Financial Mathematics
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Lecture – 51
Introduction to Risk Measurement

Welcome to the lecture on introduction to risk measurement. So in this lecture, we are going to have some concepts about certain terminologies related to risk measurement, specially the risk related to return and in that, we will have having the introduction to certain terms like what are those parameters by which we measure those risks and what is the significance of those terms and all that.

So first of all let us see how this risk measurement is important in the financial aspect. So assessment of risk and return, they are 2 of the most significant determinants while taking major financial decisions. So when we talk about financial securities like shares or bonds or other investment assets like stocks and bonds and other investment assets.

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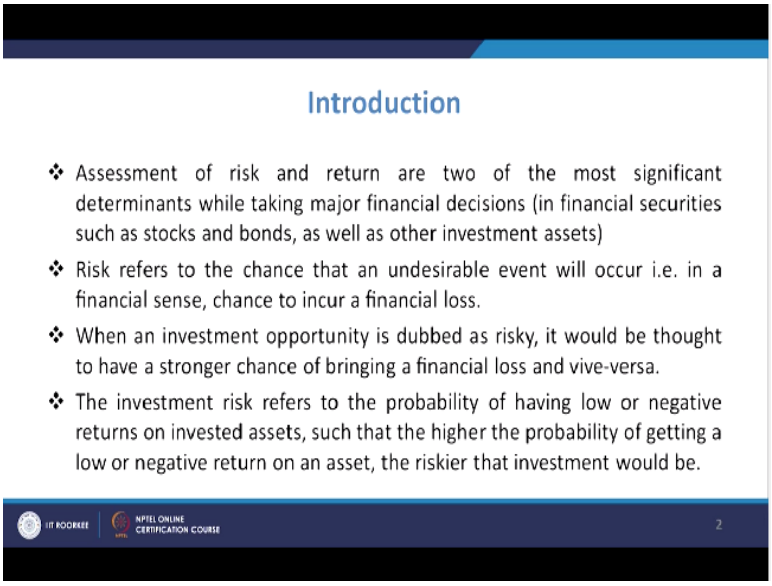

Now in those cases, everybody wants, you must have the assessment of the risk and return because many a times you are investing something for getting some return. Now in many case, you may get large return and in many cases, you do not get such a large value of return. But then there is risk associated whenever, in most of the cases when you are getting the opportunity to

have very large value of return.

At the same time, there will be risk involved. And if you have no risk involved then there is no return. So all these things are there. Risk refers to the chance that an undesirable event will occur that is in the financial sense chance to incur a financial loss. So it will be a chance that something undesirable may happen. And because of that, there is quite a possibility that you can incur the financial loss. So when an investment opportunity is dubbed as risky, what does it mean?

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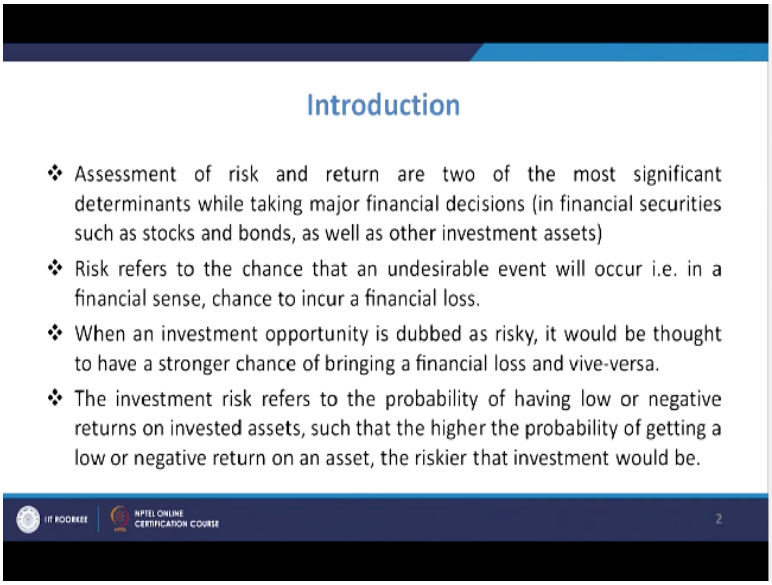

It will be thought to have a stronger chance of bringing a financial loss and vice versa. So whenever we tell that this investment opportunity is risky, it means that there is strong chance of the financial loss into that investment. Now investment risk refers to the probability of having low or negative returns on invested assets. That is what investment risk means that you may have low return or may be that you do not have any return. In fact, you are incurring the loss.

In that, there is a negative return and the higher the probability of getting a low or negative return on asset, the risk that investment will be that much higher. So that is what the meaning of risk it is. Now when we talk about the risk or when we talk about the return, what we get in that. We know that what is return, whatever you are putting some money for an investment and when you are getting something back.

So you will be getting something for that as the loss or profit. So based on that, you are getting some return and then there will be rate of return also that will be in terms of percentage. So many a times, we try to quantize it or try to express it or we try to calculate it when there is risk involved. It means there is chance, there is probability. You are putting an investment, so certainly there is risk involved.

You cannot say that you will be getting this return. You can get the return even more; you can get the return even less. So based on that, you have to calculate. So based on that, there are certain terminologies and among that measurement of risks, so that is how risk is measured. Now what are those parameters by which you can measure? So first is the expected rate of return.

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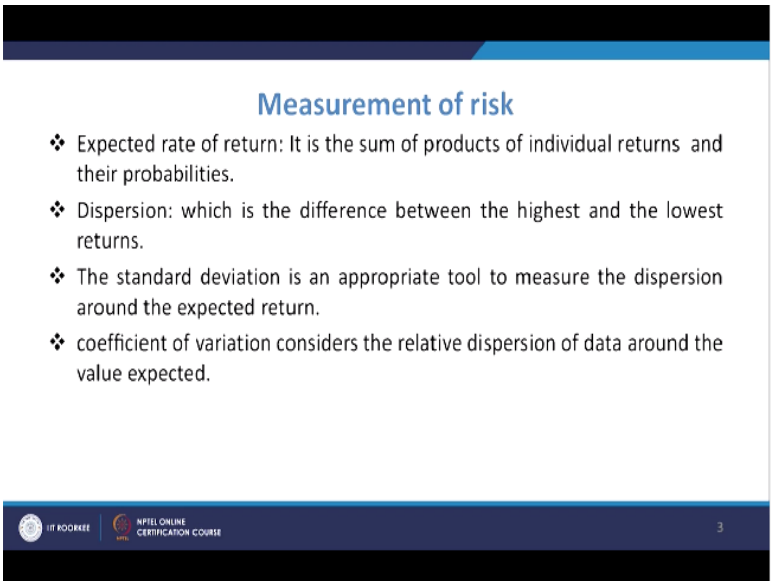

So expected rate of return means it is the sum of products of individual returns and their probabilities. So what happens that many a times, you have certain individual returns and also the probability is associated with that. So in those cases, you have to define. So this definition is there for the expected rate of return.

In that, you multiply the individual return with their respective probabilities and then we sum it. So that will be your expected rate of return. So for example, so this is basically represented as expected rate of return, Ke and K is the individual return. So Ki is suppose the individual return. And Pri is the associated probability, so that is your Pri will be their respective probabilities.

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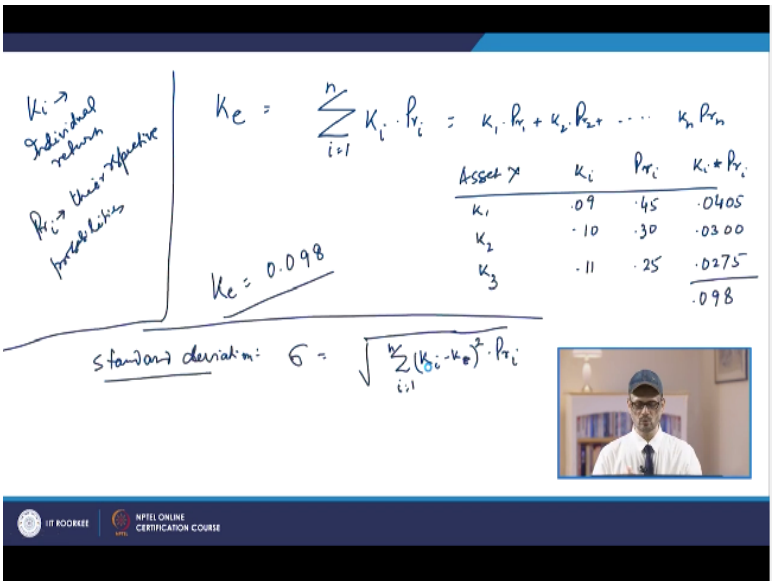

So this is for certain investment suppose. Now in such cases, there will be expected rate of return and this will be calculated as summation and i-1 to n, and then that is your Ki*Pri. So that is your ith individual return and multiplied by the probability of that return, that particular return Pri. So what will happen, that will give you, we will be taking as K1*Pr1+K2*Pr2, like that. So it will go up to Kn*Prn.

So this way you can say that you will calculate the expected rate of return. For example, suppose you have asset X is there and this is, its probability is for certain return is tabulated. So if suppose you have the K1 and K2 and K3, that is there. So for that, the return value is 0.09, 0.10 and suppose 0.11. So in that case, what will be your, and its associated probabilities are 0.45 and 0.30 and 0.25.

So suppose for getting that return of 0.09 or 0.10 or 0.11, you have the respective probabilities of 45%, 30% and 25%. All together, it is 100. So in that case, when you have to calculate expected return on asset X, so you will be getting a table that will be Ki*Pri. So Ki*Pri. So if you multiply 0.09*0.45, so it will be 0.0405. Then it will be 0.0300 and this will be 0.0275. So all together if you add, it will be 0.098.

So you can say that this 0.98, it is the expected value of the return. So expected value of the

return is computed out to be 0.098. So certainly we have seen that for 0.098, it was 45%. For 0.10, 30 and 0.11, 0.25. You can see from the Pri values also, if you see that, it is 45% and then it is 10% is here, 30% is for 0.10. So in between, it will be closer to this largest value. So it is coming as 0.098.

So this is how the expected value of the return is measured. Then the next is dispersion. So dispersion is the difference between the highest and the lowest returns. Now how does that. So basically when you have the returns of suppose 9%, 10% and 11%, so in that case, the dispersion will be basically the difference between the maximum and the minimum. So if you try to see the asset X, now its range will be something like 2.

So this will be; so 11%-9%, so that is your dispersion that is 2%. So depending upon different values, you can find these dispersion values in such cases. So accordingly if you have another investment and that is asset Y is there and in that case, if the value is suppose 5%, 10% and 15%. In that case, the dispersion will be 15-5, that is 10. So that way you will have a dispersion. So that will basically tell you that what is the range in which this is likely to come.

And then in that, if that is more than, variability is basically more, so certainly that way the probability will differ. Now what we see that, so it means the variability is more, so risk will be more. Now the next important term which is very important while measuring this variability, that will be basically; or to measure the dispersion, is the standard deviation. Now this standard deviation is basically appropriate to measure the dispersion around the expected return.

So this will be basically when you have the expected value of the return that we have calculated. Now around that, what will be the dispersion for that. This is a very important tool by which we measure with this standard deviation. And standard deviation basically when we talk about, so we define it like, we denote it with sigma. And this sigma basically will be done by the root of summation of Ki-Ke and then that square*Pri.

So this way you are getting the root and i will be from 1 to n. So depending upon the number of data, you will be calculating. This is Ki, this is your individual return values and this is your

expected value of the return. So you will be taking the squares of this and multiply it by the respective probabilities and then we sum it and then we take the square. So that basically tells you the standard deviation.

So here what we see that, Ki-Ke, if you look at the difference, the difference when the data is varying above and below the expected mean. In that case, there will be sum positive values and there will be some negative values. And if you simply take the algebraic sum, then many a times, they cancel each other. So basically thus total deviation if you look at in normal sense when you only take an algebraic sum, then in that case, it may not be the true representation of how it has varied, either it has.

So variation is nothing but either it is positive or negative which is all the variation from the mean value. So for that, what we do is do the square and then we multiply with the respective probabilities and then we sum it and then we take its; the square root. So that gives us the value of the standard deviation and that is a very effective tool for measuring the dispersion.

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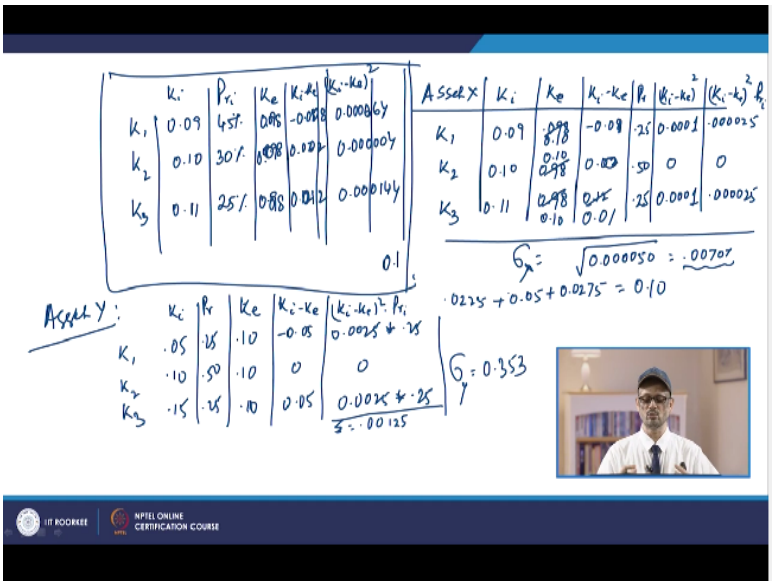

So if you see that how to measure this dispersion. Now we can have certain example by which we can see that how to calculate these dispersion. So suppose you have asset X which is there. And in that, you have the values like Ki. So Ki is you have 2 values. We know that asset X, so you have 3 opportunities and the probabilities are associated as 0.09. So that is not probability,

this is the return percentage.

So 0.09, 0.10 and 0.11. And its respective probabilities we know. But we calculated the expected value of the return and that is your 0.098. So that is what we have calculated. Then what we calculate is, we calculate the Ki-Ke. So for Ki-Ke, once it is taken the difference, it will be Ki-Ke. So -0.08, this will be 0.02 and this will be 0.12. So we know that we already defined its probabilities as 45, I mean, 0.25; 0.50; and suppose, 0.25.

Suppose these are the probabilities associated in that case. So if it is 0.25, so we can get these values like 0.25*0.09, so it will be 0.0225. And +, this will be 0.05 and then this will be 0.275. So this if you look at, so this will be 50+, so this will be 500 and this is 225 and 275. So 225+275. So basically this will not be the value because if you add them, it will be 0.10. So we can have this as the modified one, 0.10.

This is also 0.10 and this is 0.10. So in that case, you will have -0.09 and -0.10, so it will be -0.01 and similarly you have this here as 0, and here you will have 0.01. So that way, it will be varying and then you can take these squares. So Ki-Ke is squares you can take. So it will be 0.0001, this will be 0 and this will be 0.0001. So this will be multiplied with these probabilities. So Ki-Ke square*Pri.

So this will be multiplied with these probabilities, so you have 6 digits after decimal. So this will be 25, this will be 0 and again this will be 0025. So in fact, what you can do is, you can find the standard deviation by summing all these. So if you sum them, that will be 0.000050. So if you take that, it will be something like 0.007, something like that or 707 or so. So this will be your standard deviation for such cases.

Similarly, you can have, you can even calculate for the earlier cases where you had these value as Ki and the respective probabilities were 45, 30 and 25. In that case, we had when you had K1, K2 and K3, values as, so Ki was 0.09, 0.10 and 0.11. And the probabilities were 45%, so 0.45, 30% and 25%. Now that was the case in our earlier problem. So it was 0.45 and 0.30, 0.25. So in that case, your Ke was 0.98, that is what we had calculated.

And if you get the Ki-Ke, so it will be -0.08 and this will be 0.02 and this will be again -, so 0.12. So that will be your, so this is not 0.12, this is 0.12. So this is 0.09, so -0.08 and if you, this 0.11, so that is your 0.09, 0.10 and 0.11. So in that case, it was 0.098, note that we have done some mistake. 0.098. So this will be 90-, that is 98, so it will be 0.008. So if you look at these values, so it will be 90-98, so it will be 008 and this will be 100-9, 0.002.

And this will be 110-98, so 0.012. This is how your things will come up, you will take the squares of these values. And this will be something like 0.000064 and you will have this as 0.000004 and this will be 0.000144. So this way that is to be increased and then this will be multiplied with the Pri. So with 0.45, 0.30 and then you can sum it and you can get these values.

So you can check, you can calculate these respective values and you can find the expected rate of return. You can find the standard deviation in such cases. So this will be for a particular asset X. Now suppose for asset Y, if you look at for the asset Y. And if you try to calculate the value of the standard deviation and for asset Y, again the data is given like you have K1, K2, K3 is given as 0.05, 0.10 and 0.15.

This is your Ki and its probability is given as 0.25, 0.50 and 0.25. In that case, you can calculate the expected values. Expected value will be 0.10 because you see that this is a symmetry. So it will be 0.10. So you can get the value of Ki-Ke and Ki-Ke will be -0.05. This will be 0 and this is 0.05. So its square*, and then you have to multiply also will these probability that is Pri. So it will be -, so this will be 0.0025*0.25.

This will be 0 and this will be 0.0025*0.25. So if you take its sum, in that case so it will be 0.00125. So if you sum them, it will be, summation will be 0.00125. So you have to take its standard deviation. And sigma Y if you look at, it will be, you can have its square root, it will be 0.353. Now what you see is sigma X was coming as 0.00707 and sigma Y is coming as 0.353. So it has a meaning. If you try to find its deviation, if you find its standard deviation, if you find the probability density, now if you try to show that how its variability looks like.

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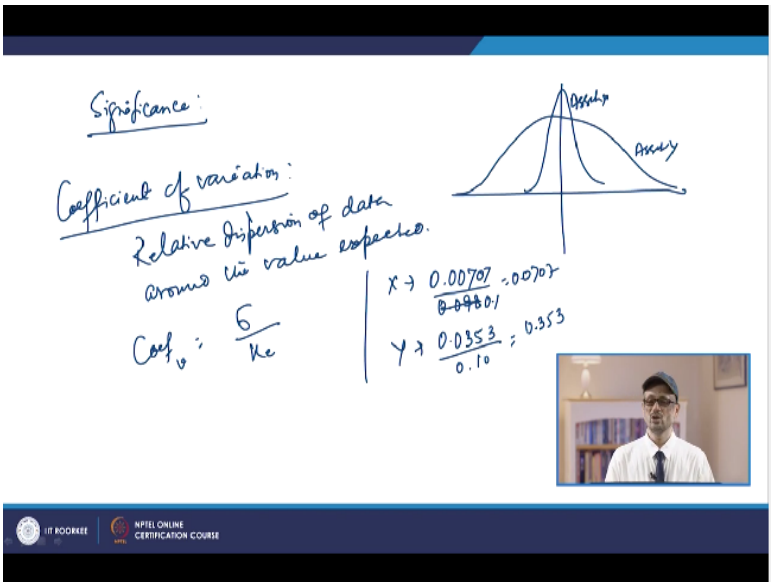

So you can show it using the curves like, this will be showing the less variability and this will be with the larger dispersion value. So this will be for asset X and this will be for asset Y. So that is how these standard deviation values are represented or they are seen. Now if we assume that these probability distribution is normal, if we assume that. Then what we see that for the asset X, 9.8% which is the expected value, expected return for asset X.

Now it will be within +- of the 1 standard deviation that is your 68.26% of the time. So what is the significance of these. So if they follow this normal distribution curve, in that case, it means that there is 68.26% of the time, this is going to be within +- one times the standard deviation. So standard deviation is your, you know that the standard deviation for that. So you have calculated the expected value.

And expected value if you have calculated, so you can see that it is, you can take the +sigma and the -sigma and for that, you will have for 68.26% of the time, it will be falling within that. So similarly if you go to 3 sigma left and 3 sigma right, then you are likely that it will be taking about 99.75% of the time, it will falling in that particular range. So that is what the meaning of these curves are.

Now there is another important reliable measure of the risk. And that is known as the coefficient of variation. Now what is, even as we rate this standard deviation even better, we call it for the

coefficient of variation. And it is basically talking about the relative dispersion of data. This is the relative dispersion of data. So this is basically around the value expected. So we call it as the coefficient of variation.

And it is denoted by coefficient of variation and this is a ratio of standard deviation to the expected value. So you have the asset X and asset Y data and you can find the coefficient of variation for these 2 values. And if you find the coefficient of variation for X, it will be, we have calculated, it is something like 0.007 and then 1 or so and divided by 0.098 or that was calculated, that we have not calculated anyway.

For the first case, we have done. This was 0.0707 and that is for expected value of 1. So that was 0.1 in fact. So this was 0.1. So that way you had calculated. So that will be 0.0707. Similarly, for Y, we calculated 0.0353 and that was the standard deviation and this was, expected value was 0.1. So it is 0.353. So when you have higher value of coefficient of variation, in that case the investment seems to be more risky.

And similarly if you have the lower value of the coefficient of variation, in that case, your investment seems to be less risky or so. So these are the standard terminologies which are basically useful when you talk about finding which of the investment is more risky or less risky and so. Thank you very much.