Financial Mathematics
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Lecture – 45
Bond Premium and Discount, Bond purchase

Welcome you to the lecture on Bond premium and discount and Bond purchase. So in this lecture we will discuss about you know the premium amortization and also about the discounting schedule and also we are going to discuss about the bond purchase. So the premium is what we have seen that in the case of premium and you know your the amount which you get B naught or B0 that will be more than you know the value.

So, so that way your premium is there so that premium is recovered gradually throughout the interest payments. And the investor will not receive this premium price at redemption.

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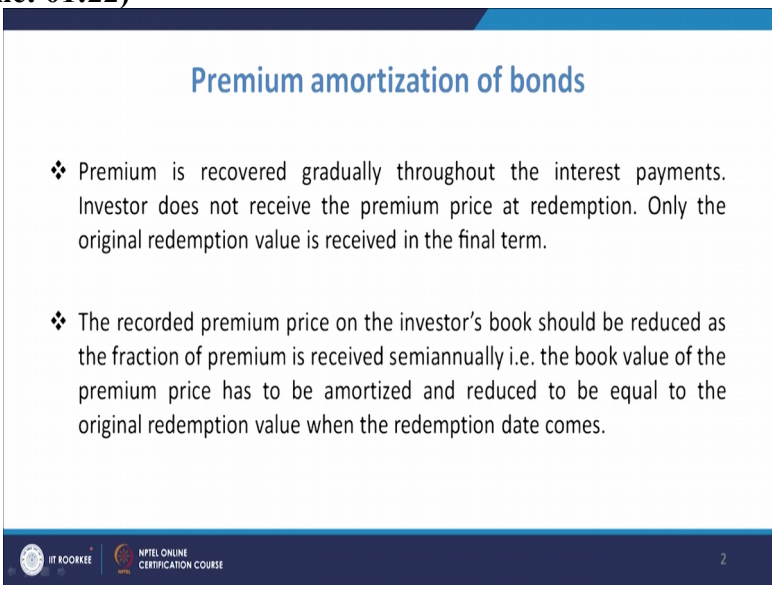

Basically now only the original Redemption value is received in the final term so what happens that your recorded premium price on the investor book it has to be reduced as the fraction of premium is received semi-annually. And book value of the premium price has to be amortized and reduced to be equal to the original Redemption value when the redemption data comes.

So what do we mean to say that he is getting that you know throughout the interest payments not at one time and that can be understood for suppose a rupees 3000 you know bond is there. So if we take for example we are taking a rupees 3000 bond so for a rupees 3000 bond.

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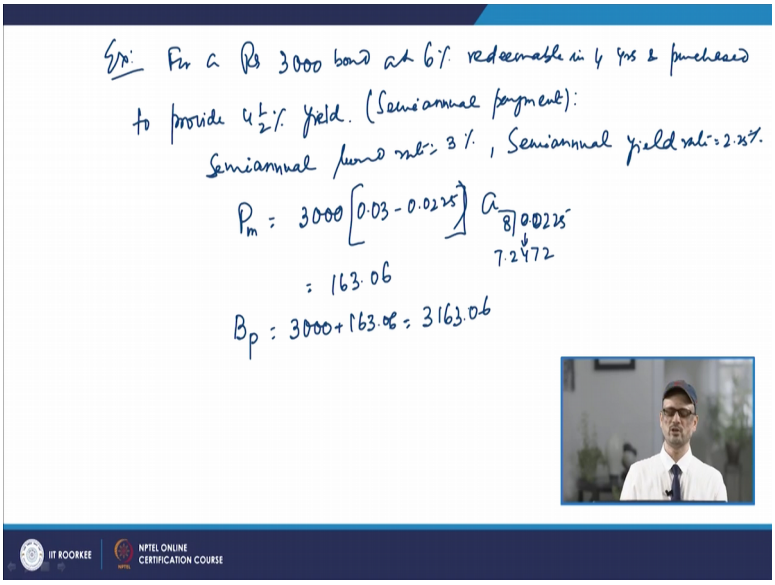

And if suppose you know it is at 6% and you know that is redeemable then in 4 years and you know the ill rate will be certainly as we know that in this case in rate becomes more than the bond rate. So no it is basically it is providing and it is purchased, so it is purchased to provide so that is in the case of discount. In the case of premium it is a smaller value so purchased to provide for an half percent yield.

So, so in that case what we see that and you have the semi-annual payment is there so in so if there is a semiannual payment. So now what we can see that you have the interest rate at the rate of 6% and it is semiannual value will be calculated. And then at 6% so it will be you know 180 or so and its half will be 90 or so. Now this you know semiannual bound rate will be 3% so semiannual bond rate will be your 3%.

So and the semi-annual yield rate is half of this. So it will be 2.25%. Now the thing is that if you have to find the premium we know that we calculate that premium and the premium will be calculated as you know we multiply this you know redemption value that is 3000 that is m. And then this will be done with R - I. So that will be your R is 3% and I is 2.25%.

So it will be 0.03 and -0.0225 and then we are to get that factor that is P / F factor for I. n and your four years is the time so you know PV by I that is the F a factor that is for I m and that will be eight. And I will be your you know that is this I so for that if you calculate 0.0225 so it will be the data that that factor is basically represented as eight and then you have 20.0225.

So this is nothing but the summation of 1 by 1 + 0.0225 raised to the power t and t will be going from one to eight. So this factor is coming out to be 7.2472. So now if you do this calculation then you are coming out to be premium out to be 163.06. So, your bond price so that bond price BP that comes out to be 3000 + 163.06. It will be 3163.06.

Now what will happen that you will have the amortization schedule. This premium amount is to be amortized and this amortization schedule will be there and for that what we do is that you will have the difference you know the interest and interest you know payment and based on the yield rate you will have the differences and that will be you know the difference which we will getting that will be your amortized premiums and hold all that amortize payments together. They must come to 163.06. So how we do this? So, we can have a table.

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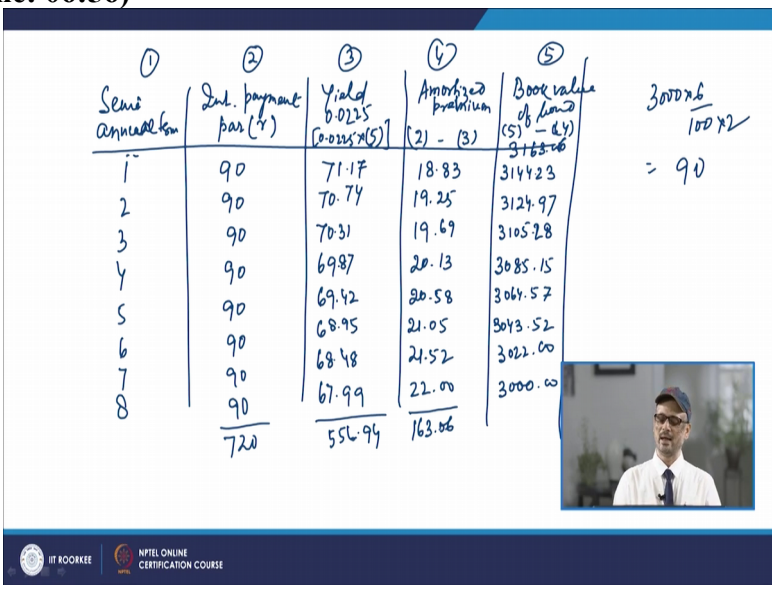

① Semi annual term	② Int. payment par (r)	③ Yield 0.0225 [0.0225×(5)]	④ Amortized premium (2) − (3)	⑤ Book value of bond (5) − (4)
				3163.06
1	90	71.17	18.83	3144.23
2	90	70.74	19.25	3124.97
3	90	70.31	19.69	3105.28
4	90	69.87	20.13	3085.15
5	90	69.42	20.58	3064.57
6	90	68.95	21.05	3043.52
7	90	68.48	21.52	3022.00
8	90	67.99	22.00	3000.00
	720	556.94	163.06	

$3000 \times \frac{6}{100} \times 2 = 90$

And in that table what we do is your; this is your semiannual term, semi-annual term. So this will be varying from 1 to 8 because it is for 8 years 1 2 3 4 5 6 7 and 8. Then you will have the interest payment. So this interest payment will be there you know for so the rate which is given like 6%. So now for that the semi-annual interest payment will be in 90. So now you know this is for you know the par value and that that is with the bond rate. So that will be your interest payment.

Similarly you have the yield and yield will be basically it is nothing it is about 0.0225 that is your 2.25% semi-annually and that will be you know multiplied with whatever you know your book value of the bond is so that will be multiplied with that. So, that will be calculated and then you have the amortized premiums so amortization of premium you know described in this table.

And here you will have the book value of the bond. So this is your 1 and this is your 2 this is 3 this is 4 and this is 5 now if you go and try to see that you have the book value of the bond which you have calculated before this before this you have calculated here 3163. So you

should have one space here. So here you have a 3163.06. This was the book value of your bond.

Now this interest payment which will be on the par value or face value of the bond and that is your R and R is 6%. So for all these you know payments you will have 6% is 3000 into 6 by 100 and then also the semi-annual so it will be by 2, so it will be 90. So all in all these cases, you will have 90 as the interest payment. Now what will happen in now this value yield which you get it will be based on this 0.02 to 5% of the book value.

So it will be basically 0.0225 and into 5 so this will be multiplied whatever book value is remaining. So based on that you will be calculating this yield and the amortization premium so that will be the premium will be there every time because generally that will be change in the interest what we calculated earlier also and it is basically the summation of that. So it will be basically this the interest minus this you know interest which you calculate based on the yield rate.

And then you will have this value which you calculate basically this 5 and 5 - 4 will be so what were amortized the premium is you know amortization is done that will be subtracted every time. So what you see that if you calculate the 2.25% of this 3163 it will be 71.17 so the premium amortization you know amortization of premium is done and that is coming out to be 18.83 and your bond value becomes equal to you know 3163.06 - 18.83.

So it will be coming as 3144.23. So say on the second you know semiannual term you will have the yield that is coming out to be 0.0225 * 3144.23. So that will be coming as 70.74. So that will lead to the difference between the 2 and 3 that is amortized pretty premium amortization is something like 19.25 and your book value remains coming to you know that is 3124.97.

So this way then you will get the 2.25% of this amount to be calculated here so that will be coming as 70.31 and your difference will be further more than this. So it will be you know 19.69 and your bond value in a book value of the bond that comes out to be 3105.18 sorry 28. So this carries you know till the end and you are supposed to get 3000 at the end. So if you calculate then it comes 69.87 and it really bums becomes like 20.13 and this will be coming as 3085.15. Further you can get 2.25% of this amount so that will be 69.42 then your difference of 90 minus 69.42 that will be 20.58 and 20.58 and so if you take the difference of 3085.15 minus 20.58 that will be 3064.57.

Even then again 2.25% of 3064.57 that will be 68.95 and you get the premium amortization of basically 21.05 and your book value will be this minus this so that will be you know 3043.52. Then again 2.25% of this amount that will be 68.48 and your this amount will be 21.52 and further you will get 3022.00 and in that last term you will have 2.25% of 3022 and that is your 67.99.

So this premium amortization is coming out to be 22 and you are getting 3000. So this way what you see that you are coming out to this you know book value of bond at the end to be 3000. Now if you look at these you know interest payment which is made now. This is altogether 90 you know made for 8 terms this is coming out to be a 720. If you add all these interest this is coming out to be 556.94.

And the premium amount which is there, total which we have calculated 163 that we get from here by adding if you add them this is coming out to be over 163.06. So we see that you know how this you know Premium you know price of whatever it is there how it is gradually reduced to the redemption value of 3000 by the end of eight semiannual term. And that is calculated to be you know 3000 in that case.

So, so this is how this premium amortization is being carried out. So this is about the premium amortization and now we are going to discuss about the discount accumulation. Now in this case of premium amortization what we have seen that you had the the larger amount of the; you know that is reduced to the smaller value. Now in this case the unlike in the case of premium price, you have the discount price is less than the face value.

So we are giving the discount that discount value we are calculating first of all you know because that yield rate will be larger. So you will have the discount of price is less than the face value and what the investor would still receive the face value at redemption. So the total deficit of the discount would be recovered gradually and in this case because that deficit which is there that you will see that it will be recovered gradually.

And little by little through the interest payments by increasing the book value to increase gradually from the discount price until it becomes equal to the face value of the bonds on the Redemption dates. So that will be somewhat in a different way in the case of this you know the discount accumulation. So that also can be further seen that in that case. So in that case you will start from smaller value and reach to the, you know larger value in the case of the discount accumulation.

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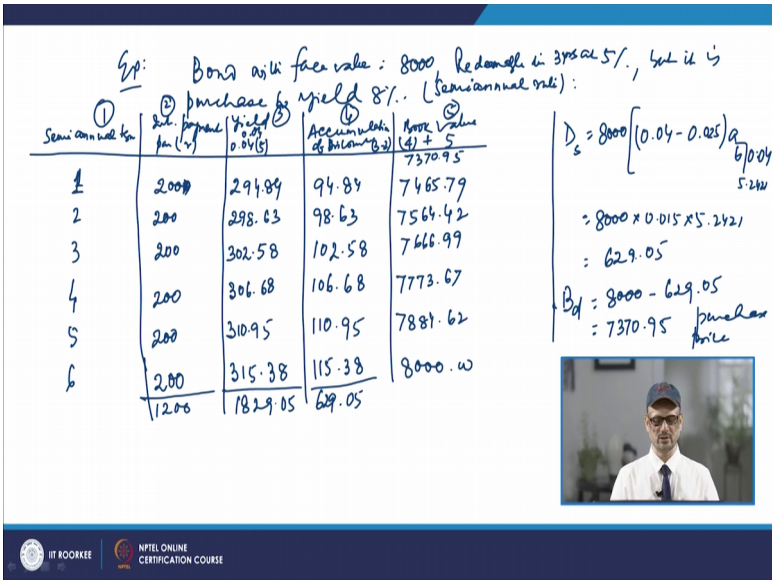

And we will see how that you know occurs so that can be seen for suppose you have a; you know bond so bond with face value. So bond you have face value of suppose 8000 and redeemable in 5 years so redeemable in 3 years and at 5% and what we see but it is purchased to yield 8%. So in that case here also we are taking the semi-annual case. So now what we see that bond rate is you know 2.5% semi-annually and your ill rate is 4% semi-annually in that case.

So know certainly when your ill rate would be larger so you will have the discount you know coming up your bond value will be decreasing and in that case what we see that you have to calculate that discount amount and we can calculate that discount amount to be like this. That you will have this discount amount it will be calculated as the Redemption value that is your 8000. And then this will be multiplied with 0.04 that is your semiannual rate for the yield that is 0.04 for 4% and minus 0.025.

So that is your this value of the bond rate and this will be multiplied with it that you know this will be multiplied with that factor. So this will be; you know a of 6 so you will have 6 terms and your interest rate is 0.04. So this is you know nothing but your this factor is coming out to be 5.2421 so you will have 8,000 into 0.015 into 5.2421 so that is nothing but you will again get 1 by 1 + I and raise to the power t.

t will be varying from 1 to 6 and I will be 0.04. So if summation will be coming out to be 5.24. Now if you take that, this amount this amount is coming out to be 629.05. This is your discount amount what we get because of the increasing the yield rate to 8%. So your the discounted book value now that Bd it will be 8000. And then it is your minus 629.05. So basically this is 7370.95 is this becomes the purchase price.

You are purchasing purchase price is smaller than the face value of the bond that is 8000. And that is why it is a case of discount. Now how this discount accumulation schedule you know varies, you know that is how that is going so that can be seen again you have we do the semi-annual term. So you have the semi-annual term so you will have you know 6 years, 3 years.

So you will have 6 terms of the match for the maturity then you have the interest payment so that is you know for using the bond rate. So this will be power into r so that will be your interest payment similarly you have yield so that yield is at 0.04% and that is your 0.04% your book value that is bond book value of the bond. So that will be your 5 so it will be again multiplied by this 5 that 5 will be so you will have 1 2 3 and 4 and 5 will be book value.

Book value of will be Book value minus the, you know discount accumulation so this is your accumulation of discount. So this will be nothing but you know 3 - 2 so in this case you will have 3 - 2. So this will be 3 and this will be your 2. So it will be 3 - 2 and Book value will be basically your whatever you get 4. And then you are + 5. So that is Book value which is getting here that will be added every time.

Now in this case what we see that you had you know bond value of 8000 and this rate was you know 5%. So semi-annually if you look at that interest 8000 * 0.05, so it will be 400 and that is for the whole year payment. So for semiannual it will be 200. So for all these you know 6 terms you will have this R value it will be coming as you know 200. So you will be starting with 200 no not 2000. It is 200. 200 is the interest amount which is calculated using the bond rate.

Now what we see that you have the purchase price of the bond is 7370 and you have to have the yield. So that will be based calculated based on the 0.04% of this. So initially you are taking the user you are getting that discount so that after discounted it is coming as 7370.95. So 4% of that 7370.95 it will be coming as 294.84. So now what you see that this is your a accumulation of discount is 94.84. And your Book value becomes 7370.94 + 94.84.

So it will be 7465.79 now again you have 4% of 7465.79 so that will be 298.63 and the difference will be 98.63 and this will be added and it will be if you can see it will be 7564.42. So again four percent of 7564.42 it will be 302.58 then difference is 102.58 and once added it will be 7667. so it will be 7666.99 and then again 4% of this will be 306.68. So difference is minus 200 that is 106.68 and what you get is again you add to it.

So you are getting 7773.67 then you are getting again 4% of this amount so that will be 310.95, 110.95 will be the difference and adding to it will be 7884.62. So finally you are getting again 4% of this so it will be 315.38 and 115.38 that is called discount accumulation and finally if you add to this it will be 8000, what is the face value what we started with.

So this is how this discount accumulation is you know calculated and if you calculate you know the interest here it will be 6 terms comes to 1200 and if you add them this is coming out to be 1829.05. So if you take if you add this also you are getting this out to be 629.05. Now this is nothing but the same discount which is being calculated here and you are getting this 629.05 which you see here.

So this is how the discount accumulation is a accumulation of discount is being computed in the case of the discount where you have the different I mean different rate of bond rate and you know this yield rate and in this case an yield rate is taken to be you know larger than the bond rate. And in that case you are basically giving a discount so, that way we calculate that. Now we wish to further mention another point that about the bond purchase.

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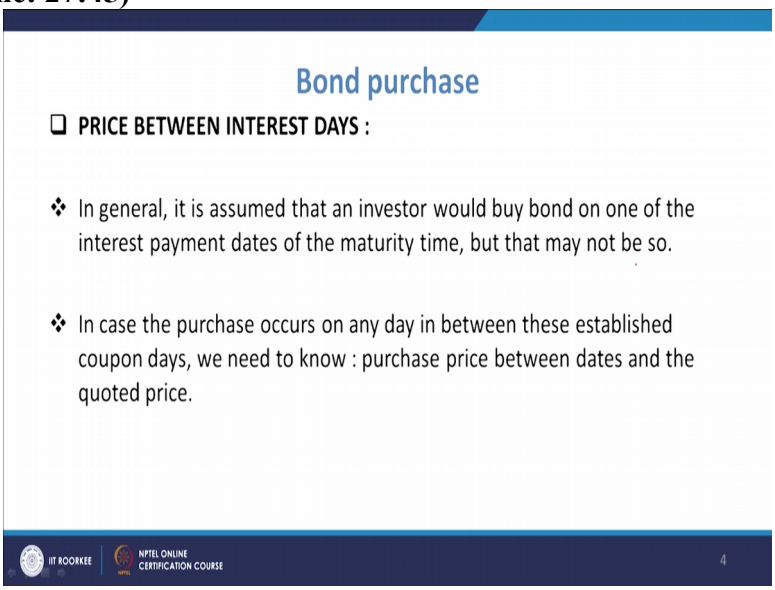

Now what happens that in general it is assumed that the investor would buy the bond at one of the interest payment dates of the maturity time, but every time it does not happen necessarily to be. So if the purchase is occurring at any day in between these established coupon days and then we need to know that, what is the purchase price between dates and the quoted price.

So for that we need to know these you know now you know purchase price between the dates. Now you know what we see that for that you have to calculate from the purchase price

of the bond and in that case the purchase price you know between the dates, if you calculate that is known as Bbd.

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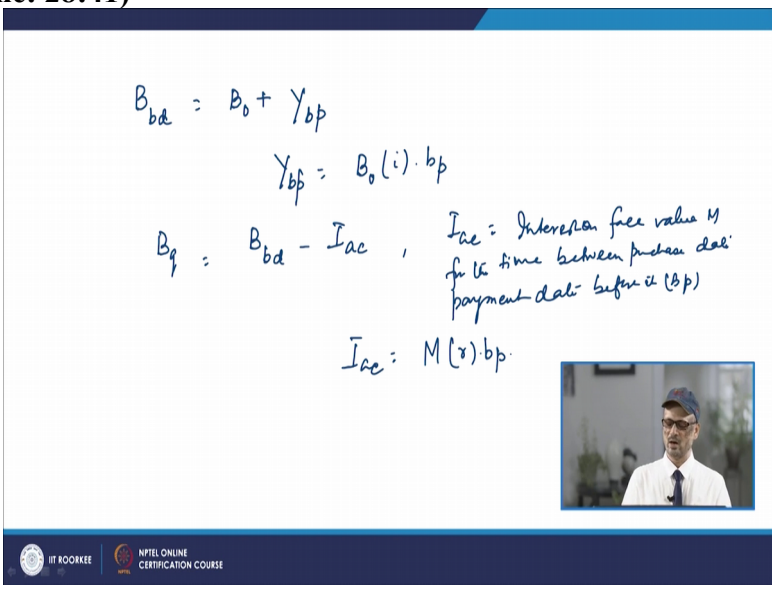

So that is your, you know purchase price between dates between dates bd, that it will purchase price between dates. Now in that case you have the B naught is given that is your purchase price and then that will be added with the portion of interest rate that is between the dates. So that is Y bp. So this Y bp is your portion of interest is there on this purchase price and this is for the time between the purchase date and the day of payment before you know before it.

So that is your Y bp and this Y bp that is you know this is the, you know portion of interest which is there. It is basically a function of B naught times I. And then it is multiplied with bp, so this is a payment before it. So that is bp and this way you get this B bd. Now the quoted price this B Q this is calculated as the B bd that is yours before date and minus I ac. So this I ac is nothing but this is the interest of the face value.

So I ac will be interest on face value m. And this is for the time for the time between purchase date and payment date before it. So that is your bp. So basically when we calculate I ac this is nothing but this is m times R that is your bond rate multiplied by bp. So this way we purchase this you know so the when we ever we take this purchase this bond between dates between interest days.

So in that case we are calculating these values like that we can understand it with the help of an example.

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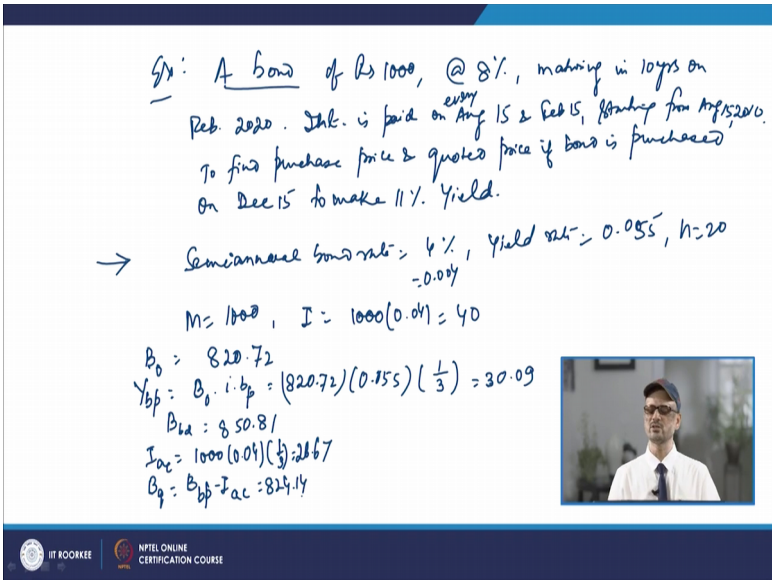

Suppose you have a bond so if suppose you have a bond and this bond you are purchasing and you have a bond of rupees 1000 and this is basically at 8% the bond rate is there and this is maturing in 10 years on you know February 2020. So suppose it is maturing you know we are starting from February 2010 and it is maturing on 2020. Now the interest is paid so interest is paid you know on August 15 every on every August 15 and February 15.

So starting from you know August 2010 so this is a; you know so starting from August 15 2010. Now, now in that case if you have to find the purchase price so if you have to find purchase price and quoted price. So for that the condition is if bond is purchased on December 15 to make 11 percent yield. Now this is the; if suppose that is how you have to understand and how to you have to see that what will be your price when it is purchased between the days.

So in this case what we see that if it is semi annual rate in that case since it is semiannual so your you know bond rate semiannual is semiannual bond rate becomes 10, 4% rate, becomes 4% now yield rate is 11% so yield rate will be 11 by 2. So it will be 0.055 it will be 0.004. Now you have 10 years of time. So n will be 20 and m is 1000 and I will be 1000 x 0.04. So that will be 40.

Now if you try to calculate you know in that so first of all you have to find B naught. Now B naught if you try to get in such cases because you have the yield rate is more so you have to use the formula. So if you use the formula if you calculate B naught and you calculate for the generate interest you are getting 820.72. You can calculate that so you will be getting this B naught and that will be nothing but your interest rate will be 40.

So this interest amount is 40 so 40 multiplied by that factor so that is your PV by FA I n that is a summation of 1 by 1 + I raise to the power t. And t varies from 1 to you know 20. So that is your 11.95 and then multiplied + 1000 multiplied by 1 by 1 + I raise to the power 20. So that is again coming out to be 0.3427. So that comes out to be 820.72.

Now we calculated you know the values like Y bp. So now Y bp that interest portion that is before you know so that we calculate we know that Y bp is calculated as B naught into I into you know bp. So if you have been not calculated I at 20.72 I, we know that it is 0.8055 and you know that if you look at so that is your December 15. So this bp coming is coming out to be 1 by 3. So you got 3 month is there so 4 months. So that is all you will have 1 by 3 coming up.

And this you know altogether if you calculate and this bp so this Y bp comes out to be 30.09 then you have the calculation of B bd. So if you calculate B bd as we know B bd is calculated B naught + Y bp. So you sum them so this will be 850.81. Similarly you can calculate I ac. So if you calculate I ac, I ac will be we have seen that I ac is calculated as m into R into bp.

So m is your 1000 and r is certainly it is 4%. So it will be 0.04 and bp is again 1 by 3. So this comes out to be 26.67. So you are you know B q value which we are intending to know it will be B bp minus I ac. So now I ac so that comes out to be 824.14. So this is how you know we are calculating these the bond purchases when we are purchasing the bond between the interest is.

In those cases we are you know when we have to find these b q. We are using these formulas and finding these values. So we must be you know clear in these concepts while we are solving such problems and you know try to solve more and more problems of such type. So that you know you can have more and more idea. And you can you you develop confidence while solving such problems. Thank you very much.