Financial Mathematics
Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Lecture – 44
Introduction to Bonds

Welcome to the lecture on Introduction to Bonds. So, we will discuss about the bonds in this class and they are one of the, you know, the major securities that are traded in the capital market. So, coming to the definition of bond, bond is a long term investment instrument usually issued and sold by big businesses or governments even to lenders for raising large amounts of capital.

So, they will be issuing that you know a bond and selling that these documents and they will be raising the large amount of capital when they need it for starting certain undertaking or some business or so. So that is issued and sold by these business groups, large groups.

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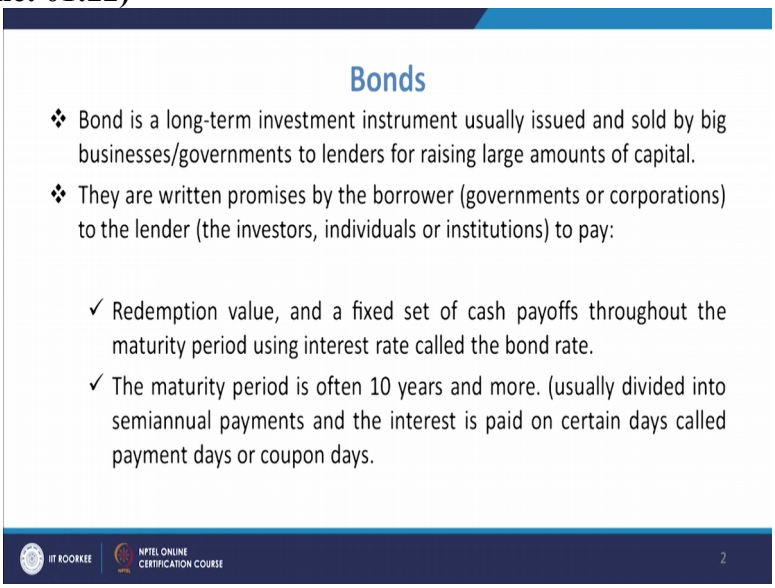

Now they are also the written promises by the borrower to the lender you know to pay. So you know once they are taking the bond then certainly there are certain conditions and these are like they will, what they will be paid is the redemption value and the fixed set of cash payoffs throughout the maturity period using interest rate called the bond rate. So whenever there is issue of bond, so there is a bond rate you know attached to it or defined to it.

So based on the bond rate he will be getting you know certain amount at frequent intervals or at periodic intervals that is basically defined. And that we will discuss and also the redemption value that is in the end he will be getting the amount also so in between we will

be getting a certain cash payoffs throughout that maturity period using the interest rate that is called bond rate and also its Redemption value.

And the so this and then the maturity period that this maturity period is normally 10 years or more so that will be the material for which he will be given the cash payoffs and then normally it is divided into semiannual payments. So if suppose you have a 10% of interest rate and if thousand is the you know value so in that case he will be getting 100 rupees every year and in that if it is a semi-annual payment than 50 rupees every six month will be paid.

So that will be paid on certain days called the payment days or the coupon days. So that is how you have the definition of bond and in that normally you have the, you know the amount which is full amount which is borrowed you know that is known as the Redemption value.

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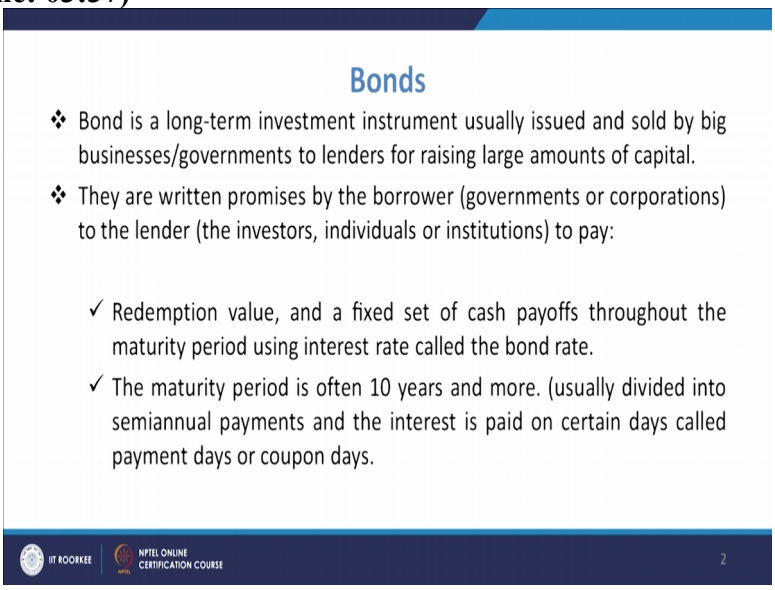

And also there are rates like bond rate or coupon rate these are you know or contract rate these are the synonymous terms which will come when we discuss these you know terminologies. Now coming to the bond valuation how the you know bond is valued how the value of the bond is calculated;

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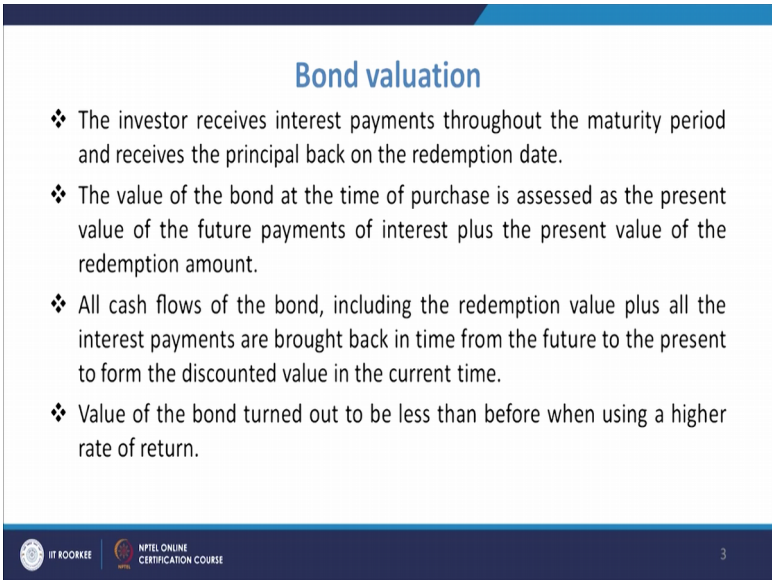

So the investor receives interest payments throughout the maturity period and receives the principal back on the redemption date. So we already discussed that he will be getting the interest payments throughout the maturity period. So you have a period that is maturity period and throughout that he will be you know whoever is investing that is the investor he will be getting certain payment.

And in the end you know on the redemption date he will be getting the principal amount also. Now value of bond at the time of purchase is assessed as the present value of the future payments of interest + the present value of the redemption amount. So it is clear that you know if something you know its value is to be seen, so the value of bond will be like whatever he is being paid you know in future.

So he will be getting that amount you know after a frequent you know some fixed intervals. So all those payments you know present worth or present value will be calculated including also that of the redemption amount and that value will be the bond value. S so all cash flow of the bond including the redemption value + all the interest payments are brought back in time from the future to the present to form the discounted value in the current time.

So that is how we are getting their value because we are getting is discounted values in the first so if it will be divided by 1 by 1 + interest rate then in the second 1 by 1 + interest rate is 2 power 2 like that. So we are getting a discounted value you know every time and then we are getting the value of the, you know, bond. So suppose, for example, if you have suppose, you have a bond and suppose, you are talking about the rupees 1000 bond.

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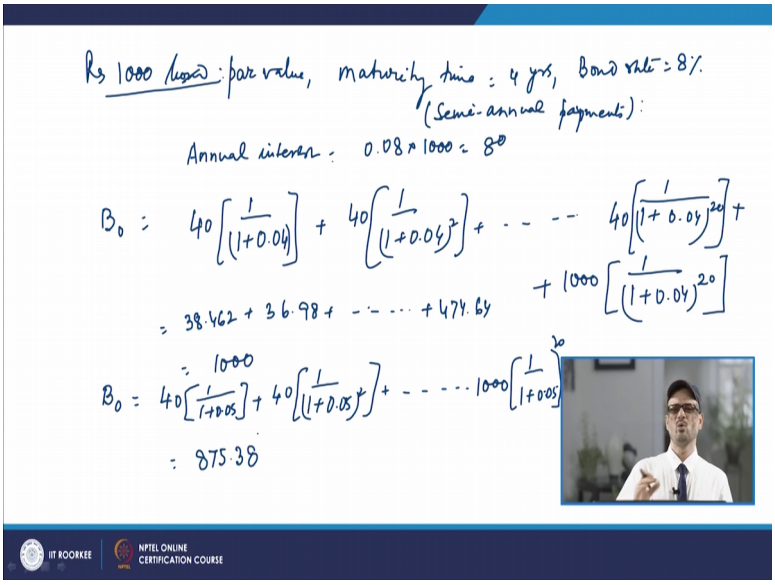

And now you know the thing is that that is that is known as a power value, so this is this is known as power value of the bond. And suppose the maturity time is 10 years so if the maturity time is 4 years and if suppose the bound rate is 8 percent so and by convention if you get the semiannual payments. So in that case as we know that bond rate is 8%. So this person will get the interest at the rate of 8% of 1000 so your interest will be rupees 80.

So the annual interest that will be 0.08 multiplied by 1000 so that will be 80. So semiannual payment if it will be 40 each. Now what will happen that this 40 he will be getting for the, you know 10 years and you know at you know 6 months of period at 6 months of gap? And in the last you know 20th installment so then we 20 installments and in that 20th installment he will also be getting you know this value also this is 1,000.

So that is original amount which is burrowed so that 1,000 will also be there. So, so in that case the bond value will be calculated so that will be nothing but you know you have 40 getting that is 1 by 1 + 0.04 so that is your 4%interest for every 6 month. So similarly you have 40 then you have 1 by 1 + 0.04 raised to the power 2. So this way will go and you will go in the end you will have 20th payment.

So that will be 10 to year here you will be getting it is you know 1 by 1 + 0.04 raised to the power 20. And then apart from that you are also getting the amount original amount borrowed so that will be your thousand so this will be thousand and it will also be having the multiplied with the factor 1 by 1 + 0.04 raised to the power 20. So if you take this if you do the calculation you will be having the calculation this as suppose, at 38.462 and then then another you have 36.98 or so.

It will be moving and ultimately for this amount it will be 474.64.So you will have all these about decreasing basically because you are you know multiplying with the factor 1 by 1 + R that if the R is the bond rate and then every time as the time is progressing and the amount is same sis contribution will be getting you know less and less so this will be coming out to be 1,000.

So this is how that you see that how these all these cash flows which is their you know including the Redemption value so all their worth, you know what when they are brought back their worth is brought back to the present value then it's coming and equal to be 0 so that is coming as one thousand. So, so that is how this bond value is calculated. Now what happens that many times now the value of the bond turned out to be less than or less than before when using a higher rate of return.

Now what happens that when we look at some other investment opportunities and we see the and we try to calculate based on that different you know rate of return that is current rate on which you know you will have that rate differing from the bond rate whether that rate may be larger or that may be the smaller than the bond rate then this value turns out to be either less or more than this it is you know par value.

So now that basically is because of the you know market rate you know of return for similar securities many a times we feel that this rate may be different than the bond rate that may be you know larger or the smaller. So you know in that case if you have to calculate this you know value of the bond that is of 8% coupon rate but then, by discontent by discounting the previous cash flow of the yield at 10% in that case what will happen to this bound rate?

So suppose in that case if you have the yield rate of if you are thinking of yield rate of 10% in that case your B0 if you try to find you know you have to you have the so 10 percent means semiannual it is 5 percent so it will be 1 by 1 + 0.05 then you have again 40 into 1 by 1 + 0.05 which is power 2 so that way it will go and in the end you will have again thousand and 1 by 1 + 0.05 and raised to the power 20.

So suppose in that case when this market rate is higher, you are taking that current rate as the higher one, in that case, if you calculate this B naught then its value is coming as 875.38. Now what you see is that these value of the bond that is turned out to be less. So you know when you are using the higher rate of return when you are comparing with that when you are using the higher rate of return in that case the value of the bond is turning out to be less.

So, so that is why it is written that value of the bond turned out to be less than of you know before when using higher of return similarly if you take the you know rate of return as the smaller one in that case the value will be more. So this way the value basically changes and we know that we use the same formula when we have the I so we will take that B naught and it will be summation of 1 by 1 + I raised to the power T and T will be varying from 1 to N and + your the Redemption value.

And that multiplied by again 1 by 1 + I to the power N. So that way we will be calculating and we will be seeing that it may be either more or less.

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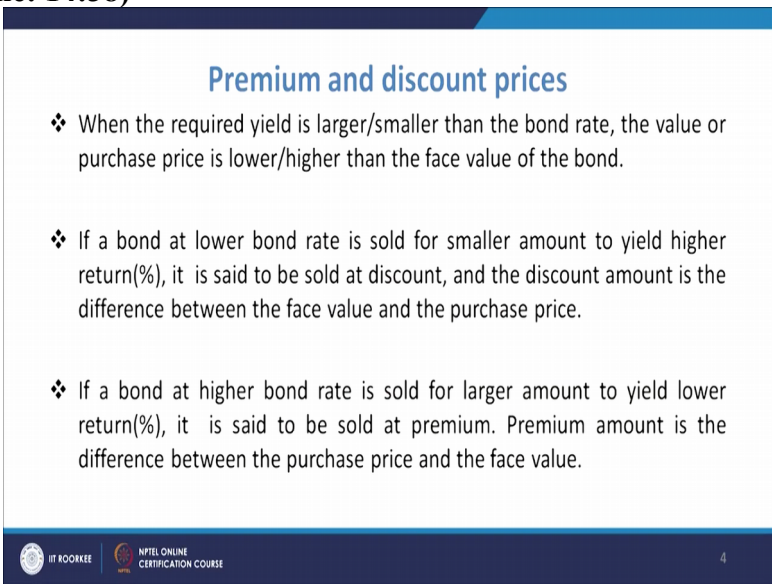

So, so that much you know, so when the required rate yield is larger or smaller than the bond rate then value or purchase price is lower or higher than the face value of the bond. Now you know how we can see from the example.

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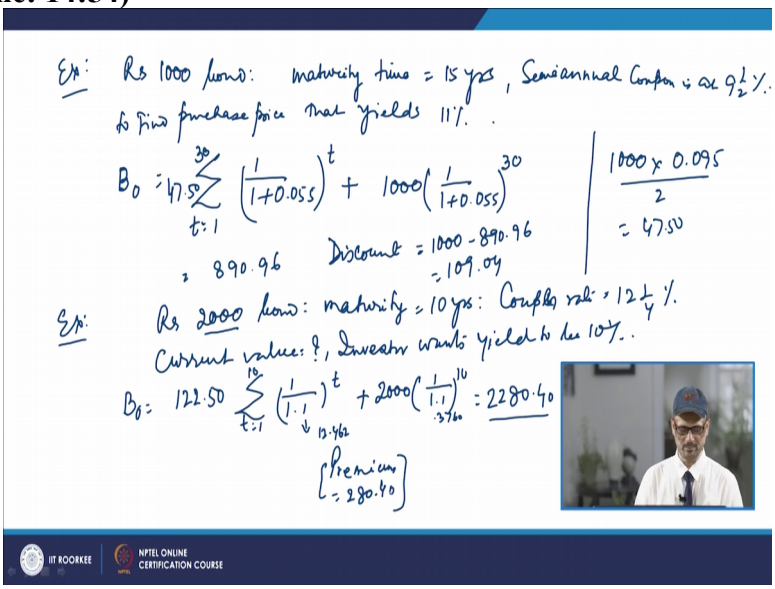

So suppose there is an example that you have a Rs.1000 bond and suppose it is maturity is maturity time is 15 years and you know semi-annual coupon is there and this semi-annual coupon is at 9 1/2%. Now in that case you have to find the purchase price so to find purchase price that yields so that will really like 11 percent. So in that case your bond rate is 9 1/2% and the current rate that yields basically is 11% in that case what will be the purchase price? So you can find this by using this I value as 11% and in that case you will do the same thing, you have, you know the semi-annual payment will be on the basis of 9 1/2%.

So you will have you know 95, 47.50 will be multiplied with so your B naught which you will be calculating now B naught will be basically calculated so you will have the summation of and you will be doing that with one by one + I so that will be 0.055 and this raised to the power t and t will be varying from 1 to 30. So because your, this is your semiannual payment and it is going for 30 years.

So you will be doing that and this will be multiplied with you know it is 9 1/2%. So if you calculate you know semiannual payment it will be 1000 x 0.095 and divided by 2. So it will be 47.50. So you are going to multiply that with 47.50 and then now further you have this as the 1000 bond. So this is 1000 and again multiplied by 1 by 1 + 0.055 and raise to the power you know t.

So this n is 30. So this is 30 so this way this will be coming out to be 0.006206 and if you calculate these you know sum. Then this is coming out to be 14.534. So you will get in be getting something like 890.96. So what you see that in these cases these purchase price will be coming out to 890.96. That is what that when you are you know this rate is higher kind of market rate is you know yield that is at higher percentage in that case the B naught value is decreasing from 1000.

Similarly if you have the you know rate is smaller in that case you know the B naught value will be larger and its example can be seen like suppose you have the rupees 2000 bond and here you have maturity of you say 10 years and your bond rate is or coupon rate is 12.25 percent coupon, coupon rate is 12 1 by 4 percent. And in this case you know you have to find the current value so you have to find current value and condition is that the investor wants yield at 10 percent.

So in investor wants yield to be 10%. So in that case if you again you can do the same thing you have coupon rate is 12 1 by 4%. So again in this case if we take the semi-annual payment in that case it will be 122.5 so and in this case since it is not mentioned that there will be

semiannual payments so we can take we have to take the annual payment itself. So your coupon rate will be 12.25%.

So you will have a 120 point you know 50. So if you calculate the B naught it will be 122.50 and then you have to find the summation of 1 by 1 + I. So one by one point one raised to the power t and t will be going from 1 to 10. So it is maturity time is 10 years and then again we are going to have the you know 2000 multiplied by 1 by 1.1 raised to the power 10.So that will be 0.3769 and this factor is 12.462.

So if this value is coming out to be 12.462 and this is 0.3 you know 760. So taking these two values you see that it is 2280.40. So what you see that these values you know they are coming in one case it is you know smaller so when your interest you know yielding yield rate is 11% it is more than the bond rate in that case of value is less than the face value. And otherwise it is more than that.

And that is what it is written that when the required yield is larger in then the bond rate, the value or purchase price is lower. So the one that required the yield is yield is your larger in this case it is larger your the value which is less than the face value and when it is more in the next case in that case you know it is it is the yield rate is yielding is the percentage which we want the yield is smaller than the bond rate in that case the value is more than the face value of the bond.

Now how we define it that we define it like this. That when you are if a bond is at lower bond rate and if it is sold for some smaller amount to yield a higher return, so if it will be sold of you know for a smaller amount to yield that higher return, in that case it is to be sold at discount. And the discount amount is the difference between the face value and the purchase price. So in that case basically when you are using the higher yielding return then in that case what we see that value is becoming smaller.

So the difference between the face value and the purchase value, that is known as the discounted amount. Now contrary to that, if your bond at higher you have a higher bond rate and you know if you are sold in selling at you know larger amount to yield higher return you know lower return basically so in that case it is to be it is said that it is said to be sold at you know premium. And the premium amount will be now you know the difference.

So in this case your difference will be there between the purchase price and you know the face value. So the purchase price will be more so the difference of purchase price and the face value that will be known as the premium amount. In such case, so this discounted you know

concept and the premium concept will be used you know in our further studies. So what we have seen that your bond is when it is you have the case of premium as well as the discount you know amounts.

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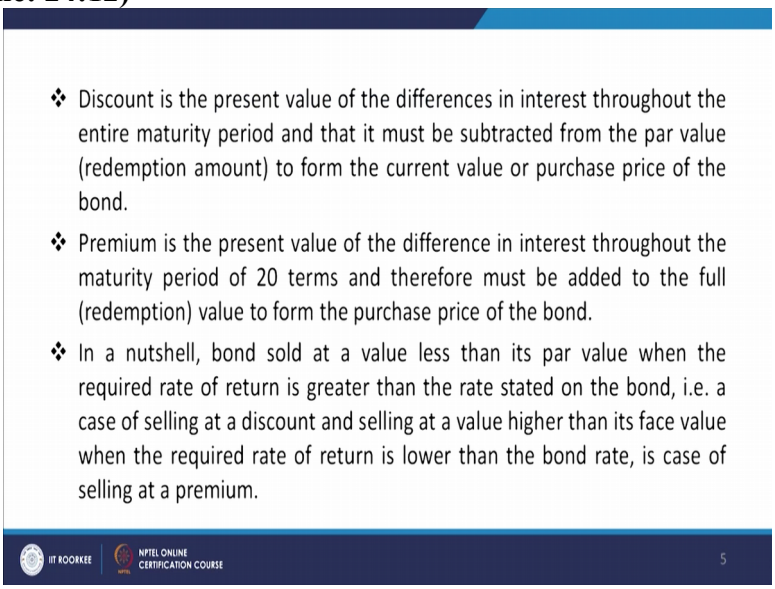

And discount is the present value of the differences in interest throughout the entire maturity period and it must be subtracted from the par value to form the current value or purchase price of the bonds. So before that what we wish to see, what we saw that in this case your bond value was you know 1000 and then ultimately this is the amount. So this is the face value of 1000 and the bond value is coming out to be 890.96.

And so this in this case this is the discount which is offered and so discount amount if you calculate this discount will be 1000 – 890.96. So 109 point you know 04. So this will be your discount in such cases. Now similarly when you are you have 2000 bond and it is sold for 2280.40 so you have 2000 bond and you are selling for 2280.40 so that you know it gives you 10 percent yield which is less than the bond rate bond rate was 12 ½.

But you are you know selling at 2280 so that the yield is 10 percent now less than the bond rate so in that case this difference, difference of you know 280.40. So this premium amount which will be you know 280.40. So this is known as the premium that is what it is defined as now what when we tell that you know so that's what we are talking that when we talk about this discount.

Now this discount is nothing but it is the present value of the differences in interest throughout the entire maturity period. And it must be subtracted from the par value so that we can you know further understand like you have seen that the you had the bond you know

value of one thousand and if you are taking for this 9.5% interest so you will have the semi-annual interest that is thousand in to 0.095 and divided by two.

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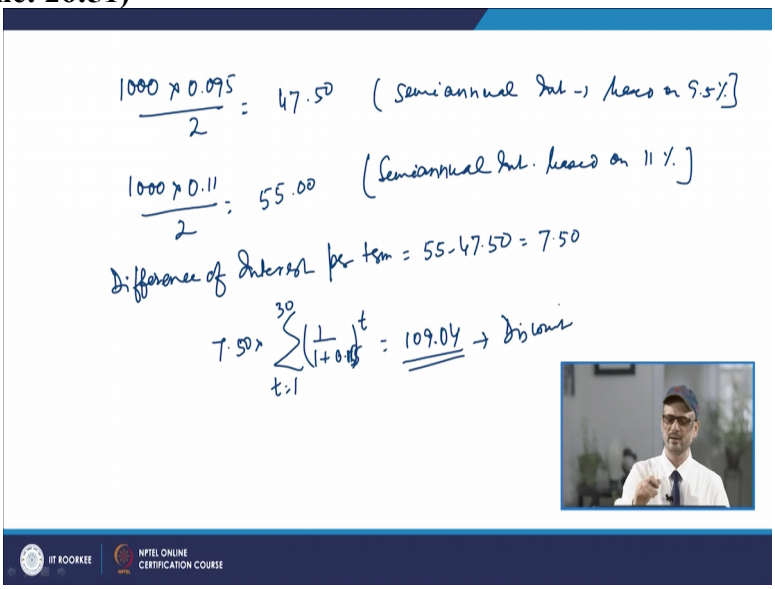

So that is your 47.50 and this is your semiannual interest based on 9.5% that is based on a 9.5%. Now, now, if you know what happens that if you take the semiannual interest based on 11 percent in that case semiannual interest on based on 11% will be 0.11 by 2. So this will be 55. So now what happens that this difference so this will be your semiannual interest based on you know this is 11% now what you see that the difference of this interest that is your difference of interest per term.

So it will be 55 - 47.50. So basically this difference is for the entire maturity period and you have 30 terms. So if you find you know the value of these so this is your 7.50 and if you take the you know its value at present is 7.50 so that will be multiplied with you know the factor that is summation of 1 by 1 + I raise to the power t and that will be at 11% so that basically will give you this 109.50 or 0 for 109.04 that is what was the discounted amount.

So you had this discounted amount of 109.04 and if you calculate this 7.50 and if it is multiplied with summation of 1 by 1 + 0.1 and that is by t and t will be 1 to 30 so this will be 0.055 basically it will be you are the semi-annual payment. So 1 by 1 + 0.055 and then if you take this value now this will be basically 14.53. So that comes out to be 109.04 and this is what was your discount. So what you see you can come to this that the discount is the present value of the difference in interest throughout the entire maturity period.

So it is the difference what we calculated the, you know interest that is calculated first that is coming out to be 7.50 and it should must be subtracted from the par value to form the current value or purchase price of the bond. So this is for the discount similarly you have the bond

also for the premium also. And premium will be defined as the premium is the present value odd of the difference of interest throughout the maturity period of twenty terms and in this case basically it is added.

So if you take the, you know, example of premium so in the case of premium what we have seen that you had the 2000 rupees of you know you know premium.

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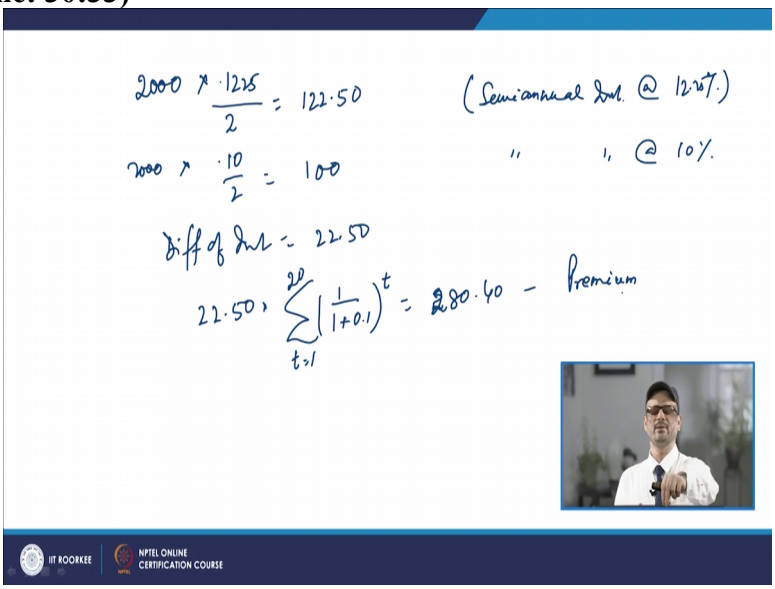

So you had the bond and here you had you know 0.1225% was there. So the 12.25% was the bond rate. So semiannual payment was you know so semiannual interest was coming out as 122.50. So this is you know semiannual interest and this is for you know 12.25%. Similarly, you have at the rate of so this is basically you have yielded is 10% only so for 10% it will be 100. So this is semiannual interest at you know 10% so that is your yield rate.

Now in this case, the difference of interest is coming out to be 22.50 and this 22.50. Now this if you calculate it is you know again its value and multiplying with the factor so that will be you know so if you multiply this 22.5 with that factor that is summation of t1 - you know and you have 40 you know 20 terms this is 10 years so you will have 20 terms 1 by 1 + and your this unit is 1 + 0.1. So if you take that and on that t. So this if you take it will be this is coming out to be 12.462 so this is coming out to be 280 this is 280.40.

That is your premium. So this way we can define the premium is the present value of the difference in interest throughout the maturity period of 20 terms. And in fact, what you do is that you are adding you know value to this you know face value to form the purchase price of the bonds. So purchase price in one case was less and in another case it was more and in a nutshell what we see that bond is sold at the value less than its par value when the required

rate of return is greater than the rate stated on the bond. So that is case of discount and when you are selling the value higher than its face value.

Then it is a case of you know you know so that is the case of a premium, selling at premium. So these are the two cases in which these bonds are basically evaluated and you know their value is calculated. So now this is about you know the premium and discount prices related to the bond valuation and that is how we calculate and it will be used this concept will be used in our you know studies related to the you know bond valuation and other topics, you know that we will come across. Thank you very much.