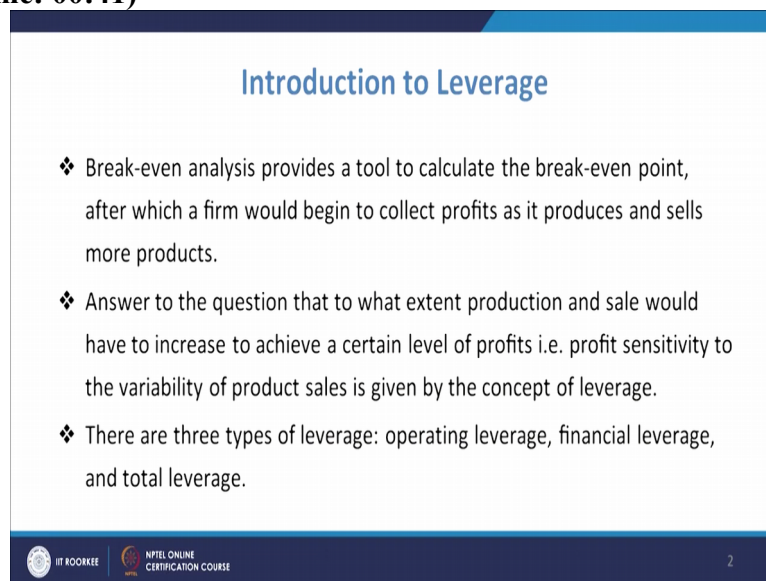**Financial Mathematics**
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Lecture – 39
Introduction to Leverage

Welcome to the lecture on introduction to leverage. So, in this lecture we are going to have the introduction about what leverage is.
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So, coming to the introductory slide what we have understood so far by breakeven analysis break-even analysis tells us that you know after that point you know the firm will start collecting profits after selling its goods beyond that point. So, that break-even analysis will tell you but then you know you know this question if the question arises that to what extent this production and sale would have to increased to achieve a certain level of profit that is profit sensitivity to the variability of product sales.

So, that you know how much you should sell you know and if you sell something what will be the you know increase or decrease in the profit if selling is increased or decreased by certain amount. So, this kind of you know sensitivity of profit to the variability of product sales you know that is given by the concept of leverage. So, leverage basically are; you know three types.

There are three types of leverage and that is your operating leverage, financial leverage and total leverage. So, you know we will have the concept about the operating leverage. Now operating leverage will be referring to the responsiveness of change in profits due to the

change in sales so that is what we have discussed that you know by the change in the sales what will be the change in profit that will be known as the operating leverage.

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So, it is the potential use of fixed operating costs to magnify the effect of a change in sales on operating income or earnings before interest and taxes that is earning before in two some taxes is EBIT. So, how you are going to change how this fixed operating cost is going to magnify that effect of change in the sales. Then there is a terminology that is known as the degree of leverage degree of operating leverage.

And it will be measuring the responsiveness of the profit as a percentage change in operating income relative to percentage change in sales so that is known as the degree of leverage operating leverage and this can be defined as;

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So, we defined this degree of leverage so that will be degree of leverage DOL normally denote and this will be as we know that it will be the percentage change in the you know so

this will be percentage change in the operating income or Y yearly income and then this will be related to the percentage in the sales so that is how the degree of operating leverage is defined as.

So it will be nothing but you know change in the operating income so if you have two points so it will be OY2 - OY1 and that will be percentage so you will have to define divide it by OY1 and then multiply with 100 so that will be your percentage change in your operating income. And similarly you will have the percentage change in the sales so it will be again S2 - S1 /S1 and that will be multiplied by 100.

So, this way your, DOL will be defined and 100 will you know cancel so it will be OY2 - OY1 / OY1 and then that whole divided by S2 - S1 / S1 so that is known as the you know a degree of operating leverage. Now how to understand you know its meaning that can be understood by with the help of certain examples let us see. Let us see that you know operating income of a business so that is increased so operating income of a business so it is given this data that it increases from $884 to $81680.

So, now you know that income has increased if you know sales are expanded that is of the product from you know $ 4200 to the $ 5880 now in that case we do that basically sales are expanded than this it was reported that this operating income is increased from 884 to 1680. Now in this case if you have to find the degree of operating leverage is what? So we can find this degree of operating leverage will be you know OY2 – OY1 so your change in the operating income is 1680 to say when 884 to 1680.

So, 1680 - 884 and that will be divided by 884 and then that will be again further by your sales you know figure that is 5850 - 4200 and then that divided by 4200 so it will be coming as so if you take its value it will be coming as 2.25. Now what is the significance of this value 2.25 now this 2.25 indicates that if there is 1% change in the sales you will have 2.25% change in the operating income.

So, if your sale is increased as 1% there will be 1% and there will be 2.25 you know% of you know increasing that you know this operating income. So, for every 1% change in sales for every 1% change in sales there will be 2.25% change in operating income that is earning before you know taxes. So, that is how you know earnings before interest in taxes EBIT defined as and you know this is what is the meaning of the operating leverage?

Now we also define this operating you know leverage or degree of operating leverage as the multiplier of the effect that a change in sales will have on the profit. So, we also you know we can it can be considered.

So, DOL you know it can be considered you know as effect of effect that a change in sales will have on profit. So, I mean we also define it as the concept of you know sales elasticity so that will be elasticity of profit with respect to; so that I mean concept of elasticity that is your elastic mean you know as you have the elasticity of profit with respect to sales of the change in sales what will be the change on the profit.

So, what we see we also express it in terms of the change in profit so expressing in terms of change in profit so because income will be changing so profit will be similarly changing so change in profit, profit is you know expressed as PI so now you know we can express this DOL as you know partial change in profit that is PI and that is relative to partial change in output that is Q.

So, basically we can write this DOL as dou PI / PI / dou Q / Q so that is your partial change in the output Q and that will be with respect to that so dou PI by Del PI with respect to dou Q / Q and if you look at the you know so this is how the DOL can also be expressed. Now DOL can also be expressed for any level of output and at any level of output now what we can express this you know if the fixed cost is constant so if the fixed cost is constant so you know what you can express is that your changing this profit so in that case is your change in profit that will be you can express it as dou Q * P -V so that is what you know we know understand that because by the expression for the profit which we have earlier defined.

We can define this change in profit as the dou Q * P - V so that is your dou PI now the profit we know this we know of the expression for profit PI so that is basically R - C your revenue minus the cost and we know that revenue is P * Q and then C also we know that it is FC + VQ so it will be FC + VQ so we know that it will be Q * P - V so we define that as Q * P - V - FC so this we know that we this is the expression for the profit.

So, what we can further define this DOL? So, DOL will be basically you know dou Q * P - V / Q * P - V - FC so this is your Q - Q * P - V - FC it is your profit. So, now and then it will be Q by Delta Q. So, this is how because it is divided by that so this Q comes in the numerator and dou Q will be coming at the bottom. So, now if we cancel this dou Q you will get the expression for the you know DOL.

And this expression comes out as Q * P - V and then divided by Q * P - V - FC so this is the expression for the you know degree of operating leverage and this is basically for any level of output that is how we try to calculate.
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Now we can see by example that suppose you have you know the you know degree of operating leverage and which has certain data like you have you know fixed cost is given as $2500 variable cost per unit is given as $5 per unit and your price is $10 per unit that is your P so these values once given and units of products suppose you know sold that is given as you know 1000. So, in this case if you are told to you know find the degree of operating leverage for that you know quantity of production.

So, you can find the expression you have you have expression for degree of operating leverage and we know that this is Q * P - V / Q * P - V - FC so if you put these expressions Q will be now we know that this Q is 1000 and P - V is 10-5 so it will be 5 similarly you will

have Q * 5 and minus fixed cost that is 2500. So, you will get 5000 here and this is 2500 it will be two. So, it means that for every one you know 1% change in sales you will have a 2% change in the operating income in such cases.

Now, if you look at this expression for the degree of operating leverage what you see that you have the expression as the ratio of Q * P - V / Q * P - V - FC so now if the FC is going to increase in that case the denominator value will be decreasing and once it is decreasing then the whole quantity will be increasing. So, your degree of operating leverage will be increasing by increase of the fixed cost.

So, this is one of the way; you know to increase that degree of leverage by increasing the first fixed cost you get that increase in the degree of operating leverage. Similarly if there is a; you know if you decrease that fixed cost in that case you know this quantity in the denominator will increase and once it will increase then in that case the whole value will be decreasing. So, if the there is increase in the fixed cost in that case you will have the decrease of the degree of operating leverage.

So, basically this is how you know when you have to increase the degree of operating leverage increase of the fixed cost is one of the way by which you try to increase the you know degree of operating leverage. Now that can be understood by taking the further you can extend this example and you can see that how that situation comes. So, if you increase the fixed cost suppose in the same example if you increase the fixed cost from 2,500 to 4,000.

Suppose so what will happen to the degree of operating leverage so in that case the degree of operating leverage will be 1000 *5 / 1000 *5 - 4000 so now it will be coming as a 5000 / 1000 and that will be 5. Now what you see that with the increase of this you know the fixed cost your degree of leverage has become more now that leads to you know so it means that if your 1% change in the sales then there will be 5% change in operating income these are the chances.

But then you know there are you know risk involved into it there are you know potential risk when you try to increase this do well by increasing the you know this fixed cost because there will be you know it will be sensitive in both ways because if there will be if you are going to decrease if you are decreasing that sales in that case there will be drastically decrease in the you know income also or profit.

So, that way there are chances of risk and also when you are increasing the fixed cost then also they will have the you know risk of not being able to maintain that increasing the sales

so that will be basically you know that that will be the risk and that can be understood if you try to see that how our body is referring to one of the table we can see this you know more appropriately how that risk comes into picture.

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Now suppose you have one example where the firm is there and you know it has so this is a firm 1 and you know for this form you know we will find half the Q value of Q R you know FC plus you know the VQ that is your total cost and then you will have profit. So, now and we will find the degree of operating leverage. So, suppose for the firm one and you are taking the fixed cost as you know one 1000 in the firm 1 case. So, if you are taking the fixed cost as one 1000 and if you are taking the you know VC so that will be your variable cost per unit is 2 and your income you know for the price of this item is $3.

So, this is your 3 now in that case what you see is that in that case if you find the Q now this 1000 which is the break-even quantity in this case we try to find the you know break-even quantity it will be FC /P - V so FC will be 1000 P - V so this is your break-even quantity and for that for this 1000 you know quantity you will have R that is equal to 3000 and your total cost will be 1000 + 2*1000 so that will be 3000 and your profit is zero.

If you go to now you know 1500 so for 1500 what you see is that your R becomes 3500 and your you know total cost becomes 4000 and profit becomes 500 but what we have tried to see that you know for 3500 if you look at or if you go to the level of 3500 now in that case your you know this R becomes 3500 *3 so it will be 10500 you have total cost of basically you know so are these days and the total cost is 8000 so profit will be 2500 in that case.

Now what we further see if you go for the second you know and if you find the DOL degree of operating leverage it will be you know 3500 so that will be degree of operating leverage

we are finding for this 3500 units we have to find this degree of operating leverage for the 3500 units and we find degree of operating leverage for the $3500 * 3 - 2 / 3500 * 3 - 2 - 1000$ that is your fixed cost.

So, in this case it becomes 1.4 now if for the second you know company if you see that your fixed cost we are taking now as we are increasing to increase the DOL so if you are increasing to 2250 and your you know this variable cost will be 1.75 and P is 3 so in that case if you calculate the DOL, DOL will be fixed cost so 35 now in this case you will have the DOL for this; now let us say now we are getting first of all for the 1000 units.

Now 4000 units we are not still at the breakeven we have if you calculate the breakeven it is 1800 units $2250 / 3 - 1.75$ that is your you know breakeven is 1800. Now so it means your breakeven will be you have to maintain that you know production to our sale to 1800 at this product production rate you will not be even going to the breakeven because you this is the risk of you know increasing the first cost because your first cost is increasing so that will increase the breakeven.

Now 4000 if you look at you will have the R as certainly it is 3 so $3 * 1000$ is 3000 and total cost will be fixed cost $+ 3 * 9$ so plus $1000 * 1.75$ so it will be 4000 in fact and in this case you have you know no profit is basically a loss. Now it means you have to go for the breakeven to 1800 so that risk is there that now you have to sell 1800 quantities for you know maintaining you know zero profit.

Now in this case your R becomes 5400 and your total cost will become 5400 because this is the break-even now you know point and your profit is zero. Now if you for the analysis this is your breakeven point. Now the analysis which we want to so for 3500 quantities if you look at now for 3500 quantities if you look at your; first of all you will find the degree of operating leverage and that will be $3500 |* 3 – 1.75 / 3500 *3-1.75$ and minus fixed cost fixed cost we are taken as 2250.

Now in this case we are having the degree of operating leverage to 2.06 your degree of operating leverage in earlier case was 1.4 whereas in this case your operating leverage degree of operating leverage has increased to 2.06 but then for that you will have to go for breakeven quantity of minimum 1800 to get the zero profit and if you look at the 3500 unit you have the income of ten 10500, $3500* 3$ and the total cost will be fixed cost 2250 plus you know variable cost is $1.7 5 * 3500$ so that will lead to 8375.

So, that will lead to your profit of 2125. Now what you see that you had a profit of 2500 with this but in this case your operating leverage has increased and certainly after that if you will move you will have the different values of you know the profits so as you move further and what you see that this is the risk because you have to go now you know to this 1800 values for getting that breakeven.

But certainly in this case with increase in the sale your DOL is increasing so for every percentage of change in sale you will have the operating leverage the value of you know increase in the percentage change in the operating income is 2.06. So, that is how by looking at this you can understand that you have the risk certainly and if you know if you have to stop before you know at smaller quantities then there are chances of more losses because degree of operating leverage is becoming more.

And in the case of losses it will be affecting more and in depth that is the point which should be understood by the you know investor or the person who is involved in to it, thank you very much.