

Financial Mathematics
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Lecture – 43
Two Stage Dividend Growth and Preferred Stocks

Welcome to the lecture on two-stage dividend growth and preferred stocks. So how to find these costs you know stock value when there is two stage dividends growth or also we also will try to have other methods for the common stock valuation. And then in the end we also will also know about how to find the cost valuation, you know stock valuation in the case of preferred stocks.

(Refer Slide Time: 01:00)

Stock value with two stage dividend growth

- If, at some point, earnings and dividends go up to exhibit a very high growth, sufficient for g to be greater than r , in such a case, the Gordon's formula would produce a negative value for the stock.
- Hence the value of stock is estimated using the two-stage growth formula, which addresses the growth in two stages.
- Stock value is estimated by combining the present value of dividends from year 1 to n with the present value of dividends from $n + 1$ to infinity.
- Other methods of common stock valuation is price/earning ratio model, book value per share method, liquidation value per share method etc.

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So as you know that you know if at some point earnings and dividends go up to where to exhibit you know a very high growth and you it you know g becomes more than r , then this Gordon's formula would produce a negative value of the stock. So basically the, you have to use, you know, the two stage growth formula. And stock value is estimated by combining the present value of dividends from year one to n with the present value of dividends from n plus 1 to infinity.

So you have to do in the two stages when the, you know g becomes more than r in that case you will have to find out you know use the formula based on that.

(Refer Slide Time: 01:57)


$$P_0 = D_0 \cdot \frac{1 - \left[\frac{(1+g_1)}{(1+r)} \right]^n}{r-g_1} + D_{n+1} \left(\frac{1}{r-g_2} \right) \left(\frac{1}{1+r} \right)^n$$

Ex: Current price: ?
 Annual dividends are expected to grow by 25% for next 5 years & slow back to 5% after that. Dividend is Rs 37 per share & required rate of return is 15%.

$r = 0.15, g_1 = 0.25, n = 5, g_2 = 0.05, D_0 = 37$

$$P_0 = 37 \left[\frac{1 - \left[\frac{(1+0.25)}{(1+0.15)} \right]^5}{0.15 - 0.25} \right] + 37 \left(\frac{1}{0.10} \right) \left(\frac{1}{1.15} \right)^5$$

= 828.69



So if you look at the formula, the formula tells like you have P naught so P naught will be D naught and then that will be multiplied by you know 1 minus then you have 1 + g 1 and so g 1 and you have g 2. So there are 2 you know growth rates so 1 + g 1 and then that divided by 1 plus r so that will be you know and that will be divided by so that will be powered n and divided by r - g 1.

And this will be again multiplied by r plus g1. That is what we had seen earlier that you have this is D1 so D naught into 1 + g 1 so that will be that. And then this is the formula for the first period. And then further from n + 1 to infinity it will, you know. So, n + 1 onwards it will go so D n + 1 so it goes for the infinity period. So you have only 1 by r that effective term. So it will be 1 by r - g 2 and then it will be 1 by 1 + r raised to the power n.

So, so this way you get this value of the current price of the stock. So this also can be you know understood with one example. Suppose you have the current price you know per share of you know is for any corporation is to be found out and so current price to be found. And in that case you are given that these annual dividends, so annual dividends are growing. So are expected to grow by 25% for next 5 years.

So this, such you know conditions are given and slow back to 5% after that. Now you know in that case it is given that the dividend is rupees 37% and required rate of return is 15%. Now in such cases we know that your this 15% is nothing but the r. So we know that r is becoming 0.15 now in the first for the, you know for first five years it is, you know, 25%. So g1 will be you know 0.25 and n will be 5.

So in that will be used in this formula for the first period and then you have the g 2 comes into picture. So g 2 will be you know it is you know comes down to so, 5% after that. So g 2

will be 0.05 and in that case your and will be certainly after that since it is going towards the you know after that it goes for you know for the larger period. So this is all about the data and you are given D_0 as 37.

So you know with this formula, you can find the P_0 and P_0 will be you know in that case you can find so it will be 37. And then you will have $1 - r$. So it will be $1 + g_1$ by $1 + r$ so it will be $1 + g_1$ is 0.25 and then divided by $1 + r$ is 0.15. So this multi you know raised to the power n so this is you know for you know five years. So it will be you know done with five. And then you have $r - g_1$ so it will be 0.15 minus 0.25.



So this term also becomes negative and this also becomes negative so that basically cancels and $1 + g_1$ will be $1 + 0.25$ then the second stage growth for that you will have this value but even is the same so you will have 37 again and $1 + r - g_2$. So r is 0.15 and g_2 is 0.05. So it will be $1 + 0.15 - 0.05$. So it would be 0.15 - 0.05. So it will be 0.10 and then it will be $1 + r$ raised to the power n .

So it will be $1 + 0.15$ and raised to the power 5. So this way you can calculate the P_0 value and it will be coming close to 828.69. So this is how you solve such kind of you know problems, when you have the two stage growth or are different stages Gordon's growths in the case of you know dividends you know and also the dividends how they grow. So that is how you calculate. Now we have to find other methods of you know cost evaluation.

(Refer Slide Time: 08:11)

Stock value with two stage dividend growth

- If, at some point, earnings and dividends go up to exhibit a very high growth, sufficient for g to be greater than r , in such a case, the Gordon's formula would produce a negative value for the stock.
- Hence the value of stock is estimated using the two-stage growth formula, which addresses the growth in two stages.
- Stock value is estimated by combining the present value of dividends from year 1 to n with the present value of dividends from $n + 1$ to infinity.
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2

And the other methods of cost evaluations include the price by earning ratio model then book value per share method then liquidation value per share method and all that so that you know we will be discussing. So, among the methods which are, as we discussed, that will be the price by earning ratio multiples method. Then you have other methods like book value per

share and liquidation value per share all that methods are there. So if we try to see that how these methods work?

(Refer Slide Time: 08:43)

P/E multiples method:


$$V_c = EPS_e \times (P/E_i)$$

Ex: A Corporation expects earnings per share to be Rs 3.10
ratio is rated at 9. $V_c = ?$

$$V_c : 3.10(9) = 27.90$$

Book value per share method:

Ex: total asset of a company : 2.7 million rupees
total liabilities : 1.2 million
Company's proposed stock dividends : 65000 Rs.
Company has 8500 common shares.

$$V_c = \frac{270000 - (185000)}{85000} = 10000 \text{ Rs.}$$


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Now, the first is the price you know by you know earning so price by earning ratio so that is known as P by E. So that is P by E multiples method. Now in this case, what is done is, the value of, you know, the value of EC that is found and this will be you know basically found as the earning per share. So that is you know this is expected earnings per share of the company. So that is why it is written as a EPS.

And that will be so this is value of the common stock so that is why we are writing V c. So expected you know this is earning per share of the expected earnings per share of you know the company. And then that will be multiplied by the P by E i. So this is price by earning ratio so that is P by E i of the industry. So this way we calculate the you know the value of the this you know common stock.

Now it is more suited for the firms which are not publicly traded. So you know you can say that suppose you have a train service corporation is there and it is expecting, so a corporation expects you know it expects its earnings per share so earnings per share to be you know peace 3.10. Now so that you know the price by earning ratio is related to 9. So that the price by earning ratio you know so is rated at 9. So and it is basically you know at a time so but not that so that is say at a time when you have the price by earning this you is rated at 9.

Now what will be the good estimation of the value of the share in that company? So in that case you have to find the VC. Now you know that we can simply find V c as the EPS so my EPS is said to be 3.10. And then price by earning ratio you know it is expected to be at 9. So you will multiply with 9 so it will be 27.90. So that way this you know price by earnings

multiple method works. Another value that is known as the book value per share method so now you know this method based on is based on certain hypothetical situation.

And in this you know it is estimated it is you know you know this gives a good estimation you know of the value of you know share in the phone and it will be thinking about the you know what will be the for the share, what will be the stockholders support capital share of all the firm's assets. So basically it hypothetically assumes that the firm's you know all the firm's net proceeds and you know all the assets are liquidated.

And all the, you know liabilities you know including the preferred stocks they are paid. So basically what it tells that if suppose all the assets are liquidated and all the preferred stocks you know you know all the, you know liabilities like preferred stocks, they are paid. In that case you know what will be the firm would be the stockholders per capita so that that finding that that way you try to find the you know cost of the share.

So, so in this case, if the value will be there you so it will be in the first books. And, and that is why it is known as the book value method. So, that is basically the case of you know hypo is it is basically a, it is a case of hypothetical in nature. So it is not that actually it has the firm has already liquidated it has you know paid you know its liabilities. But you assume and that will you calculate its value like maybe for some companies if suppose you know its total assets.

So suppose the company's total asset total asset of a company so that is estimated. And total asset is estimated to be you know 2.7 million rupees and suppose liabilities, so if you find the it has been calculated that if suppose total liabilities, they are the 1.2 million. Now in this case, you know company's preferred stock you know dividends, so company's preferred stock.

So that you know dividends and this is calculated and this is calculated as you know 6 lakhs 50,000. So and also it has 85,000 common stocks; so the company has 85,000 common stocks. So now you know what will be the value of the shares? So if you have to calculate in that case you have 85 you know this is 85,000 common stocks. So what you will do is in this case?

You are going to have the asset as 2.7 million, 27 lakhs. Now from that you have to you know subtract these liabilities that is one is your 1.2 million and then the preferred stock dividends they are 6 lakhs 50,000. So you have 12 lakhs + 6 lakhs 50,000, so 66 like 50,000 plus 12

lakhs that is to be paid. So that is we that have to be paid out of 2.7 million so that is 27 million so, so basically in this case, if you have to find a VC.

So it will be basically 27 lakh minus you are going to add these liabilities. Now liability is 1.2 million plus company's preferred stock dividend that is 6.5 lakhs so it will be you know 18500. So that will be subtracted and then for every sales you know price if you have to find our value then you have to divide with the number of shares that is 85,000. So if you calculate this value, this value will roughly come out to be 1,000.

So it is coming as 1000 because it is 8 lakhs 50,000 divided by you know 80, know it what this is 10 basically. So it like 50000 by 85000. So it is 10 rupees. So it will be 10 rupees 10 so that is the value of every share that you calculate. So and this is basically you all these are recorded in the forms of books. So that is why it is known as the book value method.

What is the value now you know which is remaining at at this point of time. So that is why it is the book value method.

(Refer Slide Time: 17:37)

Liquidation value per share method:
To find net proceeds:
 Ex: 2.3 million Rs. remaining, No. of shares = 110000

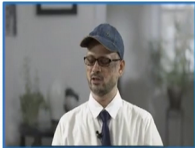
$$V_c = \frac{2300000}{110000} = 20.91 \text{ Rs.}$$

For preferred stocks:

$$P_p = D_p \left(\frac{1}{r_p} \right)$$

 Ex: A Company pays fixed annual dividend of ₹7.50 to their preferred stockholders. of $r_p = 14.5\%$.

$$P_p = \frac{7.50}{0.145} = 51.72 \text{ ₹}$$



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Then the, another method which is also important is the liquidation value. So this is liquidation value per share. Now in the earlier case using the book value method you are assuming hypothetically that the, you know liabilities are to be paid assets are to be sold. Now in this case this it is it will be similar to the, you know book value method. But the only thing is that this is for the actual case not the hypothetical case.

So now it is about the value of a share in a firm that is actually being liquidated so for that you have to work based on these data which you have. Suppose you know a firm is there and

its team is responsible for handling the firm's assets, liquidation. And it has reported that you know 2.3 million would remain after the pay mean after they pay all the liabilities.

So once you pay all the liabilities then you have remaining with you 2.3 million. So that is with you and then you have you know the all these remaining proceeds or you know to be distributed among the number of shares present. So now if the numbers of shares you know are 1 lakh 10000. In that case the value of the share can be found so you have to find you know the net proceeds to find you know net proceeds.

And if suppose you know they are finding that the team has done the homework and it has found that the, suppose the company has you know 2.3 million remaining. So remaining after all the, you know paying all the debts and all that liabilities. And you have a number of shares as one lakh you know 10000. So then, what you will do is, that you are going to divide and you are going to get the volume, you know value of the you know common stock that is there.


And it will be 23 lakhs and divided by 1 lakh 10000 so that will be something close to 20.91 rupees per share, will be the value as per these. So once your department or the term or the team of you know the experts they have worked on all these data and they found that this is how you are you are having the remaining of this much of you know amount which is remaining, in that case you are going to calculate this VC by dividing with the number of shares.



So this is all about the, you know finding the values for the, you know common stocks. Now we will come to the preferred stocks.

(Refer Slide Time: 21:04)

Preferred stock

- As preferred stock does not have any maturity date and it pays a fixed dividend for as long as the stock is outstanding, the payment of dividends can be considered a perpetuity.
- The cost of preferred stock is represented by the required rate of return (r_p).



  3

And you know what we see that in the case of preferred stock they do not have any maturity date and it pays a fixed dividend for as long as the stock is outstanding. And that is why the payment of dividends can be considered as considered as perpetuity. So as we have understood that they do not have a time of maturity and, and as long as the firm continues, they, they will be you know it is part.

So that basically is a concept of perpetuity for the infinite time they will be you know with the with the company and in that case you have the cost of preferred stock basically you have the you know when you find the price of these stocks we represent with P_p . So price you know for the preferred stock so that is P_p for common stock we write P_c . Similarly the rate of return for the common we had a studied up man you know discussed many times r_c .

But here we will arise use the term r_p and also we know that when you have the things go in perpetuity. In that case whatever be the dividend you know dividend will be divided by this rate that will be the amount of, you know the, the current price. So that will be your that is how the price is calculated; so in those cases, when you are trying to find, so in case of you know for preferred stocks.

So if you will take for preferred stocks valuation if you do. So the formula becomes P_p for the preferred stocks. Its price it will be you know D_p so that is your dividend which is being paid for the preferred stock and it will be multiplied with 1 by r_p . So that is how you get the value of for the P_p and so P_p will be D_p by r_p . So, for example, suppose a company has a, so if suppose there is a one company which has paid a fixed and relevant.

So a company so that pays fixed annual dividend so there is some amount that is supposed of dollar 7.50 and you know to their preferred stockholders. So as we have understood that company has to pay them first so that is your preferred stockholders. Now what will be the value of the preferred stock with the required rate of so if the required rate of return for the preferred stock is 14.5%?

Then what will be the, you know value of this preferred stock? So value of the preferred stock price of the preferred stock, it will be dividend, divided by the rate. So dividend is 7.50 and rate is 14.5%. So it will be 0.145. So, so that becomes you know 51.72. So 51.72 dollar is the price or the value of the preferred stock you know in such cases. Now, now we also should know something more about the preferred stocks.

When we talk about the preferred stocks there is a terminal that is cost of preferred stock and this cost of preferred stock is represented by the required rate of return. That is your r_p and

we know that from the same formula if you try to see r_p , r_p will be your D_p by P_p . So if you talk about the cost of preferred stock.

(Refer Slide Time: 25:20)

Cost of preferred stock

$$r_p = \frac{D_p}{P_p}$$

$$N_p = P_p - FC$$

$$r_p = \frac{D_p}{N_p}$$

Ex: If a Company pays out 9% of its stock par value of \$96 as dividend of preferred stock & cost to issue & sell is \$4. What is r_p ?

$$D_p = 9\% \text{ of } 96 = 8.64$$

$$N_p = 96 - 4 = 92$$

$$r_p = \frac{8.64}{92} = 9.4\%$$

So we know that now you get this r_p as you know the D_p by P_p . So this will be dividend for the preferred stock divided by the price of the preferred stock. Now instead of using this preferred value the preferred stock value as P_p , we also use the term net proceeds because you will have to, you will have to subtract the flotation charges. So basically the, you have to find the net proceeds so that will be your N_p .

And N_p will be nothing but the preferred stock price minus flotation costs, flotation cost is there while transacting with these shares. And so your actual equation for the r_p becomes D_p by N_p . So when the, you know flotation cost is also involved and when you have to deal with that in that case the flotation cost has to be incorporated in the formula to find the N_p net proceeds.

And the r_p will be the ratio of the dividend for the preferred stock divided by the net proceeds. So for example, suppose, there is a company is there and it pays out 9% of its stock. So it will be in a paying. So if a company pays out 9% of its stock par value of you know dollar 96 as dividend of preferred stock 9% as preferred stock, you know. Given that cost to the issue and sell and the cost to issue and sell is dollar 4.

Now in that case you know what will be the cost of stock? So we know that it is using the 9%. So 9% will be you know so D_p will be you know 9% of the 96. So that will be your 8.64. Now the N_p that is your net proceeds, net proceeds will be 96 - 4. So that will be your 4 is the up you know the FC that is your flotation cost so that is, that is you are pursuing for the issue and sell, so it is coming as 92.

So your rate of return if you find, if your rate of returns you are interested for the preferred stock it will be D_p by N_p , so 8.64 by 92. So if you take its value it will be coming as 9.4%. So that is the rate of return for the preferred stock and that is how you calculate the value of the preferred stock. So what we have so far understood that you know there are many you know points which needs to be kept in mind while dealing with these you know.

Now valuation of the stock or the cost of the stocks for beat the common stocks or beat you know preferred stocks and how to you know find because the floatation costs involved and how to you know calculate then. Also we studied about the growth in the dividends and growth also in one you know once the growth rate is constant or the growth rate has changed in stages.

So in those cases we have used the different formulas for finding you know the value of the price value over the price of the cost you know stock. So the all these formulas you know we have studied about the interest factors and basically they are instrumental in giving you all these expressions or or recalling you all these expressions because you are using these you know present worth values using the future you know values.

So you are basically finding those you know usual using these factors to find the; you know current price of the you know share or the stock or so. So that way you have to have the clear understanding of which formula to be used and especially when your growth rate is given and growth rate is changing. So in those cases how to use the formula that must be you know kept in mind so that you can efficiently use these formulas and get the net you know required results. Thank you very much.