Financial Mathematics
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Lecture – 52
Decision-Making Under Risk

Welcome to the lecture on decision-making under risk. So in this lecture, we are going to further elaborate on the different methods, criterion which are helping us to decide under the cases of risk analysis uncertainty. So first of all decision based on assumptions are called as decision making under uncertainty. Then the uncertainty refers to the variation in actual values due to errors in estimation because of insufficient information about the factors affecting the final outcomes.

So these are the reasons because of which there will be uncertainty or the variation in the actual values. And also the risk is defined as the variation in the results because of randomness. So many a times we do not know what is going to come when we talk about the future outcomes or what is going to happen. These are all the probability things. So the probability is associated with them.

And in those cases, you have to understand that there is randomness involved, there is probability involved and how to take into account these things. So we can say that we can understand it with one example. Then what happens that there are certain situations, certain cases by which you can think of. Suppose we talk about certain outcomes of, you have a decision evaluation matrix and it will talk about, we have any contractor is there.

So somebody has got a contract and in that, there are 3 possibilities like, you may get the contract 1, you may get the contract 2, or you may get contract 1 and 2. And then there are 5 alternatives basically. And the way you have to give these contract work to be done. And then all these alternatives like A1, A2, A3, A4 and A5. Then they will be giving you these profits. So you will have a profit matrix.

Now in that you have to decide that how to decide that which of these alternative is going to be

better. So that way this is a case of decision under the case of uncertainty. And in that, there are many criteria by which we are doing that work.

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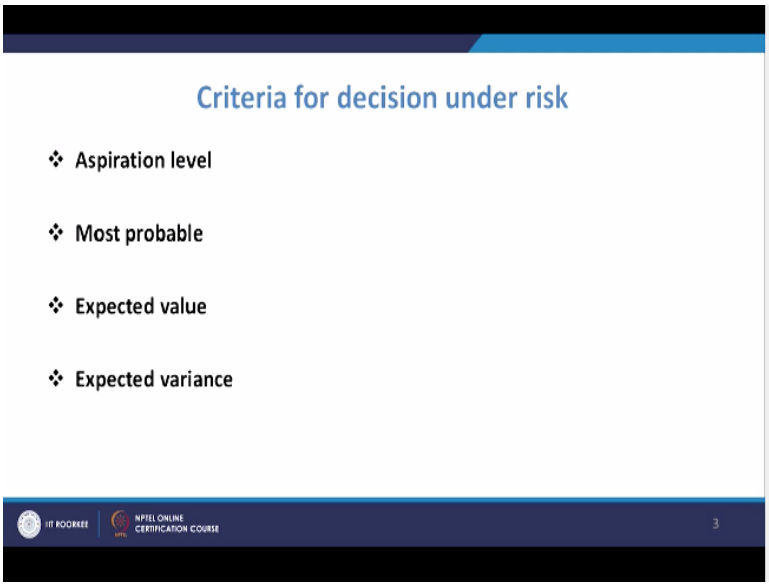

So whenever we have these alternatives and among them, we have to decide, then these are the criterias by which you will decide. So first of all we will have the dominance criteria. Suppose for every alternative you have been given certain profits and if you feel that any alternative which is chosen, it is giving the profit inferior to all of them. So that basically dominated by all the criteria, all the alternatives.

So that can be neglected. So that is known as the dominance criteria. Then comes the aspiration level criteria. An aspiration level criteria is talking about those alternatives. So basically the company has certain minimum aspiration that what should be the minimum level of profit or what will be the maximum loss which the company can sustain or think of. So all these are the aspiration level.

And that is set by the company. So once it is set, in that case among the existing alternatives, you can think of which one is not fitting into it. Because if some alternative is giving more than that aspiration level of loss, minimum, I mean maximum loss, if it is giving more loss then in that case that can be discarded. Or if something is not giving minimum, that much of profit also, so in that case that also can be just discarded.

So that way you have the aspiration level. Similarly, you have the most probable criterion. So in that we will go for those alternatives which have the maximum probability. So among them, you will have, so for suppose you have, the firm has to give 3 type of few charges on the contracts. And that can be subcontracted to 5 alternatives. So for that, whichever has maximum probability, among them the alternatives will be chosen and the alternative which has the maximum value that will be as the most probable criterion.

So which is most probable criterion, we can select those values. Similarly, you have the expected value and we know that expected value, how to calculate. So you have to, you have the probability values and based on that probability values for every alternative, you have to calculate the expected values and wherever the expected value is found to be maximum, that alternative can be chosen to be the alternative with the, the best alternative.

Then also when we get the expected value, then many a times you have, expected value may not differ in many situations. But then expected variance, so we know that the variance which is calculated, the variance must be minimum. So we can calculate in those cases where the expected value does not differ much. In those cases, we calculate the variance and based on the variance, we can prove that these are the options or these are the alternatives.

These alternative has minimum variance. So that can be preferred. So these are the different criterion for decision under risk. And that can be understood by referring to certain examples. So let us understand it by an example. Suppose one municipal corporation is trying to bid the contract and a firm has opportunity to bid on the contract.

And the firm, there are 2 related contracts. So that will be related to the different areas. Now the thing is that the contract may be given either C1 or you may be given C2. So the contract may be either C1 or C2 or both, C1 and C2. Now for this, there are probabilities associated. And for C1, the probability associated is 0.3. For getting that C1 contract, the probability is 0.3. For getting that C2 contract, it is 0.2. And the probability of getting both the contract is 0.5.

So C1 relates to something like installation of hardware and software and then C2 will be something like for the distribution network or so. So there are 2 contracts. And for a firm, either it can be C1 or C2 or C1 and C2.

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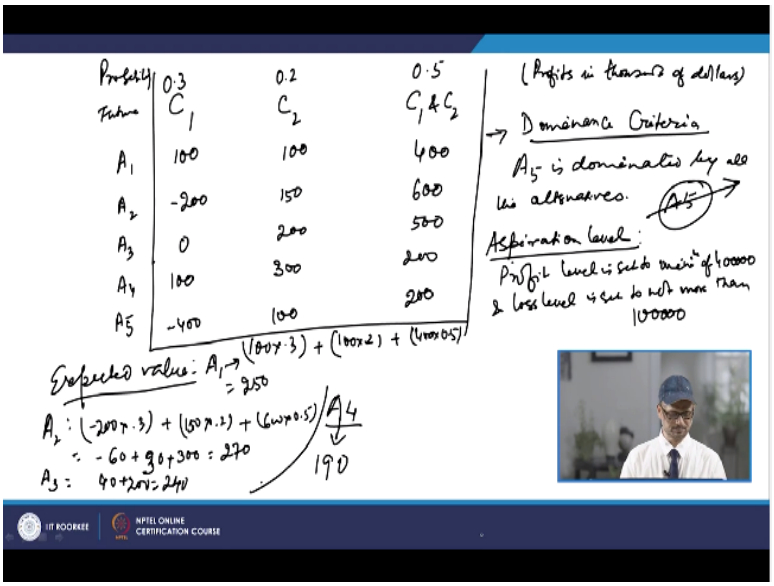

Now again there will be 5 alternatives to get this task done. And if you look at, so that is why you will have the future that is your alternative will be A1, A2, A3, A4 and A5. So you will have, these are the future and this is your probability. So probability will be for 0.3, 0.2 and 0.5. And these alternatives are given. So they will be doing some part and doing not of the parts. So that way if these alternatives are chosen, then the profit matrix, so that will lead to certain profit.

And this profit which is being found, that will be in terms of dollars or thousands of dollars, it will be like 100 and 100 and suppose 400. So that will be your profit values. So the values are profit in thousands of dollars. So these values are there suppose for the A1. And similarly for A2, it is -200, then 150 and 600. Similarly, for A3, it is 0, then you have this as 200 and 500. Then this is 100, 300 and 200.

And A5 is -400, 100 and 200. The thing is that the firm anticipates basically the 100,000s of profit if A1 is given to C1. And if C2 is secured, then it is expected 100,000s of profit. And if both C1 and C2 is secured, it is expecting 400,000s of the profit. So that is what it means. So based on that, what will be finally, how can you say that which of the alternative should be the

best alternative.

So what we have discussed about, so first criteria will be your dominance criteria. And if you take the dominance criteria to decide that which one is having the minimum value of profit for all that. So if you look at that, what you see that this is having; if it should be, it is not more than any one and in many cases, it is quite less than others. So if you look at A5, A5 is inferior to all of these.

So this is being dominated by all of the alternatives. So that is why A5 is dominated by all the alternatives. So it is inferior to each of the alternative and that can be neglected. So we are neglecting A5 here. So A5 is going out. Now we will be talking about the another criteria, so that is your aspiration level criteria. Now aspiration level tells that company fixes certain minimum of the aspiration.

Company has certain aspiration when it tries to have the profit, a minimum of the profit or maximum of the loss. So suppose that profit level is set to be at least 4 lacs. So suppose the profit level is set to minimum of 400,000. And the loss level, that is set to not more than 100,000. So if that will be the situation. Now in that case, what happens that your profit level is minimum of 400,000 and loss level is not more than, maximum value is 100,000.

So in that case, what you see that if you consider A2, A2 has the loss of 200 for C1 if it gets. For C1, you know it is 200,000 of loss. So it does not fit into or it does not meet that minimum aspiration level. So A2 does not qualify for this. Similarly, if you look at the profit also, now what you see that this A1 and A3. So A1 and A3 basically will be the one. Now this basically does not fit even in the profit.

However, from the loss account, you cannot, from the profit account also it is not acceptable. So your acceptable alternatives are basically the A3, that is having at least the minimum of 500 or you have the A1 has the 400. So among the A1 and A3, you can go for the, so you can take from the aspiration level criteria, you can choose among A1 and A3 and see that which one will be better.

Then further, to decide further, you have another criteria that is your most probable future criteria. Now most probable future criteria that is which one to be taken, now in that case, you have futures are A1, A2, A3, A4 and A5. Now in that, what you see that the most probable is for the contract C1 and C2, that is 0.5 and in that, if you see, you have the maximum value is for 600.

So that way if you look at the most probable future criterion, then alternative A2 is the preferred approach because under this, it is having the most preferred criteria. So that way A2 can be preferred but if you look into both the things like even the aspiration level in both, then among A1 and A3, it will be taken. So in that case, then A3 will come into picture. So that way you will have the criteria for taking the decisions.

Then comes these expected value criteria. Expected value decision making criteria. Now for expected value decision making criteria, you have to find the expected profit for all these investments and you have to calculate like for A1 suppose you want to calculate, then it will be like 100*0.3+100*0.2 and +400*0.5. So it will be 30+20, 50+200. So it will be 250, 250,000. So this way we can calculate the expected values for all the alternatives. So now we can calculate for A2 in fact.

And for A2, it will be -200*0.3+150*0.2+600*0.5, so it will be something like -60+30+300. So this will be coming as 270. So you can calculate for A3. A3 will be 0*0.30+200*0.2, 40, 40+500*0.5, so it will be 40+200. So it will be 240. Similarly, you can calculate for A4. And for A4, it will be 100*0.3, 30, +60, 90+100, so it will be 190. So if you go for A1, A2, A3 and A4, if you find A1 is getting 250,000, for A1, 250,000 profit, this is 270,000 profit.

A3, 240,000 profit and this is 190,000 profit. So based on that, if you see, your decision will be based on the expected value, you can go for the A2 which is giving you the maximum expected value of the profits. So that is how these decisions are in whereby affected. And in a nutshell, what we mean to come to the conclusion that ultimately you have to decide keeping the different criteria in mind, like you have come using the, dominance criteria you have removed one.

Then using the aspiration level criteria, you have removed few of them. So you have 1 or 2, is there. Then using the expected value criteria and using the most probable future criteria, you are coming to some conclusions. And based on that, you can finally decide, the person has to decide which of the decision criteria should be taken so that the expected value, the outcome or the return will be maximum in such cases.

Now as we discussed that because these are all the, so ultimately you will have the decisions to be compared and then you have to come to the conclusion that which of the criteria becomes the better one. When we talk about the use of such methods, now many a times we used in many situations like what should be the decision like, there are examples like somebody wants to make a dam of certain height in a village and on a river because every time the river is creating the floods.

And the flood is doing the damage. So you will have the histories of last many years. And damage also is shown. So damage certainly will be proportional to the amount of, the level of river which is crossing certain limit. So that way, and there is option that you can make a dam of certain height. Now this height can be of different values. If you make the dam of larger height, the chance of losses, the losses which are incurred because of the flood, it will be less.

But if the dam is, but then that making the dam of larger height requires initial investment more. Similarly, you have the, another option is that you make the dam of smaller height and that will take small investment, initial investment. But then that will certainly give you more losses in terms of floods. So in those cases, what you can do is that you have to have a pattern of the previous years that what had been the river level, what was the losses in those years.

And if you make a dam, then if you make a larger dam or a smaller dam, then how that is going to affect. And based on the present worth value, how much you invest and what will be the losses, based on that, you can come to this conclusion. So there you use these expected value decision making methods which is the most simple methods for calculating the expected value where you sum these, take the probability values.

And depending upon the probability value, you can calculate what will be the loss or what will be, or the probability of having that river level and based on that what will be the losses. So based on that loss amount, can we calculate it. So these are the uses of expected value decision making criteria. Similarly, many a times, what happens that when you come to the value of expected value, few values using the expected decision making, in that, you do not see much of the difference in 2 options.

So in those cases, you have to use these expected variance decision making when the outcome is uncertain. So that time, you have to have the value of the variance and then you have to come to conclusion. One of the very live example may be related to the introduction of new product for certain company. And if it wants to launch a new product, in that case, future cash flows which are expected.

Suppose the company is going to invest 3 lac of rupees for the product and there are few possibilities. In the first case, it may give some decreasing type of profits or so or the outcomes that is 11, 10, 9 and 8. So that will be the cash flow. Similarly, in the second case, it will be expected to give 11,000 of cash every year for the next 4 years. And in the third case, it is giving 4, 7, 10 and 13.

So 40,000, 70,000, 100,000 and 130,000. So in the case of 3 lac of investment. And here in the first case, it was 110,000, 100,000, then you have 90,000, then 80,000. So that way if these are the outcomes. Now in those cases, if you want to decide and suppose for the first case, the probability value is given like for the first, it is 0.1%, I mean 0.1 that is 10%. Second case, it is 30% and third case, it is 60%.

So in all these cases, you have to assess the present value of the investment and then you have to multiply with the respective probabilities. And that way, it will be your expected value of the present worth. So basically the company is giving certain investment and then it is getting certain return and there are 3 possibilities. So the return will be in future and the investment is today itself.

So basically that can be compared by finding the present worth. So for these 3 things you can have these present worth values multiplied by respective probabilities and then you can have the expected value of the outcome. And if it is positive, you can say that yes it is worth. Or if it is negative, then you can say that it is not worth to start such a venture. So these are the use of expected value decision making.

And as I told that depending upon the situations when you have the options, when you are not able to decide based on only the expected value, then you have to find, you have to go to other methods like you have the expected variance decision making. So you will find the variance and you will see that where the variance is smaller and those values, those options which has the lower value of variance is there, you can say with more definitivity that this option will be better.

So that way you can go for analyzing using the expected variance decision making processes. So these are the criterias for decision making under the risk. And we must be knowing that because this will be useful in analyzing the different situations. Thank you very much.