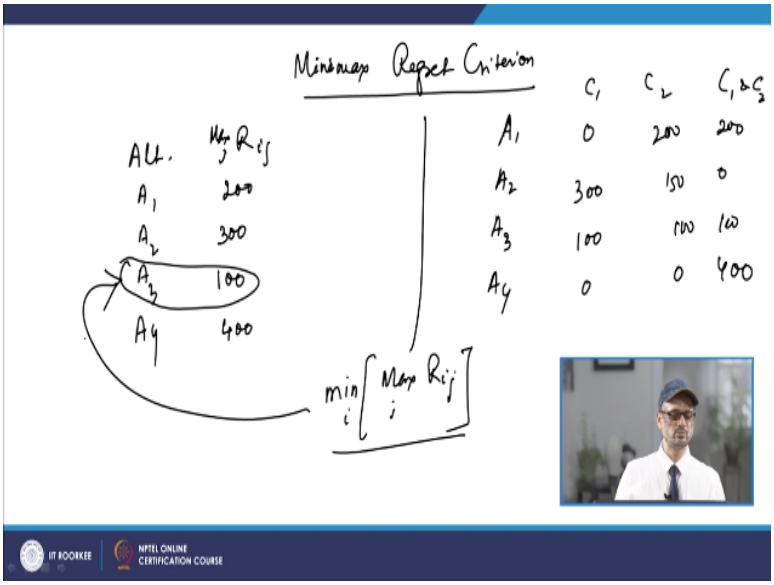

So this will be Minimax Regret criterion and in this; as you said that it will be first maximizing that you know, this regret and then among them you will be taking that minimum. So for that first of all you find the regret values and as we discussed that for alternative A1 and for C1, C2 and C1 and C2 we found the regret matrix and that was 0, 200 and 200. Similarly, this was 300, 150 and 0; and this was 100, 100, 100; and it was 0, 0, and 400.

Now in that what you do is you are getting the; so this is, you are getting the minimum of the maximum you know, regret. Now for in this the maximum regret if you look at, so if you see for alternative you know A1, A2, A3 and A4 the maximum regret that is i, j that is for maximum j, it will be for maximum 200; it will be 300; it will be 100 and it will be 400. Now what you do is you are doing the minimum of this maximized regret values, and in that case the minimum value is 100.

So you are taking this A3 as the; so basically you are telling, taking minimum you know of maximum of the regret, you know are i, j. So this values comings out to be comes out to be for the alternative A3, and in that case you have the chance of minimum regret you know, in such situations. So many a times people need to, you know they are very much you know, still they have a strong criticism for them this type of criterions are suited.

So these are the different criteria in the case of uncertainty and in fact what we do is we, then it is left upon us to decide which alternative should be taken. And for that what we do is, we summarize all these criterions and then based on that we decide which are the alternatives by most of the criteria; it is said to be the good one, so in that case we take that alternative for the decision making. So that is the cases for the decision making under uncertainty. Thank you very much.