Financial Mathematics
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Lecture – 38
Economic Order Quantity

Welcome to the lecture on economic order quantity. So, in line with what we have studied the break-even analysis similar to that many times in the inventory control department in the plant we need to find the order quantity or purchase quantity that should be the most economical one. So, basically the decision has to be made and it is to be a certain that what should be the quantity of the order so that your total cost you know for maintaining the inventory for the period this would be minimum.

And for that you know there are many things which are to be studied like you know we know that what is the requirement for the year and one thing is that you can have the whole you know requirement with you at the beginning of the year so that you can use it for the whole year. So, one extreme is that and for that you know there are many challenges so that we will discuss for us first of all let us define what is? Why we try to study this economic order quantity?

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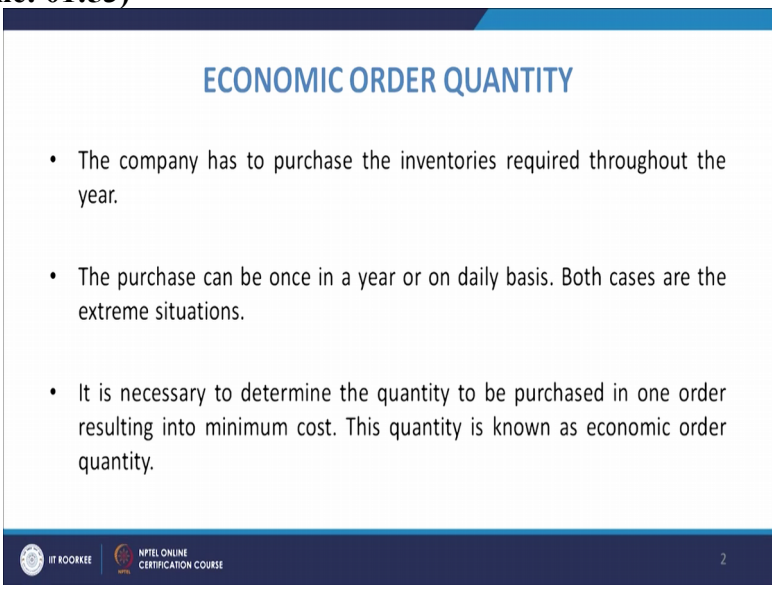

So, the company has to purchase the inventories required throughout the year so basically whenever we are dealing with any company we are in the need of these inventories for whole year and the company has to purchase it. Now purchasing can be done in a year or on daily basis both these cases are the extreme positions in extreme situations means the best thing

most easy which looks like will be that you take you know you purchase all the items once and whole year and that will serve your purpose.

The second extreme is that you every day you purchase every day you know you purchase and use it you know wherever required. Now both these things are extreme in the sense that if you are you know taking the whole purchase whatever requirement you have there in a whole year. Then it is very difficult to maintain you have a lot of holding charges you have to keep in you know a very big space you will have to maintain it so basically the holding charges will be very, very high.

And in that process there are many items which you know many a times difficult to relocate or you know some may be spoiled or so that is one extreme. Another extreme is that you take on daily basis. Now in the daily basis basically every day you have to you know process for I mean the purchase so you have a lot of charges in to that you have to make the purchases so for that you will have the cost on purchase every da.

So, that will be basically very, very high in that case so they are the extreme situations. So, what practically it is followed in the industry is that we should have the quantity which should be purchased in one order which should result in two minimum cost. So, this quantity which is you know ordered every time that results into minimum total cost that is known as the economic order quantity.

So, how to get that so that is again a type of you know something like break even bigger because that will lead it to minimum you know cost so it is on the similar line and let us see that how can we you know we can proceed for getting this economic order quantity with the plants.

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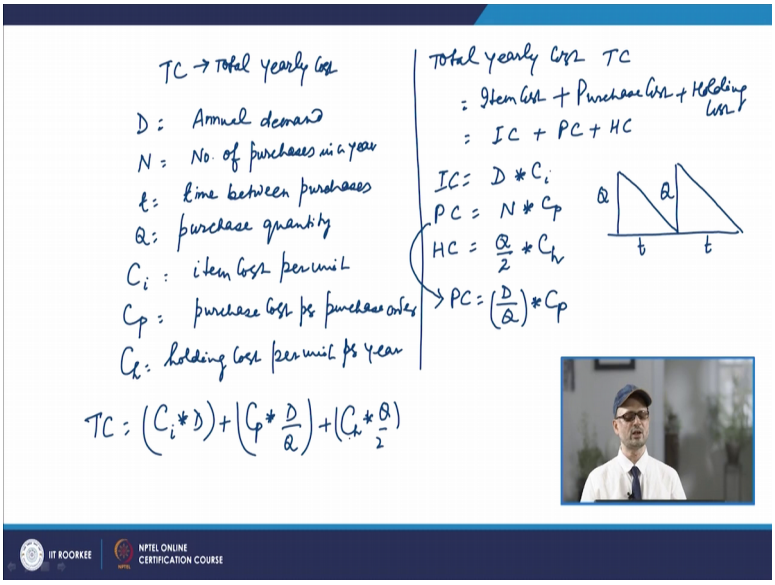

So, now let us say that you have the TC defined as the yearly cost so that will be total yearly cost of providing the item that will be TC then you have D, D that is the annual demand basically. So, demanding any you know definite time so you can write that annual demand of the item is D then you know you go for n number of purchases. So, and you can take as number of purchases in a year.

So, basically N will be their required because on N number of occasions you have to make the purchase orders you have to process that involves cost. So, cost of purchases will be there for N number of times that is N. Then t is the time between purchases so t is defined as the time between purchases and then we take this Q, Q as the purchase quantity so that will be basically we will be getting the expression for Q which will minimize this total cost and in that case we are going to have the expression for Q.

Then C i that is the item cost per unit so that is your purchase price and then, C P C P is the you know purchase cost per purchase order. So, these are the input parameters in the term of what we are going to have the expressions and another term will be the C h that is your holding cost so this is the holding cost you know per unit per year. So, basically it is you know made up of many items like interest, insurance, taxes, storage all these cost components are or spaces or handling all these components are into this holding cost.

So, then what we need to know that we have to have the expression for the total yearly cost and if we find the total yearly cost you know total yearly cost which is defined as you know TC. Now this TC will have three components one is you know that will be the item cost so this will be item cost then you have the purchase cost and then you have holding cost. So,

item cost will be you know cost of the item so you have you know annual demand is D and called item cost per unit is C i.

So, first of all we write item cost as IC purchased cost as PC and holding cost as HC. Now item cost will be basically you have the total demand is D and that will be multiplied by cost of the item per unit. So, item cost per unit is C i so C i * D will be your item cost. Now purchase cost purchase cost means you will be dealing with the purchases many number of times and we know that we are doing the N number of purchases.

So, for every purchase order we are having you know involving a cost of CP and we are doing N number of purchases so purchase cost will be number of purchases in a year that is N multiplied by the purchase cost part versus order that is your CP so that will be your purchase cost. Now we are talking about the holding cost and holding cost will be nothing but you have the inventory which is stored and for that basically this is your inventory you know holding charge per unit for a year.

Now what is there that what you see you have the inventory flow like this so what you do is you are ordering for Q quantity and the ordering you know time between orders is t. So, after t time you are ordering again and you are here you have certain assumptions and the assumption is that there is no time lag between you know; so, here there is no time lag so once your order you are you are thinking that you are getting the material in time.

So, there is no time lag in between and here again this Q quantity comes in so what you see that this Q comes and it replenishes in t time. So, your average inventory level is Q / 2 and so the holding cost will be average inventory level that is Q / 2 and that will be multiplied by the holding cost per unit so you know it will be C h. So, this is your expression for the holding cost. Now coming to the expression for PC if you look at the PC is the purchase cost and this is N so and you know that this N is number of purchases in the year.

And number of purchases in the year will be demand divided by the quantity that is what will be the number of purchases. So, you have total demand of D and you are ordering for only Q at one point of time so altogether you have the N as D / Q and then that is multiplied by CP. So, on the whole you have three expressions IC equal to Ci * D and then PC as D / Q * CP and holding cost will be Q / 2 * Ch.

So, the expression for total cost total annual cost will be C i * D first part then you have CP * D / Q and then that is your purchase cost and then the holding cost that is Ch * Q / 2 so this is how your three components look like. Now what we have to see we have to find you know

what should be the quantity you know what should be the quantity ordered so that your total cost becomes you know minimum.

So, we know that this is a function which is shown in terms of Q and this function which must be having the minimum value for that we have to take its derivative and this derivative value should be equal to 0. So, once we equate that to 0 in that case we will get you know a condition which will give you the minimum total cost.

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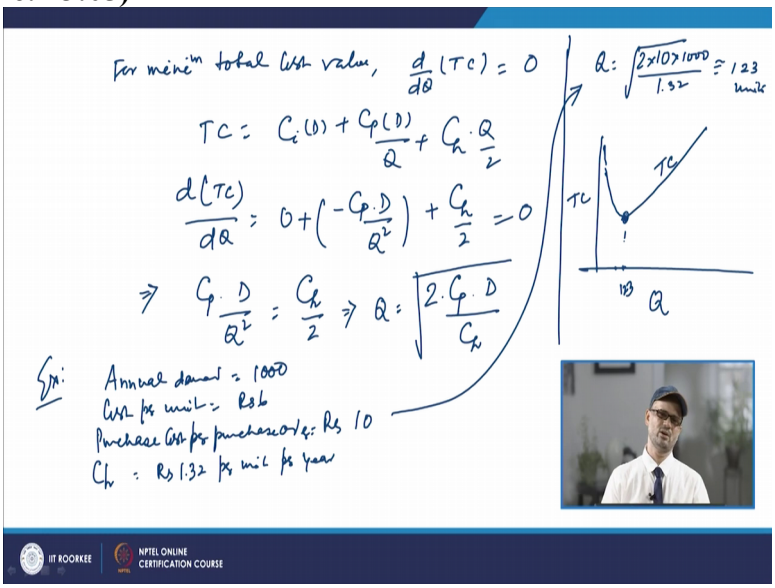

So, we can write that you know for the minimum total cost value your d / dq of total cost should be equal to 0 so for at minimum value the slope will be 0 so we know TC is you know now Ci * D + CP * D / Q + Ch * Q / 2 so if you take d of TC by dQ that will be equal to this will be 0 then this will be minus of CP * D / Q square and then this will be Ch / 2 so that has to be equated to 0 so now this leads to CP into D / Q square will be equal to Ch / 2.

So, now Q will be equal to 2 * CP * D / Ch and whole you know under root so this is the you know economic or this quantity which you get this quantity is known as the economic order quantity. So, you can have you know examples and that examples will you know these data's will be given and once you have this data then you can get the value of economic order quantity.

At this value of Q you will have the so if you find the expression for or the graph of the total curve total cost line in that case at this value of Q if you take the different values of Q then if you find the graph so for that value of Q you will have the minimum value of the total cost. So, let us say if you if you have some examples like you have you know annual demand for certain item is you know it is given as 1000 units and suppose the cost per unit is given as rupees 6.

And then purchase cost per purchase order is also supplied so purchase cost per purchase order is given as rupees 10 and then cost of holding the inventory Ch the cost of holding the inventory in one year it is estimated to be you know rupees 1.32 per unit, so, for per year. So, if suppose these values are given we know that these you know this is very you know clear from this expression that we can get these values.

So, for this problem if you try to find the economic order quantity then simply you put in this expression that is Q will be under root and you have to and then CP is lifted as 10 and then you have demand is 1000 and divided by Ch. So, Ch will be 1.32 so if you do that it will becoming something like 123 units. So, this will be so it means that if you order once 123 units so you will be going something like 8 times you will be doing the ordering.

So it will be 123 * 8 is 984 so it will, becoming something close to you know 8.12 or so. So, in that case your total cost is minimum and this you can verify by you know plotting a graph so if you plot the graph of total cost with respect to the you know purchase quantity Q so what you will see that the for this you know 123 it will be because it will come something like you know this kind of curve will come.

So far this 100 it is about 123 you will be having the minimum point so this is a TC line so at this line you will having be having the minimum value of total cost. So, that talks about the you know economic order quantity which results into the minimum of the total cost when mostly that is used in the case of inventory management where you require to order and how much to order once you know how many times over to most order and what quantity so that your total cost is minimized.

Now similarly the line you will also come across certain terminologies in plants and that is your economic you know production quantity. So, many times in the plants you have to determine the optimum production quantity so that that results into the total yearly cost to be minimum. So, now so in those cases you know what should be that production lot you know in that this you know production should be made and that is known as the economic production quantity.

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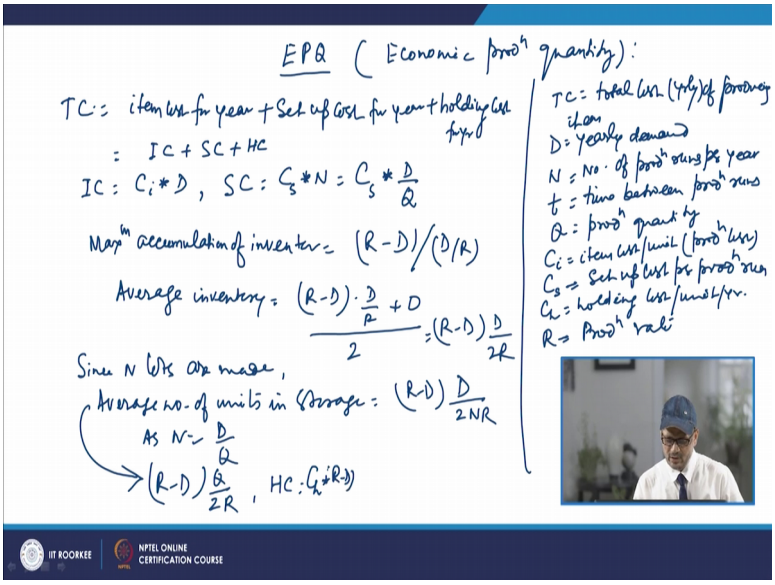

So, that is your EPC economic EPQ that is economic production quantity. So, in those cases you will have the data again you will have the data like TC so you have the data TC as the total yearly cost of producing item. Then you have D as the yearly demand of the item. Then you have N as the number of productions runs so that is number of production runs per year. Similarly you define t as the time between production runs and you have Q as the production quantity that is what is optimized value we have to find so that is your production quantity here.

Here again you have the definition of Ci the item cost per unit or production cost so that is your production cost in that earlier case you had that was purchase cost then Cs in this case will be setup cost because every time you have to setup so that will be a set-up cost per production run. So, in this case you have to have the setup and in that you need to have the to put into the cost.

Then you have again here as Ch that is your holding cost per unit but here so per unit for years again it is meant for the similar type of costs like storage insurance and all that. Then you have R as the production rate. So, this is your production rate of you know of producing the item. Now again in these cases we are assuming that the demand basically is constant and also the production rate is constant during that period.

So, you have to have certain assumptions this is optional valid for the earlier cases and again we also assume that the lead time is zero here production lead time is zero and there is no shortage. So, these are the assumptions which were also there during the expressions which were found for the economic order quantity. Now in this case again we will be having the

expression for the total cost and the expression for the total cost in this case will have again you have the item cost for the year.

And similarly you will have the set-up cost for the set-up cost so you will be having to set up many times so that is your set-up cost for a year. And similarly you have a holding cost for the year. So, this way you will be having three cost components and we define it as IC + SC + HC so this is what we did in your earlier cases. Now in this case again IC will be you know now that is CI into your demand so Ci * D.

Now set-up cost you are doing the set up N number of times you are going for the products angles N number of times. So, it will be again in a similar line setup cost will be set up you know causative for every time is Cs so Cs multiplied by N and again N will be your demand by production quantity so it will be CS * D / Q. Now comes the holding cost for the inventory and in this case what is happening that you know the production rate is R.

So we know that you know you know if the items are added you know to the inventory at the rate of R units per year and taken from inventory at the rate of D per year. So, in that case you know the net rate of accumulation is R - D per year so that will be your net rate of accumulation R -D and you know the time required to produce these D units at the rate of R so that will be your D / R.

So, in that case what we see that so what happens that the maximum accumulation of the inventory will be basically R - D / D / R so that way your maximum accumulation of inventory so you will have you know R - D so R is present rate and these your demand so that will be accumulation and then your since that rate is D / R so it will be divided D/ R. So, this is your maximum accumulation and then minimum accumulation will be you know zero.

So, since because there is no storage in fact so your average you know accumulation so average number of inventory so every inventory since minimum is zero so average inventory will be R - D * D / R + 0 and then by two so it will be R - D * D / 2R. So, this is your average inventory in the case of these production runs. Now since you are making N lots so since N lots are made so now your average number of units in storage.

So, it will be divided by N so it will be R - D * D / 2 N R so this will be your average number of units in storage and this multiplied by the holding costs because holding cost is for every unit you know we are paying the holding cost as Ch for every unit and every year so this is to be multiplied with Ch and that will give us this holding cost for the year.

So, you know in that case so this is your average number of units in that storage now in this expression also again since if you take this n and will be nothing but D / Q so as N is D / Q so your average number of storage; this becomes R - D * D / 2 * N is D /Q so Q will go there so Q / 2 R so this will be your average you know number of units in the stories and then at HC holding cost will be this multiplied by Ch so it will be Ch multiplied by R - D * Q / 2R.

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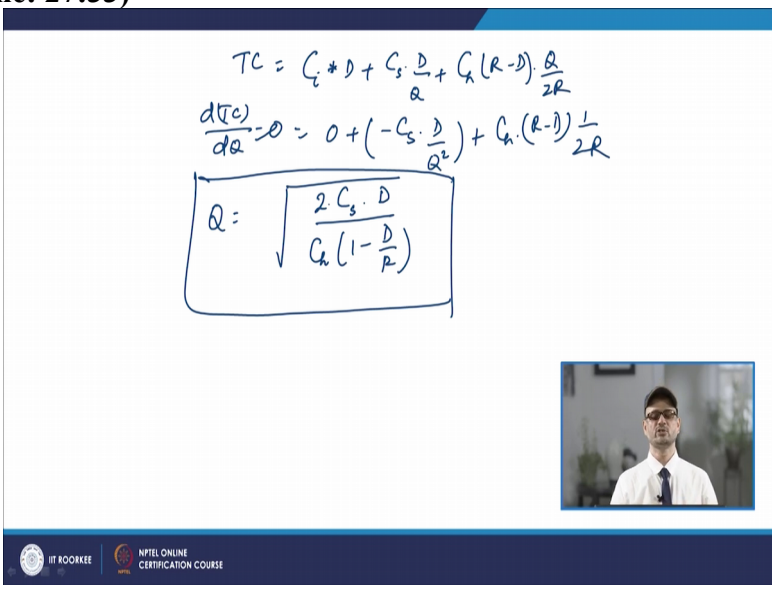

So, this is your holding cost and then what you do is that you get the expression for the total cost and then you are differentiating that with respect to Q so TC will be Ci * D + CS into D by Q + Ch * R - D * Q / 2R so that will be there and then you further find the dTC / dQ as an equated to 0. So, if you equate it to 0 so it will be this will be anyway 0 and then it minus of CS into D / Q square and in this case you will have Ch * R - D into 1 by 2 R.

So, we have to equate the two you know that is to be equated to 0 so you will be getting the expression for Q and Q will be you know 2R * CS into D so, that way it will be coming so you will find the expression for Q as and under root 2 into CS into D / Ch * 1 - dy /R so that is what finally you get the expression for you know the economic production quantity this much of production should be made you know in 1 lot.

So, that the total cost is minimum so this is how you use and similar to that previous you know you make a little quantity question you may have these data's you know you know for the problems and you have to put these values into this expression and it will give you the economic approximate quantity values so that can be evaluated and that leads to the you know value of that quantity which gives you minimum total cost.

So, this concept is very much utilized for you know for forgetting these you know values of quantity which give results into the optimized cost optimize total cost minimum total cost and

it is based on that analysis which we have studied in our earlier lectures, thank you very much.