Financial Mathematics
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Lecture-04
Statistical Measures

Welcome to the lecture on statistical measures, so we will be try to be conversant with many of this statistical parameters, many of these statistical terms which are use to when we study the financial mathematics. And most of them are related to like finding the mean, average, inner average, then you have a standard deviation, variance. Then we also find certain terminologies like covariance, correlation.

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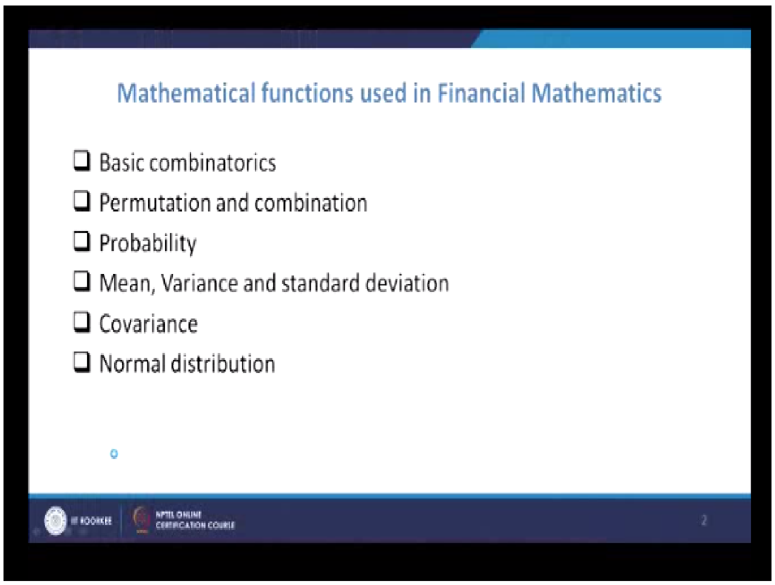

So, all this so we will discuss first of all we will have some standing about the basic combinatorics and in that basically when we talk about the basic combinatorics it basically talks about the fundamental rules mathematical rules and concepts of counting and reordering and ordering. So, basically there are many times you have to order, you have to do the arrangement, so what are the different rules fundamental rules for that.

So that we will try to we acquainted with then we will talk about the permutation and combinations, so you must have the idea about the permutation and combinations. So, in the permutation how you have to arrange, then in combination how you are making that group. So, you know there are certain rules by for having the permutation and combinations , we will also have some idea about the probability.

And then we will discuss about these statistical terminologies like finding the expected value or that is also known as mean. Then finding the variance, so how the variance is calculated, how it is variability among the data. So, you how that is calculated that is quantified, then there is a standard term standard deviation, so that also we will know that how to find or calculate this standard deviation.

Then we will try to have the idea about the covariance, covariance is further we calculate you know depending upon certain expression we find the covariance. And then finding the correlation from there basically it will talk about the you know how the 2 random variables, how they are going to behave. If 1 is increasing, the second is also increasing or if 1 is decreasing, how the second is behaving.

So, depending upon that so depending upon this correlation and correlation coefficient you can predict that how these 2 are behaving . Then we will also discuss about the normal distribution because it will have the use of you know the mean value, then standard deviation values and all that. So, all these terminologies we will try to be acquainted with. So coming to the basic combinatorics rules where we are trying to get familiarized with the basic rules, so as we know that there is a n factorial rule.

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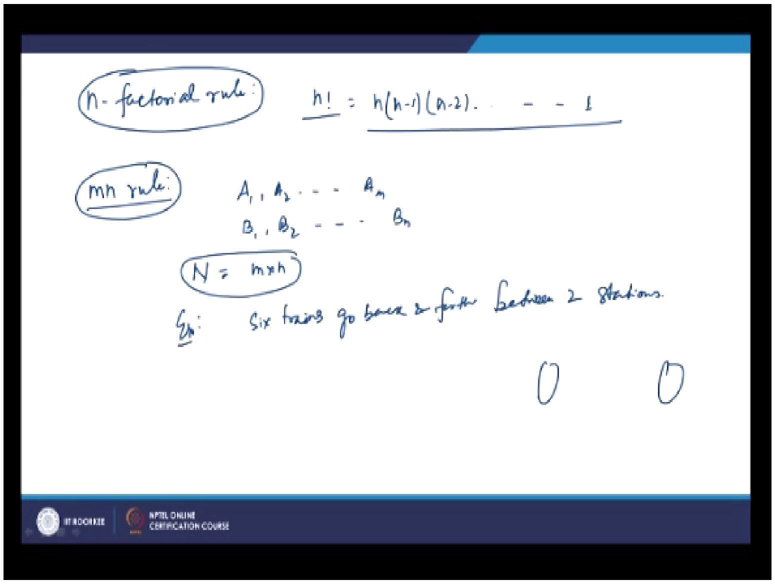

So, you have the n factorial rule now in this case what we do is that if suppose you have you know n objects ad you want to you know order. So, in how many way you can order them, so that time you use that n factorial rule, so you have n factorial value is calculated. So, this is n factorial, so they how can the ordered you know, so that is n factorial.

Suppose you have 2 objects, so they can be in order in 2 way, so AB and BA similarly if you have 3 objects. So, that way how can you know order them, so A, B C, so you have ABC, ACB. Similarly B will be there at least some place SE and CA, so C will be at just 1 place AB and BA, so that way you will have 6 type of you know ordering is possible.

And that is why when you have n objects taking all at a time, so if you want to find the total number of ways in which they can be ordered. So, that will be your n factorial, so n factorial will be n*n-1*n-2 it will go up to till it counts as the 1. So, that way you find these this is known as n factorial rule, similarly you have the mn rule. So, if you suppose you have a m elements and n elements.

So, the total you know you can form total of groups that is mn, so if suppose a has m elements so A1, A2 and Am. Similarly if you have a B as B1, B2 up to Bn, so in that case you can have if you have to have a group you can have a total of so, A1, B1, A1B2, A1B3, A1Bn similarly A2B1, so that way you can total of n elements that can be you know form a total of n you know groups and that will be equal to m*n.

So, this is known as m by n rule, so mn rule, so **so** this way we try to see that how these you know rules are being applied in those cases and this will be used many a times you have also to use to your own judgment suppose you have 2 stations and in between people have to be stop. They have to stop at 1 place, second place or so or take to the you know example that you have you know go from 1 station to another station.

And you have you know so that way you have to exclude the your own station, so that way you have to go and then you have to come back. So, in those cases you have to find in how many ways you know suppose you have to go from station 1 to station 2 and you have 6 trains example is like you have 6 trains which can go back and forth between 2 stations.

Now the thing is that if you put the conditions like you are going from 1 station to second station and you have 6 trains and the condition is that when you are going from **1** by 1 train you have not to come back by that same train. So, so in how many ways you can do that, so you can go by 6 way but you can come by only 5 way.

So, that way it will you will have a 6*5 and that comes out to be equal to 30. So, that way you can have the you know what way in how many ways you can come back that will be 6*5 that is 30. Then you know comes another term that is that will be used will be the permutation.

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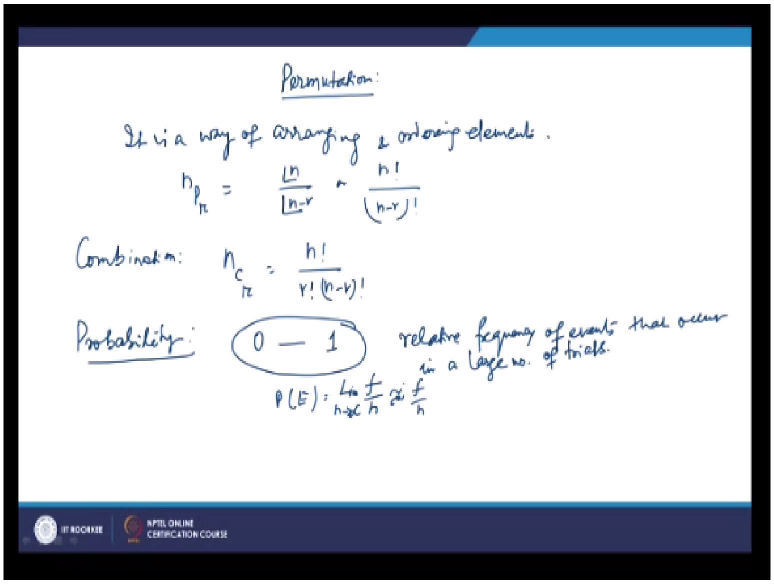

And as we know that permutation is presented by npr, so when we talk about you know arranging an ordering elements. So, **so** it is a way of arranging an ordering elements, so you know it is basically defined as the number of ways by which you can order or arrange them. They can r at a time, so in that case it will be npr, so you have total n elements and you have to have r at a time and you have to arrange and order.

So, you how many you know ways you can make them and this is known as npr and this is as you know you must have studied about all this. And this is defined as factorial n/factorial n-r or factorial is also defined as this. So, this is known as the permutation then you must have the idea about the combination and as you know that you have in case of combination .

It is aby of you know again here you have the you have to see again this is also the way of arranging in ordering elements and here it is defined as ncr and this will be n factorial/r factorial and then n-r factorial. So, that this known as combination formula and this is ncr. Now in this case what is there in this case the difference as you know that ab and ba will be taken as different one in the case of permutation.

But they will be taken as the same in the case of combination, so that is why the in the case of combination you get the values ncrs n upon factorial r into factorial n-r. So, that way we calculate these values of the permutation and combination. So, suppose you are given certain suppose number of people and among them you have to make a 5 member committee.

So, in that case in what how many ways you have to make the committee because suppose you have to make a committee of you know you have abcdefghi and j and of the there you have to make a 5 member committee in how many ways you can make that committee. So, in that case permutation will not work because either it will be abc or **a** acb or abcde and bcdea, so it will be same.

So, that is why we call at the same combination, so it will not be the different combination. So, that way you can have 10c5 it will be factorial 10 by factorial/5 factorial 5, so that way we calculate these you know permutation and combination values . Next is about the probability, now as we know that you know is the measure of uncertainty and it is estimation of the likelyhood our chance that an uncertain event will occur.

So, you know you can say if you toss a coin and you have to tell that what is the probability of getting head certainly you can say that either it will be head or a tail**.** So, probability is 50% so depending upon that when it is not uncertain now depending upon the total outcomes and all outcome you desired to have you can always predict the probability.

And probability is basically the number of favorable you know chances number of times it is favorable divided by total you know outcome which is you know total number of outcomes which is there. So, the ratio of that is known as probability we all know that the probability value is lying from 0 to 1. So, it will be lying between 0 and 1 and there are many axioms to this probability theory.

And when an event is certain to happen then we say that it when it is completely 100% certain then we say after all it is 1. And if it is not at all certainty 0, so if there is likelihood of having it you know and with not whether it is 0 or 100 in that case it is value will be between 0 and 100 or 0 and 1, 100% we talk about, so it will be as a 0 or in 1.

Then probability of if you that they are many event all the events of there are possibility of many event taking place. Then the so either of the event will take place suppose then probability of having all these event all together if you sum them they have to come as 1. So, that also we know also probability you can define as the relative frequency of events that occur in a large number of trails.

So, this way we define the probability and if you have you know n trails you are taking the n trials of the experiment and for an event e you know which occurs f times. So, in that case probability of that event e you can have defined as f/n and limit that is n tending towards infinity, that is how the probability is defined f is the number of you know times the event has occurred.

And when l will approach towards infinity then we can say that this is the probability of that event e. And you know many a times the probability formula is can be simplified and we can directly take it to be f/m. So, this is basically we are simplifying it and we define this probability as to be you know f/n so that way we define this probability and this probability as we know that when we talk about the different kind of events where things are not certain.

Then we bring into picture these probability aspects many a times it will be used we can when we talk about the risk uncertainty cases. In those cases if suppose you have many you know forums and you have to give the tender and which forums to be give him how that depends upon how they are you know going to give you the profits certainly and that also can depends upon many factors.

In those cases you will have certain values of the probabilities associated with all those options and that can be used. So, that way this probability comes into picture, you can also you mean you must have the idea about these probability concepts for other cases like I mean the probability of having you know a jack when you draw 1 card and the probability of having a jack you know. So, out of 52, so based on that so you can find these probabilities in that sense . Now we will discuss about the you know other statistical term parameters.

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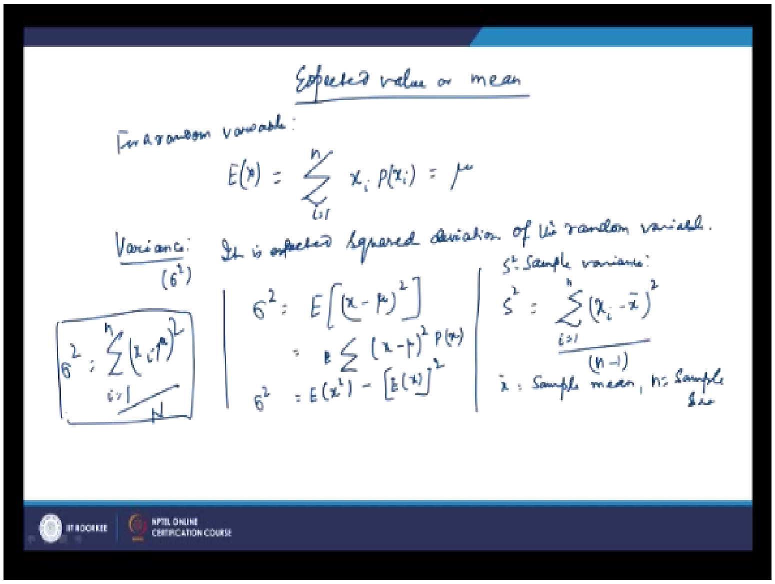

And one of them is the expected value or mean, so as we know that we defined this probabilities as the relative frequency. So, it means that the one we talk about the probability distribution, so is basically the distribution of long term frequencies how many times you know based on that . Now so if you this probability distribution function if you look about if you try to find these mean of the probability distribution of a random variable.

So, that will reflect the centrality of the distribution, so that basically is known as the expected value or the mean value. So, we call it as so you know for a random variable, now this expected value I mean if you do the you know the value which you are expected to occur. So, that is what is the expected value because any value which you are getting it has certain probability associated with it.

So, when you find these expected value of the random variable, so it is nothing but you for you know and trails. Now for this you are getting any value which has certain probability associated. So, this value basically will give you this mean value and that is why it is known as the expected value or mean. So, you know that basically we will talking about the you know weighted average.

And it will be representing the mean of that distribution, so this mean value will be when we talk about the different type of distribution functions. Then your mean has to be defined and depends upon the you know type of that distribution how it comes out to be and we will discuss few typical distribution curve that is normal distribution where it will be clear. Then the next term which is coming further to our notice is the variance.

Now this variance basically the variance understand a deviation these are the 2 very much commonly used terms which are used when we analyze these financial you know studies. And it is basically it is the expected, so it is expected a square deviation, so basically what is happening that for every entity. Now if you look at the you know the values which is there from so that value now in that distribution you have x1 to xn values.

So, you will get certain mean of it now every mean has certain you know deviation from you know that particular value. So, now this so it is a square term that will be indicative of the variance, so basically it is expected I mean a square deviation. So, basically you are taking the difference and you are making it is a square and of the random variable. So, you are finding the expected value or the mean value of this square deviation of the **the** random variable.

And that is basically represented by the term sigma square and that is why you define the sigma square as expected value of you know x-mew square. So, this is how you define the you know variance and you can have this further defined as expected value. So, it will be summation of x-mew whole square and px, so as you know that once we try to get the expected value you are finding the sum of the every entity*px. So, this way we find the you know variance talks about the variability of the data from it is mean.

So, basically you are getting the you know variation from the mean and you are squaring it, so that is known as the variance. And it is also defined as expected value of x square-expected value of x and x square. So, this is also we can find these you know variance as you can find from this formula that is e x square . So, this will be the expected value of the x square terms if you have the values of x.

Once you have the values of x you will find it is square and then this is the ex that is the mean and then that mean term will be squared and it is difference it will be talking about the variance of that particular data. Now we can have we can solve in the coming lectures we can solve the problems based on these and we can see that how the problems based on certain such kind of formulas are solved.

Now the thing is that when we talk about the random variable, now the variance of the data is either in the context of the population or you know in it is sample and population context. So, it maybe for the samples then when it is for a particular sample then we talk about sample variance and if you are talking about the population whole population then we call it as the population variance.

And when we talk about the sample variance then we represent it terms of s square, so this is known as s is sample variance that is you know s square is sample variance. And for a particular sample when you calculate the sample variance, in that case we calculate from i=1 to n and we calculate xi-x prime bar square and that is divided by n-1. So, this is how we calculate this sample variance and in this case x bar is the sample mean.

So, if you have depending upon the sample size we calculate this sample mean and n is basically the sample size. And when we talk about the entire population then we talk as population variance otherwise it is the you know you know it is known as the sample variance. Now when we talk about the entire population in that case we call it as population variance and that is defined as summation i=1 to m and then xi-mew and that square/m.

So, that basically normally we talk when we deal with the population variance or so. So, these are the different you know formulas which are used for, now in this case the capital N is the population size and mew is the population mean. So, this is mew is population mean and this is known as the population size, so that way we calculate these you know sample mean, sample variance of population mean, population variance.

Now another you know very important parameter which is required to be understood is the you know standard deviation which is an important parameter. So, the standard deviation basically when we talk this is basically talking about dispersion from the means of.

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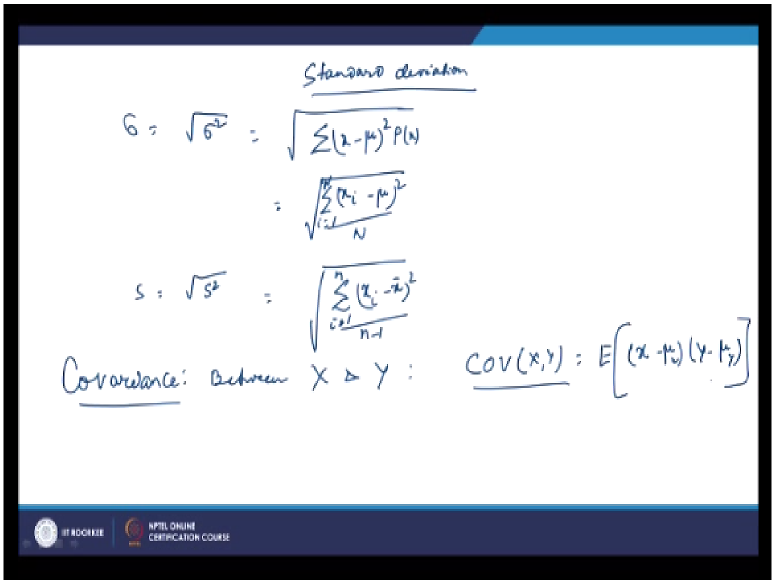

And a standard deviation is calculated by finding the square root of the variance and larger will be the value of the standard deviation, so larger will be the dispersion in the data and higher will be the variability of I mean from the mean. So, that is how the concept is and the standard deviation is represented by sigma and this is nothing but this square of the square root of the variance.

So, we know the formula of the variance, so you will have summation x-mew square*pxi so that way you can find these values of the standard deviation. You can have may formulas and you will also have formula like square root of summation of i=1to n. Then you have xi-mew then square and this divided by n and then it is square roots that also is another formula by which you can find the standard deviation.

You can have standard deviation for this sample also and when we try to find the standard deviation for the sample. So, again it will be square as you know that it is formula is different you use the term and -1 and in that case what you do is you have a square root of summation i=1 to n and here xi, -x bar square/n-1 and then you get the square roots when we talk about the sample you know standard deviation.

So, you get this formula and once you get the variance you can calculate that standard deviation and you **you you** use them . The further there are another term parameters which are of importance one of that parameter is the covariance. Now covariance is basically between 2 variables between x and y and it will be represented as COV xy if you have 2 variables COV x and y, so what is the covariance.

Covariance between x and y is basically defined as this expected value of you know x-mew x and into you know y-mew y. So, this is basically known as the covariance you know and if the covariance value is more than 0 then it means the 2 variables are going in the same direction and if it is less than 0 then they are going in the opposite direction. And if they are r they are basically equal to 0 then they are not you know linearly related that is what the meaning of the covariance term is.

And there is another you know parameter which is normally used which is of importance is. When this covariance is divided you know there is a term known as correlation and this correlation is the measure of the you know linear relationship. So, this correlation shape is further defined and it will be the correlation between the 2 random variables.

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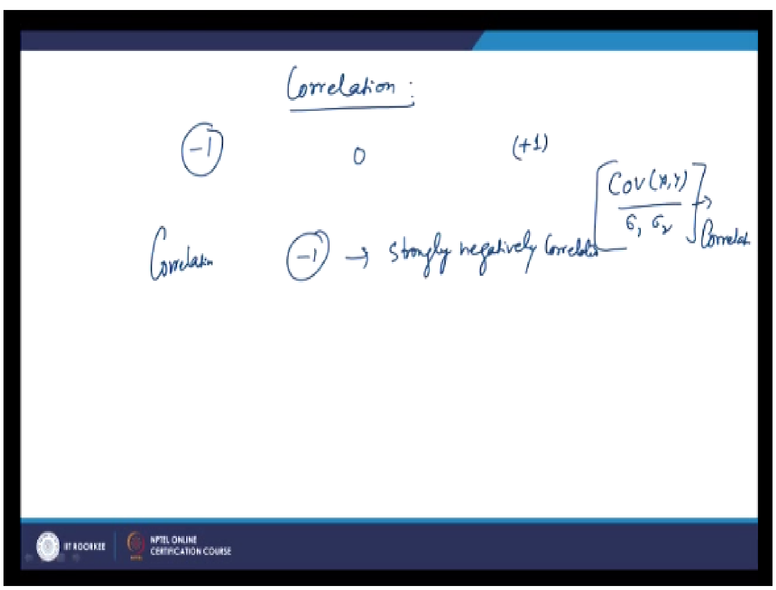

So, how they are correlated, now in that case what we do is that correlation value will be varying from -1 to +1. So, what is done in the case of you know correlation is that you divide this covariance with 1 quantity which will basically be giving that to be in the range of -1 to +1. And if the correlation is you know from towards -1 if the -1 is value of the correlation.

It means they are very much you know negatively correlated and if it is +1 then they are positively correlated and if it is 0 then they are not correlated. So, that is how these you know correlation is defined and what is done is that you know here you try to divide it with the standard deviations sigma 1 and sigma 2. So, covariance value divided by you know sigma 1 and sigma 2, so that gives you this correlation coefficient so.

Now this value if it is o correlation coefficient if it is you know coming out to be -1 then it is strongly negatively correlated means if 1 is increasing another is decreasing. If 1 is largely increasing and another is largely decreasing like that and if it is +1 means it is strongly positively correlated. So, that way 1 will be increasing another will be also be increasing, so that is positively correlated and if it is 0 it is uncorrelated independent.

So, this is how so if you have the data given you can calculate these value of covariance and then further defined divided by the product of the standard deviations of that 2 you know random variable. And then you can find these correlation value and you can say is that 1 is how they are correlated. So, these are the statistical measures or terms which are used going to be used for our studies, thank you very much.