Financial Mathematics
Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology – Roorkee

Lecture – 37
Analysis of Break-Even Time and Dual Break- Even Points

Welcome to the lecture on analysis of break-even time and dual breakeven points. So, you will talk about the break-even time and also the dual break-even point analysis in which you have 2 points of breakeven now many a times when we talk about so we have talked about the break-even analysis in terms of break-even quantity so that much so what will be the you know you know product quantity.

And at that particular quantity of production we assume that there will be breakeven. Now there may be situation when you are interested to know the time at which this breakeven will occur and certainly that will depend upon what is the production rate. So, if you know the production rate in that case you can predict the break-even point and the time in that case will be so it will be expressed in terms of breakeven time.

So, you know what happens that so when we try to define it is basically the time you know required to produce you know that much amount.

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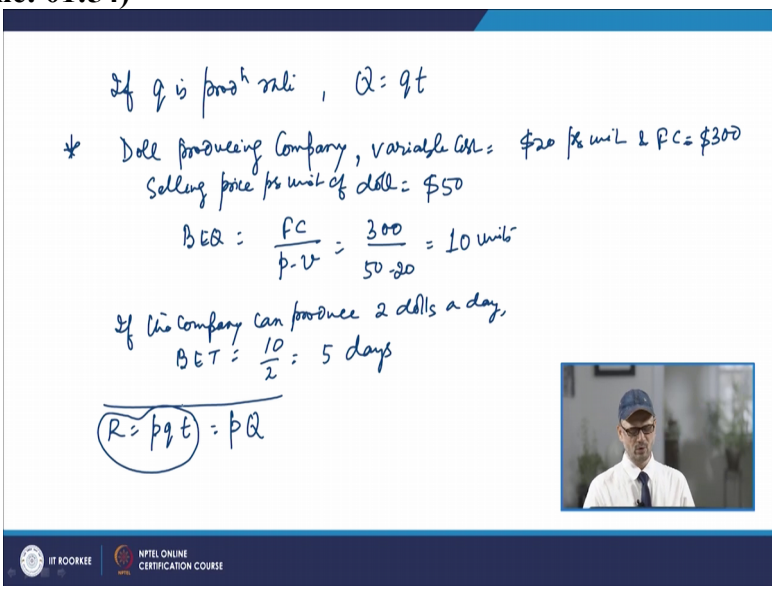

And if suppose you have you know q is the production rate so if q is a production rate so in that case you know but that if the if t is that time to produce that size so your q will be qt so in that case t will be the break-even time which will be you know of interest to know that that will be time I mean assuming that the production rate will be constant for that particular time

but then there may be other cases that we will discuss also about the delay in selling and in that case there will be time lag when it is produced.

And then there is because anyway you are getting the money only after this sold so you have may have a time lags that also we will discuss. Now we can just see one example that suppose somebody has a home business and they are producing a doll you know dolls they are producing and the doll producing company is there and they have the costs and they have you know variable cost.

So, variable cost is $20 you know per unit so per doll they are having the $20 of in a variable cost and the fixed cost is you know $300 that is your fixed cost now selling price of the doll selling price per unit of doll is $50 so this is data which is therefore the company's as we have studied now in this case we know that we can have the break-even quantity and we know that break-even quantity will be now our point will be FC / p -V so you have 300 and your p is 50 and V is 20.

So, now it is around 10 units so that is 10 units is the break-even quantity. Now you know but if you try to find the you know break-even time and if you know that if the company can produce 2 dolls a day then what will be the break-even time so in that case a break-even time will be you know 10 / 2 so it will be 5 days. So, if the production rate is assumed to be fixed in that case after 5 days the company will reach at the break-even stage when the company has neither profit nor loss.

Now the thing is that what we discussed that in reality we assume that in reality basically it cannot be assumed that it will be always same there may be; so when you are producing and then it will go to market and then it will be sold so only after selling you are getting the income so then only you can say that now you have to you know compare with this cost line and then you can come to your point of break even where there will be r and the total cost will be same or the revenue line and the total cost line will be meeting.

So, basically there are you know a factor that is your time lag now in the normal case what we have seen that your R becomes p * q * t so that is what because R is nothing but p * Q so p is your price per unit and Q is the quantity produced. Now this is as this q is qt, so q has become qt so R will be pqt.

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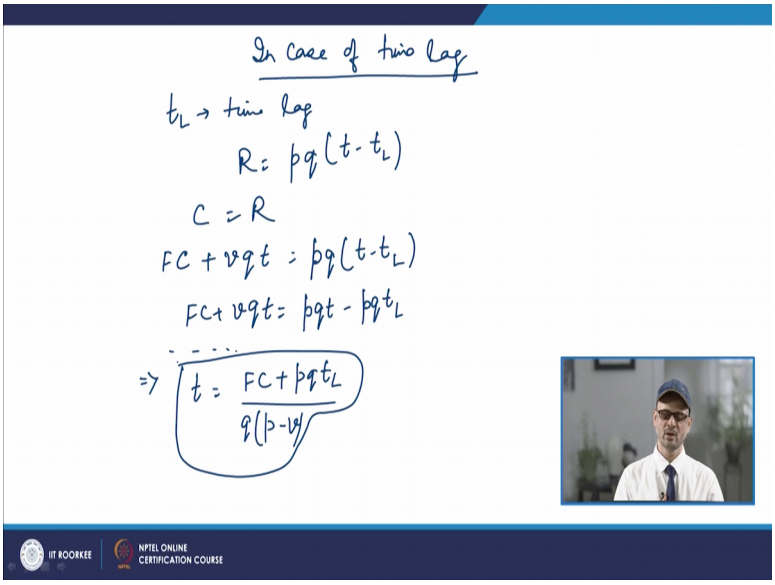

Now if you talk about the time lag so in case of time lags if there is a in case of time lag so if that is denoted by t L so that is basically well that time difference between the point or the time when it is produced and the time when it is sold so that is your t L it is a time lag. So, if you define this time lag and if you keep that into the loop then what we can write these are basically we be pq * t – t L so basically that will be your time lag.

So because of this time lag your revenue equation will be coming like this because your revenue will come only once the product is sold you know so that time lag this t L has to be subtracted from that t. Now in that case if you see that if your cost line and the revenue line has to be increased I mean or to be equated in that case this cost which will be fixed cost plus a variable cost.

So, variable cost will be Vqt and R will be pq * t – t L so you can write further so FC + vqt it will be pqt - pqt L so you can further write so from here you can come if you do certain analysis you will get this equation for this t as FC + pqt L and divided by q * p - V so that is how this you know breakeven time will be expressed in terms of fixed cost the unit price of the product then the q is the production rate t L is the time lag and then you have V as the variable cost per unit.

So, you can express in such cases whenever you have the situations of these time lags in those cases you will have the; you know you can find these values of this time you know this breakeven time as you know the function of all these components of the breakeven you know analysis and you can find this time in case of there is you know the there is time lag I mean in such cases.

Now we are going to discuss so you can solve the problems you may be having the data and you will be told to solve such problems where the time lag will be given so you can simply fit those time lag you know values into the equation and you can get the you know a break-even time. So, that way you can solve such problems.

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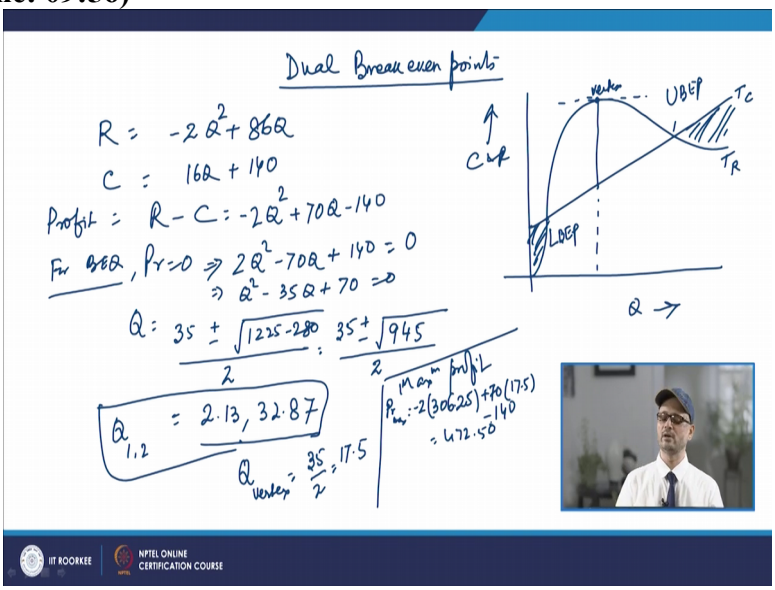

Now we will discuss about another important you know you know terminology that is your dual breakeven points. Now what is that will break-even point because we have studied about something where we assumed that the total cost line or the revenue line or the income line they all are straight lines but that may not be you know the actual cases the these lines may be you know non linear there may not be necessarily; they are not necessarily linear especially the you know the revenue lines.

So, in those cases they are so well since they are linear they are assumed to be linear so they are you know cutting each other at one point which is the point where the two things are same and that is why we are telling that there is one breakeven point. But if they are of different shape in that case if they are cutting at two places then that is a case of dual breakeven analysis the very break-even point.

And in that case you will have two lines basically so if you have some situations like if you have with a total cost line goes like this and if your; the revenue line goes like this so what you see that you have two lines you know one line is here one region is here another region is here so this reason and this is your profit and this is your; so that way and this is the cost which is so this is the cost line and this is your income line.

So this revenue generated TR and this is your TC so this is cost and this is the cost and this is your profit now so cost and R will be like this and you have quantity here in this cases. Now

what we see that under such cases you have this point is known as the lower break-even point and this is known as the upper break-even point. So, lower break-even quantity or upper break-even quantity that may be defined as and you can have a point where you have maximum profit and at maximum profit what you see you can see that this is a maximum profit point is the one where you are slope is zero so that way you can have this point also.

Now let us say so this is the example of these you know dual breakeven points now you can have an example suppose these you know line of the revenue which is generated it is basically not linear it is -2Q square plus 86Q and your cost line is 16Q + 140 cost line is there is a straight line and this is not a straight line. Now in that case we know that we find this profit as R – C.

So, profit will be R -s C so you have to do subtract R - C so will be -2Q square + 86Q - 16Q so it will be +70Q and -140 so that will be your profit line and we know that the breakeven points can be calculated by equating this profit to be 0. So, once we equate this profit to be zero in that case for break-even quantity your profit has to be 0. So, this will imply that you know 2Q square -70Q +140 will be equal to 0.

So that way you have to you have 2 values of Q this is a quadratic equation in Q so you will have 2 values of Q and Q will be found as so you can find I know it can further be written as Q square -35Q + 70 =0, so Q will be -V so that is 35 +-V square minus 4ac so 1225 - 280 / 2 so that way you will have 35 +- it will be under root and it will be basically 945 and divided by 2 so it will be something like 30.75 or so.

So, if you calculate these values this will be; you have 2 values one is 2.13 and another is 32.87 so this is these are the two values Q 1 and 2 are these values. So, it will be 30 you know something like 30.75 so once you do that you will get these 2 values. Now what you see that so that is why you have a Q1 here and you have Q2 here. Now after that so in this region you are likely to get the profit.

And then after this region you are likely to get the losses so that is the you know a sense of these type of curves where you are getting these values. Now the maximum value of these profits you have to get and that will be - V by 2A. So, the maximum value can be found and it is a this can also be found by you know taking the derivative of this line with respect to Q and that equated to 0.

So if you take the derivative of this equal to 0 so it will be 2Q - 35 that will be you know that is will be equal to 0. So, for that the Q value comes out so Q you know for that the vertex so

this is the vertex point and Q vertex basically will be nothing but you know this will be the its derivative which will be 35 /2 so it will be 17.5, so this is the point which is having Q as 17.5 here and then you can find the maximum profit once you get this point then you can have the point of maximum you know so profit.

So maximum profit which can be you know obtained will be by taking this value into the profit equation. So, if you take that profit equation it will be so maximum profit maximum will be -2 * Q square so it will be 306.25 and then you have a +70 * 17.5 - 140 so if you calculate these values you will be getting 472.50. So, that is how you can calculate the value of the; you know maximum profit in the case of these dual breakeven points and we must you know understand how to analyze these you know dual breakeven cases where you have to breakeven points.

As you know that towards the left hand side of the lower breakeven point you have the total cost line is above the you know revenue line so again you have the loss case and here also it is going below the total cost line so again you have the loss region but in between you have the profit region which is increasing slowly and it is getting maximum at certain point of time and then you can have the calculation of you know the maximum profit in such cases.

We will also discuss about we discussed about the applications of these breakeven points and it is typical application can be in the cases of stock selling where while selling as well as purchasing when they are bought or they are sold you know at different prices and then you have commissions which are there in those cases and these commissions are paid through transactions.

So, basically you this break-even analysis or concept can be useful you know to find that you know point beyond which the investor will basically will be in a position to sell the stocks you know to make again. So, in those cases these breakeven analysis are useful it is a very good you know decision-making tool when to sell the stocks so that she can have maximum profit and so that we will study.

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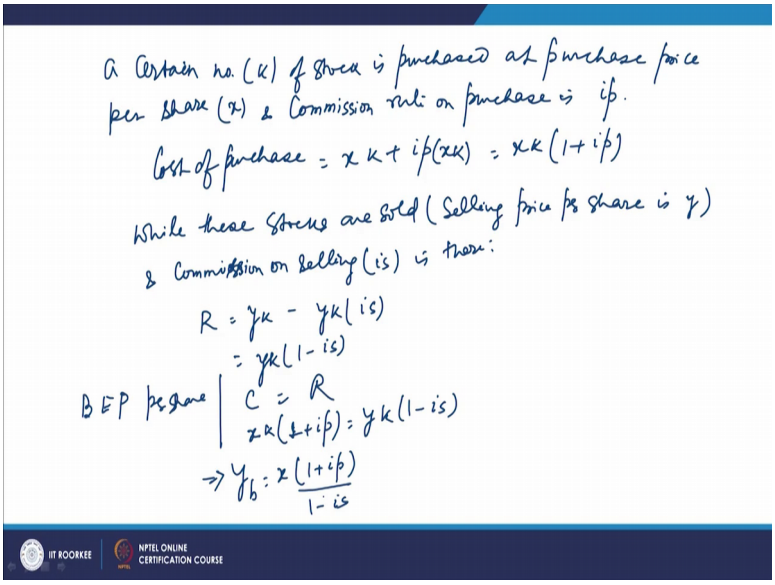

So, you know suppose you know when we talk about the stock selling suppose you know a certain number of stock is a number of stock of stock is you know purchased and you know at purchase price there will be some purchase price per share and that we take as x and you have Commission rate so basically the Commission will be you know used while purchasing or while selling.

And the Commission is basically Commission rate on purchase is ip so that is your ip is the rate of commission you know so ipx basically you know whatever ip into that amount that will be your Commission for the purchase. So, cost of purchase so if you talk about the cost of purchase now in that case the cost of purchase will be x k + ip * xk and you know so we have a partial prices x and then you are taking the you know number of k so that will be x k and then ip x k will be your cost of purchase.

So, x k will be common and it will be 1 + ip so this will be your cost of purchase similarly we have the revenue equation and this will be basically we can write this revenue of stock. So, you know if these stocks they are sold at certain selling price so you if you are selling the price stocks at certain price. So, suppose know while selling while these stocks are sold now the selling price so per share is y.

And again the commission will be there even again on the selling so if the; and in commission on selling so this is Commission on selling that is, is there in that case now what we have the revenue will be you know yk - so in this case whatever revenue will be there it will be less than what it is actually because you are paying the Commission in this case. So, yk * commission on the selling. So, it will be yk * 1 – is. So, in that case when you talk about the break-even point you know per share so now in this case y will be so in that case when

you are talking about break-even point now at break-even point you have the cost of purchase same as cost of you know the revenue.

So if you equate them it will be x * xk * 1 + ip will be same as yk * 1 – is, so now the y you know you know breakeven you know what should be the selling price that should be you know known and in that case it will be x * 1 + ip / 1 - is so basically it tells you that what should be the new you fix you know the price of the selling price of the share. So, that you are coming to a break-even position where you will be having the no profit no loss kind of situation when you are putting these commission charges you know on both the transactions like purchasing of the share or selling of the share.

So, in those cases you can have you know the idea of fixing your selling price of the share in such cases. So, now in a nutshell we studied about this breakeven analysis this can further be used for many applications it is also used for the early retirement plans where many a times you are given certain benefits before retiring. So, in those cases you can have the assessment of those benefits whereas there will be some losses if you are not continuing with the jobs in those cases also you can do the analysis and you can have a break-even analysis and that will help you in judging whether you should take premature retirement or not.

So, in a nutshell we studied about these you know break-even analysis its users are many like we it will evaluate these capacity of firm to cover its operating cost so that it is able to make the desired profit. So, that is one of the; you know primary use of the break-even analysis. Then it will also assess all the ways in which the profit will be relating to the sale and it will also measure its responsiveness you know to fluctuation on the sale values.

So, that way how the sale value will be changing if you know price is changing and I mean price is changing then how the profit will be changing so that also will be you know clear by doing this analysis. And it will also be talking about the business risk also it will be providing you that you know knowledge of the business risk and when basic especially when you are thinking of you know the variability of investment returns on assets and also the you know operating leverage that we will discuss later.

Where we will see that by selling how much what will be the change in the you know income and all that so that analysis that is also basically idea that idea is also you know coming from that analysis. And overall it will also give you an idea whether to launch the you know possibility of launching a new product or understanding the business better because you know that if you are launching a new product then how much time it will take how much

units it must you must be able to sell so that you are at a stage where you can say that you are neither at profit or nor at loss.

So, that will be these are the different types of uses because many times you are over ambitious and you think of starting you start you know in the new product in a flick and that may lead to the losses rather than the profits which you are aiming to you know generate. So, these are the; you know uses of these break-even analysis there may be limitations also. Then one of more important thing which is required to be understood in the case of break-even analysis is the you may come across this term like sensitivity.

So, sensitivity of the variables so we have already discussed that the variables how they are going to affect this break-even quantity so we have studied that there are many parameters which are important you know and how they are going to change how they are going to change the break-even point or break-even quantity. So, these variables are like you have seen that when we call define the break-even quantity the FC / V - p – V.

So, you have fixed cost and then you have variable cost per unit then you a price per unit then you have marginal cost so that is a contribute margin of contribution so contribution margin which you caught which you defined so all these things have some say fixed cost if it is increasing then the break-even you know quantity and break even revenue you know that so a break-even quantity will be more so in that case.

Then if the variable cost per unit is increasing you know so that we also your break-even quantity will be more. Similarly your and so break even revenue will also be more then price per unit if you are increasing in that case your revenue will be more so in that case price per unit increasing the break even revenue will be less and your break-even quantity will also be less.

Contribution margin if it is increasing in that case the break-even quantity will be decreasing and break even revenue will be decreasing. So, these are the you know this is known as sensitivity analysis where we try to see the effect of the variables on certain output and how they are affecting these values so that is known as the sensitivity analysis of you know these variables on the BQ and BR.

So, that should be clear into your mind and that can be you know when you are solving the problems you can see that how they are changing how they with the change of one of the parameters other things are how changed so that can be understood, thank you very much.