Financial Mathematics
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Lecture-05
Problem Solving on Mathematical Functions and Statistical Measures

Welcome to the lecture on problem solving and it will be related to the topics of mathematical functions and statistical measures. So, we have discussed about the mathematical functions we talked about the behaviors of the functions then mathematical terms. We also studied about the statistical measures in the previous lectures, we studied about the mean and standard deviation, variance and all that.

So, we will have some problems which will be solving and that will help us in also solving the assignments as well as for the exams. And it will also make us our concept clear about these kind of problems which you intend to get in the exams and you will have the concepts clear about it.

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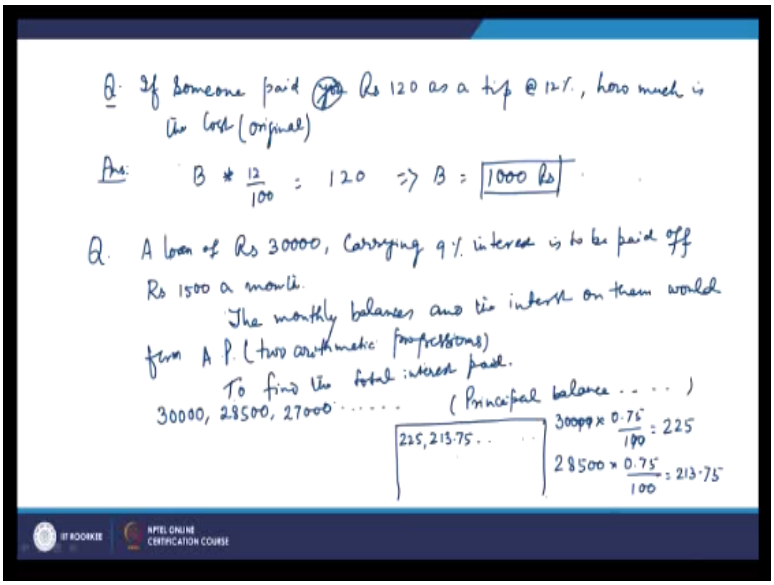

So, let us go to the first question and suppose we discussed about the interest you know rate you know or percentage with the percentage we will have a question. Suppose there is a question like if someone paid you so paid you rupees 120 or someone paid not you but anyone. So, someone paid you a rupees 20 you know as you know a tap that is you know at the rate of 12% ok.

So, and you know at 12% he has somebody has received 120 and it is at the rate of 12%. So, you know how much is the original cost, how much is the cost. Basically you know original cost of any service you have you know suppose in the dinner you are going and you are giving a tip you know of is 120 , so what is the cost of dinner suppose.

So, that you can find it, so you have to use the you know concept of the % and if suppose x is or any amount original amount is suppose b and you know b*12%. So, 12/100 and it should be coming out as 120, so for this way you can get b as 1%. Suppose 1000 rupees this way when you are that is the concept of percentages which you know can use to get these amount.

Next question similar to 1 is suppose you know you are taking a loan and that loan is of rupees 30,000. Now there is a interest of carrying 9% interest, now it is to be paid off you know rupees 1500 a month. So, what you see is that you have to you have a loan that is of rupees 30,000 and you have the 9% interest is to be paid and also you are paying 1500 a month.

Now what we see is here is that the interest rate is basically on the amount which is remaining. So, the monthly balance now what do we see is that every time you are going to pay the you know interest and then that way you will have the balance leftover that is remaining that is lesser. So, your interest paid will be again lesser, so that will be buy in amount and that will faraway progression.

So, you have to find the total interest, so the monthly balance and the interest on them would fall AP. So, basically 2 arithmetic progressions, now you have to find the total interest paid. So, what we see is that you have this 9% interest and this 9% interest is for the year, so for a month you have to calculate that amount. So, now what we see is that you have 9% interest for the years, so you can here roughly estimate that you are paying.

Basically you know 9/12 that is 0.75 interest rate/month, so what we do is that your balances if you look at the balance is in the first month if it is 30,000 is remaining. Now how much will be remaining so you are paying you know 1500 a month. So, your that will be forming this 28500 then again 1500 will go, so this will be 27,000, so the principle balance will be having this series.

Now the thing is that the interest will be paid on these balances, so if you try to see the interest which is paid. Now that interest will be 30,000*0.75/100, so 0.75% so if you look at the monthly interest that will be you know 30,000 into 0.75/100. So, that is how you will find the interest, so if you look at that so it will be you know this 2 will go from here and you will have 225.

So, you are getting 225 rupees similarly if you look to the next term you will have 28500 and then again 0.75/100. So, this way you will have so here it will be 213.75, so thus the progresses, so the amount of interest which you are paying basically it will be going like 225 to 13.75 or so. So, your interest so is the interest has a series and interest has a series of 225 213.75 then that way.

So, you know the first term and you know the difference and difference will be 213.75-225, so total interest paid how much will be. Now what will be the number of times you are paying, now number of times you are paying is 30,000/ 150 or say 1500, so it will be 20. So, basically you have to find 20 times, so if you find the total interest which is to be you know paid, so it will be so total interest paid will be you know, so 20/2.

And then your 2a+/-1*d, so 2*225+ 20 terms are there, so 20-1 that is 19* the difference, so difference is basically you know 11.25 and -. So, you know – of a 11.25, so this way if you do that calculation you will be getting the you know final value of the sum. So, that way and it will come something close to maybe 2362, so this you can have it so if you look at this will be 10 and this will be 450.

And here it will be – something about 200 10 or so. So, then that will be multiplied, so it will be coming as 2362. So, what I mean to say that this way you can have you can use this progression concept and you here we have found the interest how interest is going to be charged. And then that is for it forms a remidic progression, so we got the sum of the n terms of the progression, next we have another problem.

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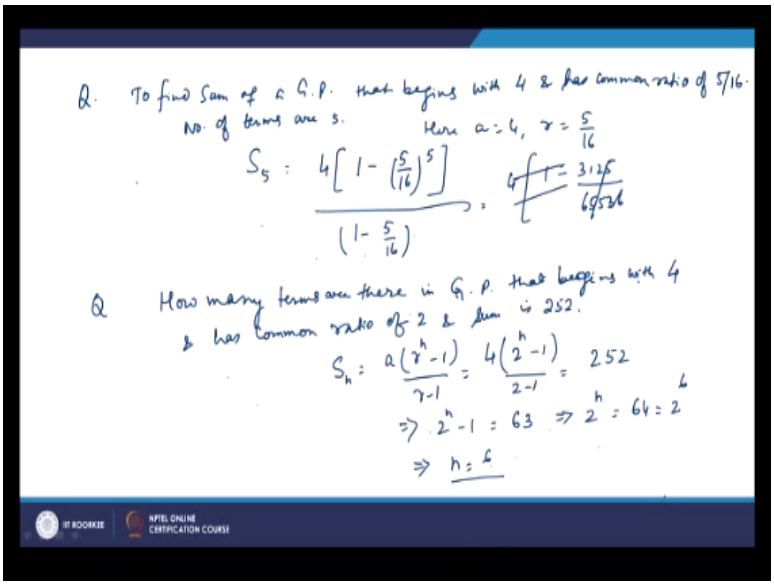

And here we will deal with the problems of the geometric progression and suppose you have to find the sum of a geometric progression GP and it has that begins with 4 and has common ratio of 5/16. And number of terms are 5, so for such problems as you know that you have a geometric progression series in that case if you find the sum of the 5 terms it will be a*1-rn by divided by 1-r.

Because r is less than 1, so we are going to have that, so the first term a is 4, so here as we know a is 4 and r is 5/16. So, as 5 will be 4*1-5/16 rise to the power 5 and divided by 1-5/16, so this way you can calculate and you can get the you know values. So, this way I mean if you look at here it will be something like 4* you know 1- 3125/so that will come something like you know 65536 you know.

It will be something different you can use the calculators and you can solve such problems it is just arithmetic problem, so that way you can solve it. The next question maybe in this that suppose there may be a question like how many terms are there in GP that begins with 4 and has common ratio of 2 and sum is 252. So, suppose such is the question so again you have to use a same you know equation.

Then here you will use Sn will be a*rn-1/r-1because r is you know more than 1, so you are using the term rn-1. So, it will be first term is 4 and you have common this is 2n-1/2-1, so it is basically

you know 1 itself and that is equal to 252. So, 2n-1 is basically 252/4 that is 63, so 2 rise to power n is 64 and 64 you can say that 2 rise to the power 6, so that is why again you have this to be equated, so n becomes S6.

So, this way you can find, so if you have to know that how many terms are required to find a geometric progression which has the first term has 4 and the common ratio as 2. In those cases you have you can use the formula and get the answer as you know has been calculated by the formula.

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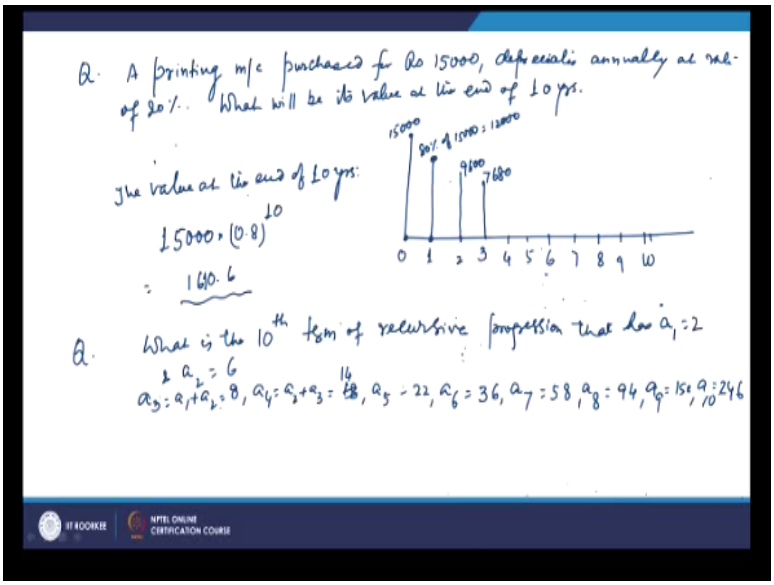

Next we have may have the questions like suppose a printing machine purchased for rupees 15,000. Now depreciates annually you know at rate of 20% what will be it is value at the end of 10 years. So, now again here the things will go in a you know geometric progression series because it is purchased for rupees 15,000 and it will depreciate annually at the rate of 20%. So, in the first year if you look at you know in the depreciation amount will be in the first year 20% of 15,000, so it will be you know 3000.

So, it is value will be something like 12,000, so if you look at you know so you can make a horizon and if you look at, so this is your 0 year. And at so you can take that at the end of here, so 0 year you have this value as 15,000. Now in the at the end of first year this value will be you

know depreciation is 20%, so it is value will be 80% of the 15,000. So, 80% of 15000, so that is 12000 similarly second year you know this is first year.

So this is second year and second year it will be coming it will be 80% of this 12000, so that is 9600. Then in the third year it will further come down 80% of 9600, so it will be you know 7680 something like that it will go. Now you have to find that what will be it is value you know at the end of 10 years. So, basically you are going to find it in the you know beginning of 11th year, so at the end of you know 10 years if you have to find.

Now in at the end of first year it is 80%, so you can directly find the value at the end of 10 years. So, what you see is at the end of first year it is 15000*0.8. So, similarly in this case you will have 15000*0.8 times 10 that is 11-1 basically it is the 11 star if you look at in this progression. So, it is 11^{th} term it is 11-1 is 10, so it is a progression series and there are this is 11^{th} term, so 11-1 that is 10. And you can have it this value and if you calculate this it will be coming something close to 1610.6 or so.

So, you can calculate from here also like it will go further 7680 then it will be multiplied. So, this will be you know *8, so that will be you know after 56 and 56144, so that will be that one so that way it will be moving and you can find the values. So, if you come at the 4, 5, 6, 7, 8, 9, 10 now in this case you will come here and you will get the value as 16 10.uh 61 .

Then we will go to the next question further now will have to be dealing with a question on the progression that we have studied. Now suppose you are asked that what is the 10th term of recursive progression that has a1=2 and a2=6, so if you have to find the 10^{th} term of the recursive progression, you must know the 8^{th} and 9^{th} term of that progression.

And once you know the 8th and 9th term then the 10^{th} term will be the summation of the 8th and 9th term. So, before that we are going to move further, so you will have a1 and a2 given, so a3 will be a1+a2, so that will be 8. Similarly a4 will be a2+a3, so it will be 16 then a5 you know, so know this will be 6+8, so that is 14. Then you have a5 as a3+a4, so it will be 22, a6 will be a4+a5, so a4 14 and a5 22, so that is 36.

So, then you will have a7, a7 as a5+ a6, so that will be 58 and a8 so a8 will be again a6+a7 so 36+58 it will be 94, a9 is a7+a8, so 58+94 so 152. And a10 will be a8+a 994+ 152 so that is 246. This is how if you are given a recursive type of progression then you can find the 100th term of that recursive progression, now we will move to another type of question.

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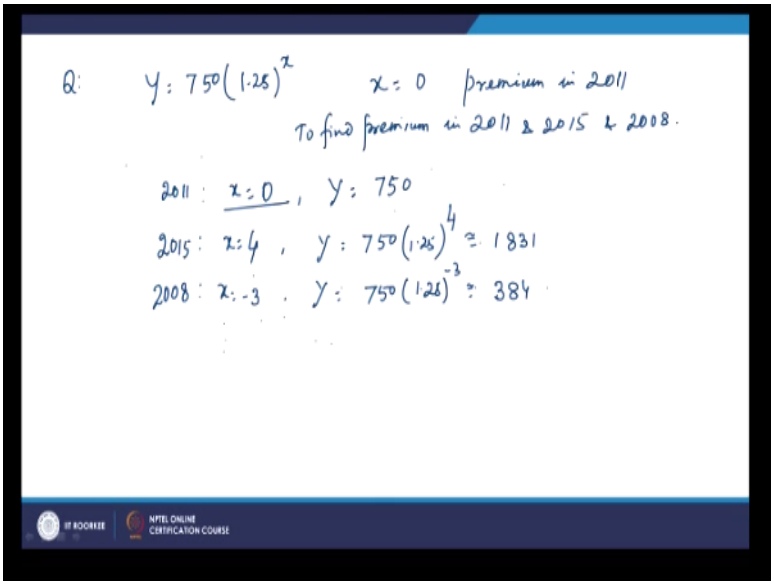

Now suppose you are given a problem where it is told that you have you are giving the annual insurance premium for the you know business transport. And it is estimated by a function that is y=750*1.25 rise to the power x, so this is the for a small business to you know to transport this product to the distributors and this is estimated by this function y=750*1.25 rise to the power x. Now in this case x will be 0 in a certain year and that is you know premium in 2011.

So, 2011 is a reference year at which so if you x is 0 it means in that year it is 750 and it maybe you know in any unit. So, it maybe into 100 of rupees then you know if you take x=1 it will be for 2012 x=2, so it will be or so, so, or if you take f=-1, so it will be you know in 2010. Now you have to find the premium in to find premium in 2011 and 2015 and also 2008 so at in 3 years you have to suppose calculate.

Now you though that when you are moving in the you know in the future in that case x will be increasing, so for 2011 if you look at x will be 1. So, for 2011 it will be x=1, so no it will be 2011

is x=0, so 2011 will be x=0, so you will have your y will be =750, that is all. Because on this rise to the power 0 is 1, so that is 750 units, now if you look out to 2015. So, for 2015 if you look at for that x will be 2011, 2015 you have 4 years of you know advance period.

So x will be 4. So, y will be 750* 1.25 rise to the power 4. So, if you take this value it is coming out to be something close to 1831, so 1831.05 is coming. Then if you go to 2015, so that is 5 now you are coming to 2008, now if you look at 2008, so 2011 is the reference year. So, for 2008 you have x=-3, so y will be 750*1.25 rise to the power -3, so it will be decreasing and then if you calculate that it will be coming as 384 use to 384.

So, this way you can use the function to calculate the insurance premium for an organization when it is following certain function. Next we will deal with some other type of problems suppose you may have a problem related to probability.

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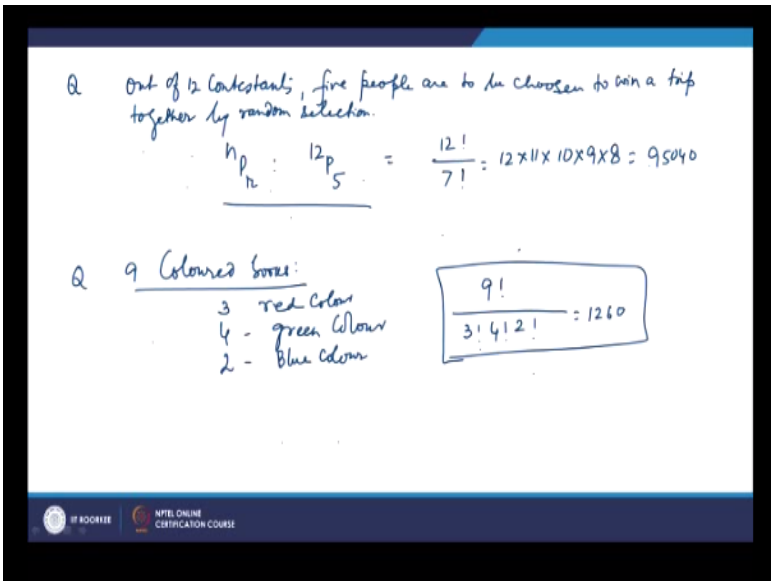

And there the problem maybe of certain types suppose you have 12 contestant, so in order of 12 contestants. Now in this you know 5 people are to be chosen you know they are to be chosen to win a trip together. So, now that is by random selection ok, so by random selection now in this case you have to determine that in how many ways you can choose this 5 winners.

So, as you know that here the concept of the permutation will be used, so in that case you have n as 12 and r is 5. So, the way in which you can choose the person that will be npr that is 12 p5. So, that way you can calculate this 12p5 and you know that 12p5 will be 12 factorial/12-5 that is 7 factorial. So, you will have this value as 12*11*10*9*8, so this value comes out to be 95040, so this way this is how you can do it.

Another you know things another type of problem you know type of problem can be encountered in case of permutation or combination. Now here suppose you have to see that suppose you have the book different colors and you know you have to see that you have 9 colored books and among them 3 is red. So, you have 9 colored books in that you have 3 is red color similarly you have 4 is green color and you have 2 as blue color.

Now in this case the number of ways by which you can have this arrangements you can have 3 groups. So, in those cases this r comes in 3 groups, so in such cases if you have to see that in how many ways you can arrange them. Now we have to have the group, now what we do in this cases that we have to arrange it will be 9 factorial total if you have 9 books you have to arrange that it will be 9 factorial.

However since they are in the groups, so in those cases you have 9 factorial/you have all these groups to be divided. So, this is 2, so this value when they are in groups and you have to arrange them, so that comes out and this way you get something like 1260, so this also is to be you know that concept needs to be clear in your mind how to solve the questions when you get such problems.

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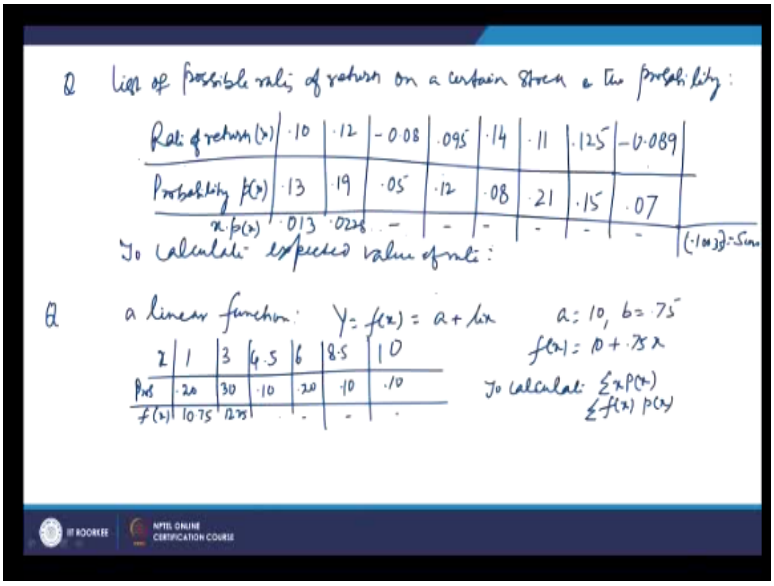

Next I have a problem may be based on this statistical you know parameters suppose you have a table which talks about the possible rates of return on certain stock and what is the probability associated. So, you have a basically you know a list possible rates of return on a certain you know stock and the probability. So, if suppose such is the question so suppose rate of return is given and also the probability is given.

So, this will be x, so this will be px, now suppose the values are given like rate of return is 0.10 that is 10%, now it is probability is 0.13. similarly 0.12 has the probability of 19% that 0.19 – of 0.08 it has the probability of 5%, so 0.05. Then you have 0.095, so it has the probability of 12% or 0.12 then you may have something like 14% return 0.14 and it has the probability of 0.08 that is 8% then 0.11 has the probability of 21% , 0.125 has the probability of 15%.

And – of 0.089, so it has the probability of you know 0.07 7%, now such maybe you know, so if you add there is 32, 37, 49 and 57 this is 78, 93 and 100. So, this all together comes out to be 1. Now for such cases suppose you have to find to calculate expected value of rate. So, as you know that once you have to find the expected value, now in this case you will have summation of xpx.

And if you find so x and px will be you know multiplied, so you will have another column and it will be x*px so you have to you know write down. So, it will be something like 0.013 then it will

be 0.0228, so this way you will be doing these calculations and then ultimately you are going to have this sum of these and this sum which you get that is the mean value what expected value and if you even do it.

And it is coming out to be 0.10035, so basically 10% is the expected return which you can think of during this. Now this is how you find the you know mean your expected value, there may be question of type when your function is given. Now based on the value suppose you have a discrete random variable and what you see is that you have a linear function if you have a linear function.

So, the linear function y=fx and it will be a function of a+bx, now a and b if you know then according to a and b as the x value will be changing the fx will be changing. And then you can also find the expected value of fx, mean value of fx. So, suppose you may have something like x value is given like you know 1, 3, 4.5 then 6 then 8.5, then 10, so based on that for this the probability is suppose given.

Then suppose this probability is given as 20%, so 0.20, 0.30, 0.10 then 0.20, 0.10 and 0.10. Now if you have to find the you know expected value of fx in that case you have to find first of all the fx. So, fx value if you look at and if a is given as and b is 0.75, so fx becomes as 10+0.75x. So, if you look at this it will be 10.75 then it will be 10+0.75*3, so it will be you know 12.25 like that. So, this way you can find fx, now based on that you can calculate you know you may have the to calculate what you do is you will find xpx.

So, if you have to find the mean of x, in that case xpx will work. If you find you know fx px, so it is summation will give you mean value of x and is summation will give you the you know mean expected value of fx. So, that way you can find the these expected values, you may also be you know dealing with a questions, so we have discussed about how to find suppose the value of the standard deviations.

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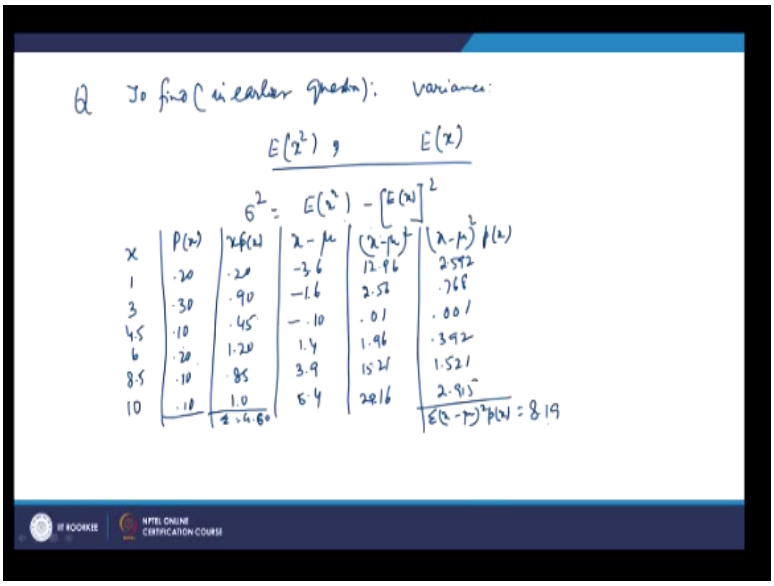

So, there may be questions based on that and if suppose you are given for the same thing you know 1, 3 4.5, 6 and he you know this 8.5 and 10 values for that you know the px values and then you know also xpx. So, now if you have to find in the earlier question so you have to find suppose the variance, now in that case what we know is that e of x square it will be basically you know you have to find ex square.

And you have find e of x and then once you know this ex square and ex then you can find these sigma x square. So, sigma x square, so sigma x square will be e of x q- ex square, so if you look at that table. Now if you look at that table how thus table looks like, so you will have x value of 1, 3, 4.5, 6, 8.5 and 10. Similarly you will have px, px is 0.20 then you will have 0.30, 0.10, 0.20, 0.10 and 0.10.

Then you come to x px, so xpx will be as you know that 0.20, 0.90 0.45, 1.20 then you have 0.85 and 1. Then you will find the x-mew, now as you know that mew will be you know summation of xpx and if this mew here summation of this xpx that will be coming as mew. So, if you sum them it will be summation is coming as 4.6, so 4.60 is the mew, so x-mew you have to find from all the values.

So, it will be coming as 1-4.6 with -3.6, -1.6, -0.10 then you have 1.4, 3.9 and 5.4, now you will find the x-mew square. So, from here also you can get the values and this will be 12.96, 2.56,

0.01, 1.96, 15.21 and 29.16. So, x-mew square px if you find now if you find this x-mew square px is will be 2.592 this is 0.768, 0.001, 0.392, 1.521 and 2.915.

And if you look this summation x-mew square px, so this comes outer to be you know 8.19 and that is what is the sigma square. So, that is how you get you can also get through this formula and you can also get through the px values these values of these type of deviation. So, this is how if you deal with the different type of questions you have to solve the different questions and get your concept more and more clear, thank you very much.