Financial Mathematics
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Lecture – 47
Performance Measures

Welcome to the lecture on, Performance Measures. So in this lecture, we are going to talk about the performance measures with respect to the mutual funds. So we discussed about the evaluation you know mechanism for the mutual funds, how they are valued, how the evaluation is carried out. Now there are certain performance measures, you know, by which we can, you know, assess their effectiveness, their usefulness.

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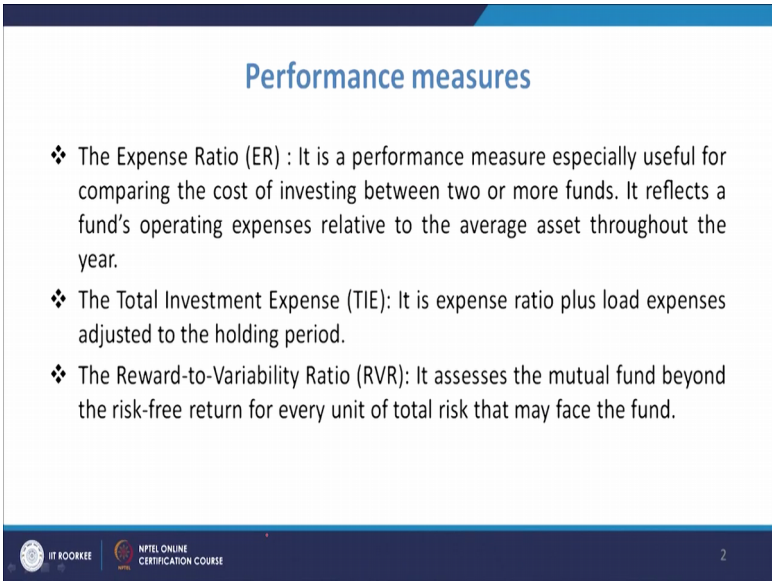

So among them, the first one is the, Expense ratio. So that is also denoted by ER. Now this is basically useful for comparing the cost of investing between two or more funds. So basically you require to, you know, invest and for that, some cost is involved and based upon the, you know, assets of the two fund, we basically calculate this. So it reflects a firm's operating expenses relative to the average asset throughout the year.

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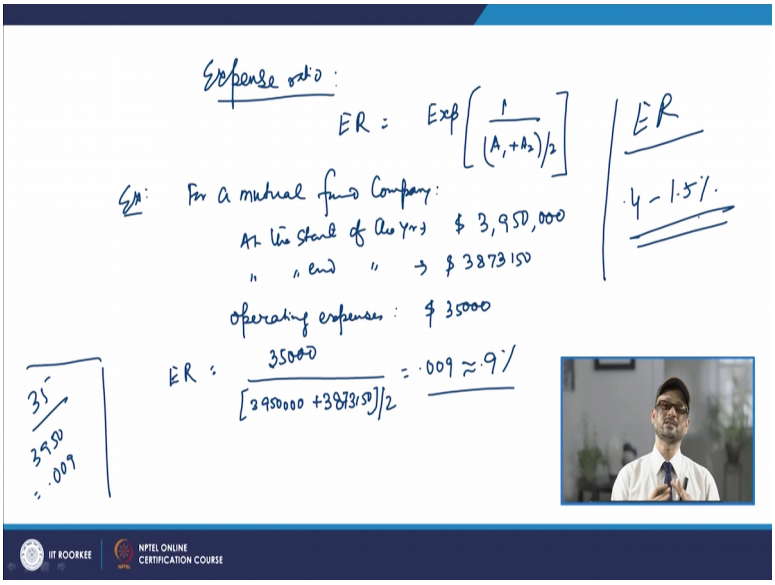

So now what we do that, when we have to talk about the expense ratio, so this is defined as so this is denoted by ER and ER is defined as the you know expenses which you know occur and then that will be multiplied by one by average of the assets so a1 plus a2, you know by 2 so average of the net asset that will be in the denominator, and then your expenses will be, you know the operating expenses they are basically, so that will be at the numerator, so and net average asset will be at the denominator. So this ratio is normally known as the Expense ratio and it is normally expressed in terms of the percentage.

So for example if suppose for any you know mutual fund company it is showing that you know its total asset at the start of the year so suppose for a mutual fund company you know at the start of the year its assets so that it will be given so at the start of the the asset is you know it is told to be of the value of 3,950,000 and then at the end of the year so at the end of the year it is estimated to be 3873150, that is dollar.

So, suppose and also if you are told that the operating expenses so if the operating expenses have reached something of the order of suppose dollar 35000, in that case, you can calculate the expense ratio and expense ratio will be 35000 divided by the average of these two. So it will be 3950000 plus 3873150 and divided by two. So that way, you calculate these expense ratio it will becoming something close to 0.009 or that will be 9%.

So this will be point you know 0.9 percent so that way, we calculate these values I think it will be 0.09. So you can calculate that so it will be, you know, 35 by 3950. So it will be if you do the average, 35 by 0.3950, so it will be something close to 0.009. So it will be 0.9 percent five so that will be you know you can they say that it will be is a 0.9% of is the expense ratio.

Now if the expense ratio is more than it's going to take away from the investment so that is what you know the meaning is and normally the expense ratios values its value varies between 0.4 to 1.5%. So this is the usual you know range in which these expense ratios should fall and if it is you know higher in that case it will be taking that return from the investment. So your return will be smaller.

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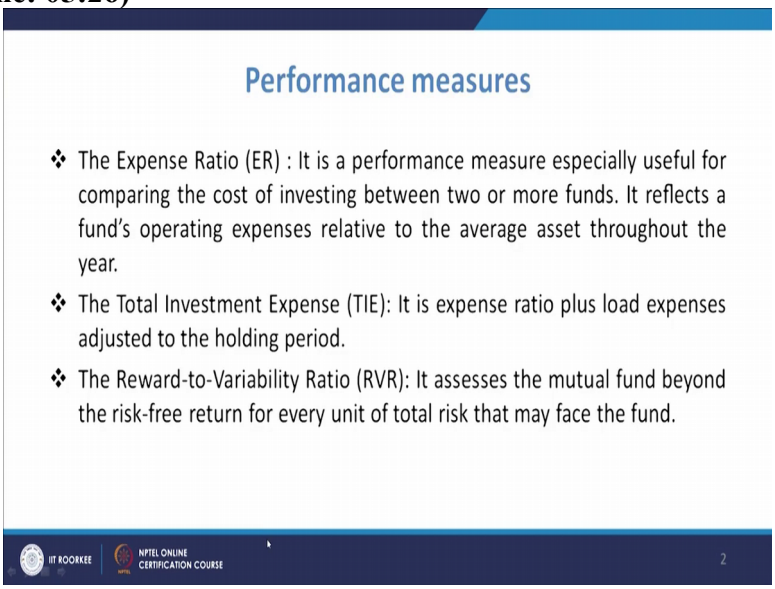

Then the next, you know performance measure which is an index of measuring that you know how good you know you will be able to get the return. So that will be total investment expense and that is TIE. Now this total investment expense it is nothing but the expense ratio plus the load expenses adjusting to the holding period so you also take into account the, you know, load you know expenses in that, so that once that is considered then it becomes the total investment expense because the load charges are there anyway and they are to be taken into account.

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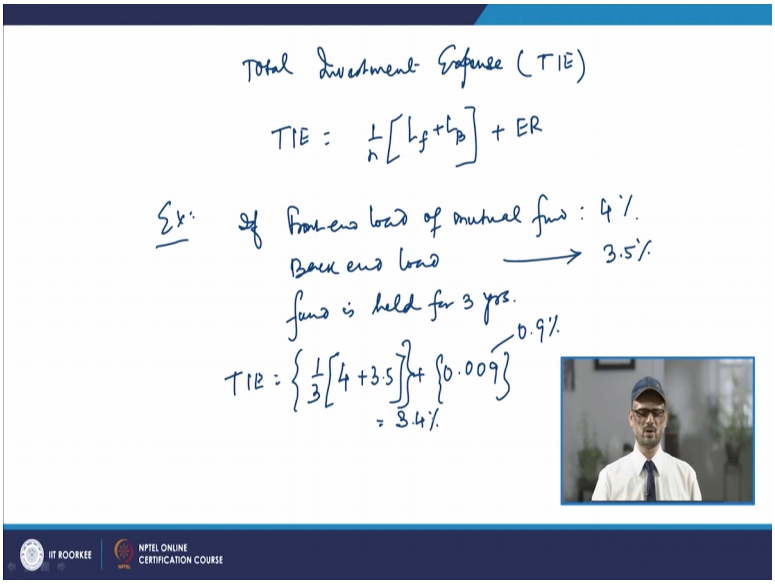

So the expression for the total investment expense so that is total investment expense that is TIE this is denoted by TIE. So this is basically calculated as 1 by n of LF plus you know LB. So that is, you know, this is 1 by n of LF + LB. So that will be the energy here and then you have so they are holding for any years and then plus you have ER. So this is how it is you know defined that is load expenses will be you know adjusted for the holding period that is n, so so that will be your total investment expense.

So for example, if you know company is you know it is said that the front-end load so if we if you know the front-end load, so that is of mutual fund, so that is given that is given as four percent and back-end load suppose that back-end load is given as the 3.5% and also the fund is held for three years, fund is held for three years. So in that case you calculate this total investment expense so if you calculate the total investment expense.

Now if you take this you know total investment expense in this case so your value was for the earlier problem itself so it will be 1 by 3 so these will sit for 3 years and then you have 4 plus 3.5 so it will be basically 2.5 and then you have you know 0.009. So that way, you know, you can calculate, these values and it will be 2.5 and then it will be 0.9. So basically this is, this is also to be taken in terms of percentage. So it will be 0.9% so basically this will be 0.9% and this is 2.5% so this will become 3.4%. So that is how you calculate these total investment expense.

Then the next you know, performance measure is the reward to variability ratio that is RVR. Now this will; this you know performance measure will assess the mutual fund beyond the risk-free return for every unit of total risk that may be faced by the fund. So basically you are assessing that fund beyond that risk free return and the risk free return may be

Something very standard one maybe for municipality or from the; you know a special type of specific government bonds. So they offered you that is free return. So based on that you are calculating these you know this performance measure that is reward to variability ratio, because it will talk about, how much you know, varying it is, how much the variability it has, so what you do here is that, you are you know, getting the; you know, this return and you are having so-so return on certain you know fund and then your return on the, you know, so there is free return value and that is basically divided by the standard deviation of the return on the fund.

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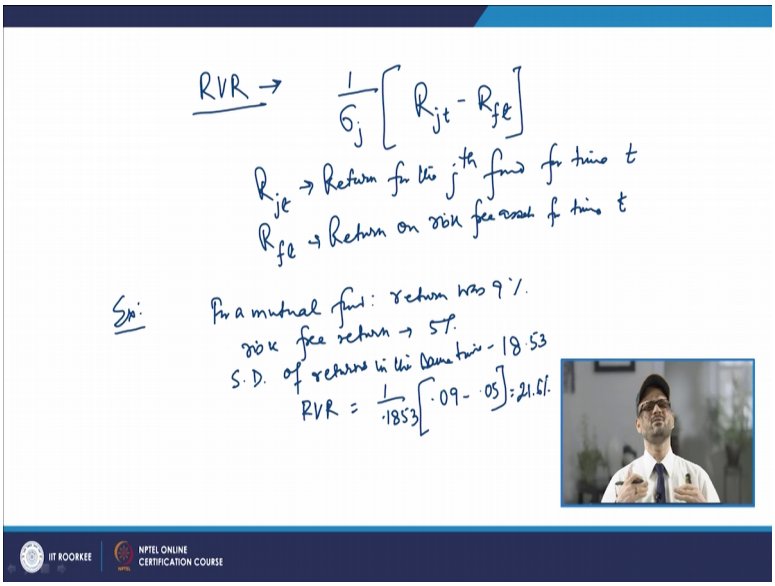

So that way we calculate this performance measure so your risk to variability ratio that is RVR and this RVR is basically computed as 1 by Sigma j s that is the standard deviation for the j th unit, you know and the fund and then we are finding that or you know jt minus you know Rft so that is you know return for the j th fund, you know, for time t.

So RG t will be the return for the j th fund, you know, for time t and the Rft that is your risk-free return, you know, that is, return on a risk-free asset, for time t, so that is return on risk free asset, okay. So that is ft for time t, so what happens that you are going to take their, you know, difference and then it is divided by the standard deviation of the j th fund. So that way you calculate these risks to variability, you know, ratio. So you can understand that by referring to certain, you know, example.

Suppose your person has, you know, you have a mutual fund which is giving you return was so far falling which were found suppose the return was nine percent so return was nine percent then also you are given that risk-free return is five percent and if the standard

deviation in that time and if the standard deviation you know of returns during that time so you know in the same time so that is given as suppose 18.53.

So in that case you can find the risk-free return so that is a risk to variability ratio and that will be 1 by 0.1853 that is be in terms of percentage, so you will divide it with this 0.09 minus 0.05. So that will be coming out to be something close to 21.6%. So, you know it means that the fund is providing 21.6% return, beyond that is free rate.

That is what it means that beyond the risk-free rate this fund is giving you the 21.6%, you know, return. That is what the meaning of the risk to variability ratio it is. Then, there is a term, that is, so you can find the rate of return on the fund. So further if you have to calculate the rate of return of the fund in that case you can calculate the, you know, fund rate of return.

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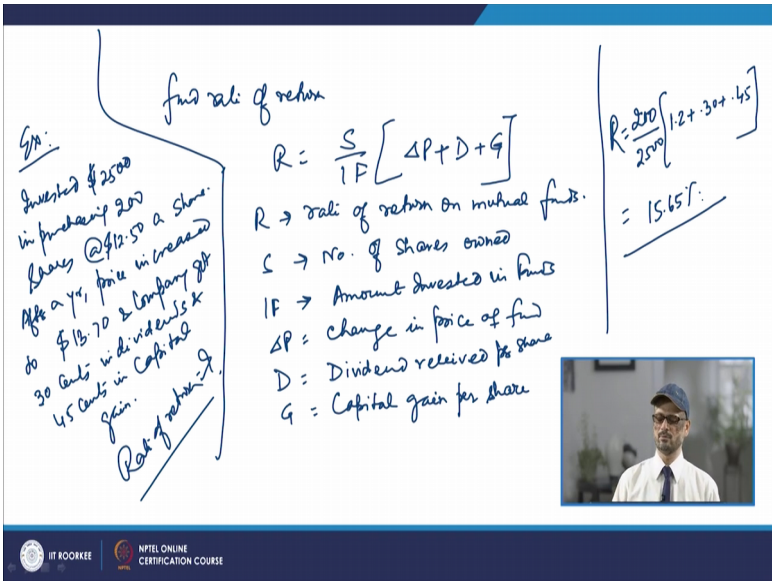

So that will be R so R will be S by IF and then you have to have the values of Delta P plus D plus G. So in this case the R is returned on the rate of return on mutual funds. Then you have S is the; you know number of shares which is owned. So S is number of shares owned, and IF is the amount, you know invested in funds. So this is amount invested in funds, and the value Delta P is the change in price of the fund. So that is Delta P and D is the dividend which is received per share.

So this is dividend received per share, and G is the basically capital gain per share. So what you see that, we are talking about all the gains which we are getting so you have you know whatever you have invested that is the investment, that is, IF is amount invested then certainly you have the number of shares, so that is there. Then depending upon the number of

shares, you are going to get something you know how the changes are there. So that is, Delta P that is change in the price of the fund that may be positive or negative.

So, based on that you will have all these values, so that will be multiplied with the number of shares. Similarly dividend received per share and that will be multiplied with the number of shares, and same way the capital gain per share, and that will be multiplied with the number of share. So that will be talking about what is the gain or overall gain and then that will be, you know, divided with the investment amount, which you have invested and this value amount invested in fund, so that will be, giving you the value of the rate of return.

So you can use when this formula for a certain example, suppose you have invested, so suppose you have invested, you know dollars, you know, 2500, you know, in purchasing 200 thousand shares, so in purchasing 200 shares basically, then it is at the rate of 12.50 dollars per share. So that is the rate and after a year suppose, price has increased, so after a year the price increased to increase to dollar 13.70 share and company got say, you know, 30 cents, you know, in dividends so that is the dividend you know you know the gain from the dividend and and and 45 cents in capital gain.

So this is this mean I mean if you have these values then you know in that case what will be the rate of return? So you can calculate these values, you know, directly by putting simply the values into the formula. So rate of return will be basically you can calculate as S by IF. So S is the number of shares owned and we know that we are purchasing 200 numbers of shares so it will be two hundred divided by investment so how

Much we have invested, so we have invested 2500 dollars, then we are keeping Delta P that is your change in price and if you look at the change in price from 12.50 it is going to 13.70, so change is 1.2. So it will be 1.2, then you have the dividend price. So dividend price will be 30 cents so it will be 0.30, and similarly capital gain is 0.45. So this way you can calculate these values so it will be 1.95, so it will be something like 400 by 2500, so 1 by 6 or so. So it is coming close to 15.65%. So this way you know you can calculate the rate of return.

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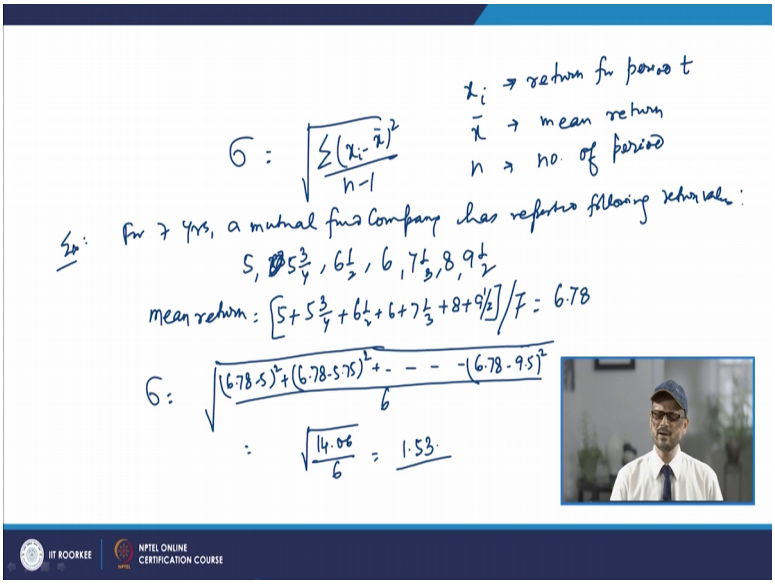

Now another thing you know which is important to be calculated will be the standard deviation and the standard deviation can be calculated based on we know that standard deviation is calculated, based on summation of, you know, x minus x bar that is square by n minus 1. So you know, so this is xi, so you have xi is the return for period t, and x bar, it is the mean return, and n is the number of periods.

So that way, so it is basically, a standard deviation is normally, used to measure the extent of risk you know you know in when we talk about the flow of returns in that case you know if being more it we will be talking about more risk. So it will be more riskier and having less value it means the risk is less. So that is how these you know the standard deviation values are calculated.

So if suppose you have you know you track down the seven-year returns for a particular specific mutual fund and what you find that for seven years, you know, a mutual fund company has registered or it has reported, so mutual fund company has reported, following returns, so you know following return values. So suppose the values are, suppose 5 then 3 so 5 is 5 then 5 3 by 4 then 6 and a half suppose 6% then 7 1 by 3% and 8 and then you have 9 and half.

So if that is, you know, the type of the pattern in which the investment I mean rate of return that is shown for any mutual fund company. So suppose we have to find the you know standard deviation in that case what you have to do is you have to first find the mean return, so you have to find the mean return you can calculate them so mean return will be calculated, and mean return will be like 5 + 5 3/4 that is 5.75 + 6 1/2 + 6 + 7 1/3 + 8 + 9 and 1/2 and this will be divided by 7 so this comes out to be something close to 6.78.

Then what you will do is you will find the summation of this value. So Sigma will be basically under root and then you will be doing that 6.78 - 5 square, similarly 6.78 - 5 3/4 square, like that we are going to have the difference in its square so it will be 6.78 - 5 squared plus 6.78 - 5.75 square. Then so that way it will go and lastly it will go 6.78 - 9.5 square.

So, this way and then also divided by n - 1 so you have n entries that is 7 so you are going to divide it by 6. So that way you are measuring the standard deviation and in this case you can find the standard deviation values by working on this. So you will get this standard deviation from this formula. So that if you do the computation this summation comes out to be you know 14.06. So, 14.06 divided by 6.

So it will be something like 2.3 and 4 or so. So if you take its under root it will be 1.53. So 1.53 is the standard deviation. Now many a times you have so since the standard deviation is smaller so you have lower amount of risk involved low level of risk in in this case. Now many a times when we talk about the portfolio investment so in that you have a lot of, you know, diversification, you have degree of diversification of the funds in the investors portfolio.

So what we do is that we find the you know relation coefficient correlation coefficient, and Pearson has given this Pearson you know correlation coefficient that is correlation index and you know this is about the degree of diversification that will measure the degree of diversification for the, the investor's portfolio.

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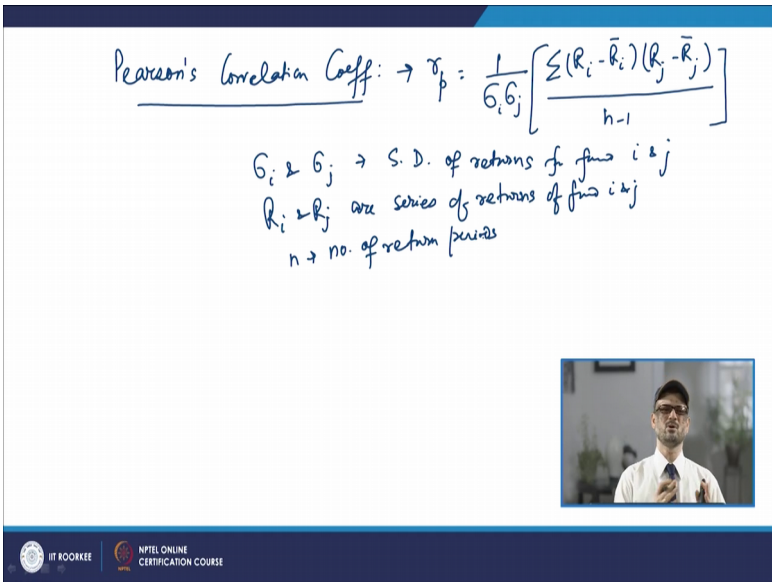

So in that there is an another, you know, performance measure and that is Pearson's correlation coefficient and this correlation coefficient that is denoted by rp and this rp is basically given as 1 by Sigma i Sigma j, and then that will be multiplied with you know this is summation of Ri - Ri bar, and then that will be multiplied with Rj - Rj bar. So you have returns on this I attend you know j th fund and then you will be dividing that with n - 1.

So that will be talking about the Pearson's correlation coefficient, in this case Sigma i and Sigma j these are the standard deviation of the returns for fund i and j and you know Ri and Rj , so they are the return value. So Ri and Rj are series of returns of fund i and j. And you know apart from that you have any the number of return periods.

So what we do in this case is you calculate, you have, once you have, you know, the for the two returns you know the, so the return values are given, then you have to calculate the standard deviation values and you will be calculating the Ri - Ri prime, similarly you will be calculating the Rj - Rj prime, and based on that and then with the,

So if you have five ratings, you will be dividing with four and you are going to calculate the standard deviation of the two readings so you have two funds so for the two funds, you will be calculating the standard deviation, and you are going to compute the value of Pearson correlation coefficient you also compute the correlation, you know coefficient, when you have, when we talked about the random variables, so in that when the correlation coefficient values, so that will be normally varying between - 1 to + 1.

So when it is minus 1, it is said to be extremely negatively correlated and if it is towards plus 1 it is the extremely positively correlated and if it is 0, then they are said to be independent. So it all depends it will tell you that how these returns vary if for fund i, in a particular year the sum return value is there and if ij has the S has some other return value and the way they are changing every year, if one is changing.

And all and the second is also changing proportionately in the, you know, if one is increasing other is also increasing one is decreasing other is also decreasing, then it is positively correlated and if one is increasing and second is decreasing that is negatively correlated and when we talk about the diversification of portfolios there were certain assets in those cases the negative correlation is more preferred. So that is what you know are the performance measures we normally discuss about.

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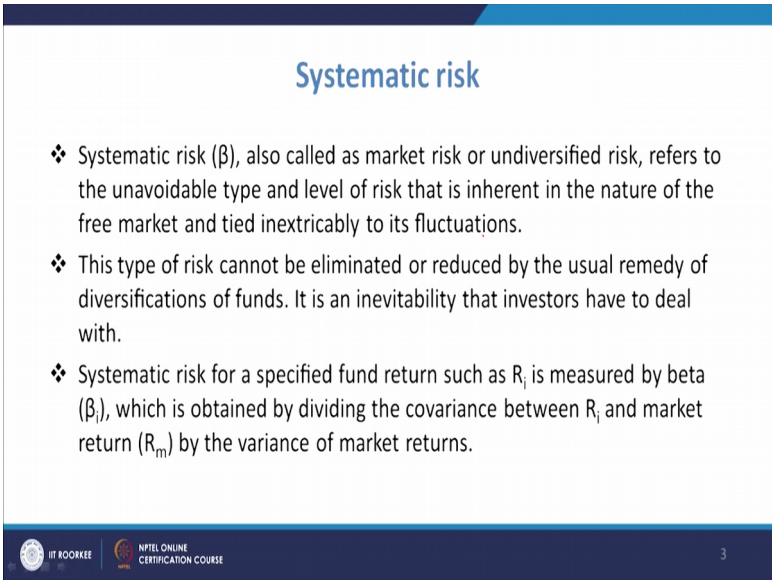

Now another thing which is important when we talk about you know these investments so you have the risk and we know that you have the two types of risk, a systematic risk and the unsystematic risk. now we talked about the systemic risk and here you normally talk about a parameter, that is beta, and this is also known as, so systemic risk also known as the market risk or undiversified risk, and this refers to the unavoidable type and level of risk which is inherent in the nature of free market. So you cannot control it and it is tied inextricably to its the fluctuations.

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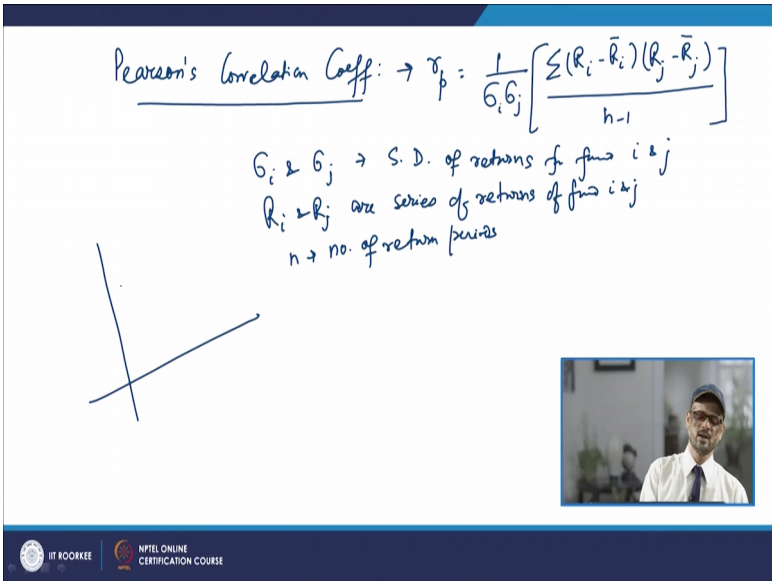

So there are certainly, so risk has basically two components, if you talk about the risk so if you look at the total risk you know varies like this and in this part it is, this part will be your risk which is basically the fixed one on which you do not have any control and this

component of risk is basically the component where you can control and this control can be done using these diversifications.

You can have the investment of fund at in the different sectors, you can have different types of shares, you can purchase and that way when you combine the assets, you will see that, there, you know, risk is there the risk is decreasing so this risk is in this so risk will be decreasing by you know by the diversification of fund and but this component is always fixed so this part is the systematic risk part.

That is what we are discussing. So this is this type of risk cannot be eliminated or reduced by the usual remedy of diversification of funds diversification by diversification of funds we can do something you know in the upper part so that's what we discussed this part will be you know in our control but this part is not in the control because this is governed by you know many things like war, political stay, you know, situations how the market is moving, so all these local disturbances, on which you do not have any control. So that is there.

And the systematic risk for a specified fund return such as Ri, it is measured by beta that is beta i and it is obtained by dividing the covariance between Ri and market return Rm by the variance of market returns, so you will be finding the covariance between Ri and you know return Rm. So that there is a formula for covariance in that we know that we are calculating the you know Ri and Ri values and then Ri mean values.

So that way it will be multiplied and then multiplied with the probabilities and then it is you are getting the covariance values and then it is divided by the variance of the market return so if you do that you will be getting the value of the systematic risk. So this we can you know do by practicing on certain questions when we deal with them in the coming lectures. Thank you very much.