

Financial Mathematics
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Lecture – 34
SOD and UOP Method of Depreciation, Depletion

Welcome to the lecture on a SOD and UOP method of depreciation. So, SOD the sum of digits method or sum of years digits method and UOP units of production method of depreciation and depletion. So, in this lecture we are going to discuss about these depreciation methods and also some other methods of depreciation of time permits. And also the depletion method which is used for these you know financial you know sectors.

So first of all we will come to the sum of digits method of depreciation and this is also known as the sum of years digits method as SOYD so many a times you may see the name as a SOYD or it may be something like SOD. So, how this method works this is again also explained a method of depreciation whereas the depreciation amount is higher at the you know initial periods and then they decrease as the time progresses.

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Sum of Digits method of depreciation

- ❑ **Sum of Digits method**
 - Similar to technique of rule of 78
 - Rate of depreciation for each year is a fraction of denominator which is sum of digits representing years of an asset's life (n).
- ❑ **Units of production method**
 - Depreciation is assumed to occur on the basis of amount of work performed, without regard to duration of asset's life.

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So some of digits method it is similar to the technique of rule of 78 as we had seen we have seen that in the case of rule of 78 basically when we have to calculate for one years of being 12 months we calculate $12 * 13 / 2$ so that way we get 78. So, similarly here you know now in this case rate of depreciation in for each year it is a fraction of denominator which is sum of digits representing years of an assets life. So, what happens that when you try to calculate the depreciation if suppose there are n years the life of the asset.

So, in the first year the depreciation amount or percentage will be n by sum of digits. So, it will be something like so when we talk about the SOD or SOYD method of depreciation. (Refer Slide Time: 02:47)

SOD / SOYD

$$SOYD = \frac{n(n+1)}{2}$$

Depreciation in yr 1: $\frac{n}{SOYD} (C-S)$
 Depreciation in yr 2: $\frac{n-1}{SOYD} (C-S)$
 ...
 Depreciation in yr n: $\frac{1}{SOYD} (C-S)$

Example: $C=5000, S=2000, n=5$
 $SOYD = \frac{5 \times 6}{2} = 15$
 $D_1 = \frac{5}{15} \times (5000 - 2000) = 1000$
 $D_2 = \frac{4}{15} \times 3000 = 800$
 $D_3 = \frac{3}{15} \times 3000 = 600$
 $D_4 = \frac{2}{15} \times 3000 = 400$
 $D_5 = \frac{1}{15} \times 3000 = 200$

So, in that case you know that if you have n years of life then SOYD becomes $n * n + 1 / 2$ so if you sum these from 1 to n then it becomes as you know SOYD that is some of years digits that we will be adding $n * n + 1 / 2$ and in that case the depreciation which we calculate the percentage will be so R 1 will be you know n by sum of years digit so this will be you know you can calculate like this.

So you will write depreciation in year 1 it will be n by you know SOYD * $C - S$. So, if you have all P - F whatever you take this the first cost and S will be the salvage value. So, based on that you will have these values and by SOYD * $C - S$ similarly depreciation in year 2 so it will be $n - 1 / SOYD$ and then into $C - S$. So, this way it will go and depreciation in year n so that will be basically $1 / SOYD * C - S$.

So, see the first cost of the asset S is a salvage value of the asset and based on that we calculate the sum of using a; sum of years digit method we calculate the depreciation charges and we can have the examples very simple examples like suppose you have you know the first cost of the asset as 5000 and you know this S and C will be 5000 rupees and S will be 2000 rupees and n is 5 years.

So the depreciation in the first year will be so first of all you will calculate SOYD. So, SOYD will be $5 * 6 / 2$ so this will be 15 now in the first year it will be $5 / 15 * C - S$ so that will be $5000 - 2000$ so it will be 3000 so it will be $1000, 3000 * 5, 15000 / 15$ so it will be 1000 so similarly you can calculate D_2 will be $4 / 15 * 3000$ so it will be $200 * 4$ so 800. D_3 will be $3 / 15 * 3000$ so it will be again 600.

D 4 will be 2 by 15 * 3000 so it will be 400 and D 5 will be 1 by 15 * 3000 so it will be 200 so if you take all together if you sum them there were 18 + 6, 24 to 28 and this is 3000 total depreciation it is coming as C – S. So, this is how the depreciation amount is computed using the sum of years digit method of depreciation.

Then we have another method of depreciation that is known as the units of production method of depreciation. Now this units of production method of depreciation means many a times the depreciation of the asset will be depending upon how much you are using how much you are producing irrespective of the assets life. Because if suppose you have taken a machine and it is subjected to you know the use and suppose it can produce suppose a laser machine we have taken and its capacity is that it is life time it can go for turning of 1 lakh pieces of the job.

Now that one lakh pieces can be done in a year or even in can be done in 20 years does not matter so under those circumstances what we do is that we use this unit of production method.

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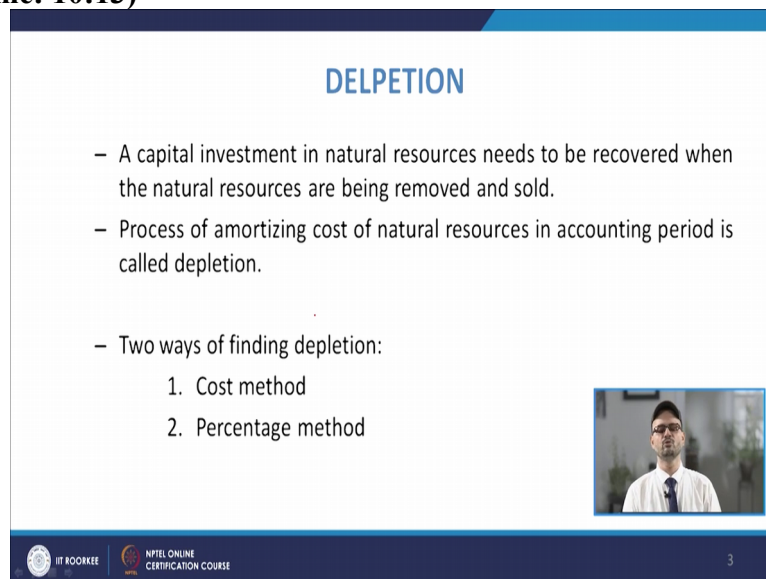
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Where it depreciation is assumed to occur on the basis of amount of work performed without regard to duration of assets life. So, assets life does not you know it is not that much material of importance. In fact like it will be depending upon; so, the depreciation charge will be calculated based on how much it has been used. Suppose we have a trench machine which is used to trench and its life is that it can go for maybe one lakh feet of the trenching work it can do.

So, if you know so it may have a certain first cost and now depending upon that depending upon number of feet's you have been trenching you can calculate what will be the depreciation charge. So, suppose something in a year 25000 you know feet's have been trenching has been done in that case 1/4 of the depreciation will be and there. In the second year it may be that there is only 10000 so it will be for 10% only like that so or if the whole year 90,000 of the feet has been trenched in that case 90% is depreciated.

So, that way these depreciation charges are calculated and this method is known as the unit of production method and that is very much used you know depending upon the case be it.

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The slide is titled "DEPLETION" in blue text. It contains three bullet points: a general statement about recovering capital investment, a definition of depletion as amortizing cost, and two methods of finding depletion: cost method and percentage method. A small video inset shows a man in a white shirt and tie. The footer includes logos for IIT KOOBEKKE and NPTEL ONLINE CERTIFICATION COURSE, along with the number 3.

DEPLETION

- A capital investment in natural resources needs to be recovered when the natural resources are being removed and sold.
- Process of amortizing cost of natural resources in accounting period is called depletion.
- Two ways of finding depletion:
 1. Cost method
 2. Percentage method

Now we will come to another topic that is depletion. Now depletion is something which is used when we talk about the depreciation of Natural Resources. So, when we talk about the investment in Natural Resources like in the mining sector or coal sector or oil sector or so now there a lot of investment is being done and they are required to be recovered when the natural resources are being removed and sold.

So, basically now when we talk about the depreciation of the normal physical assets then the depreciation amount which we are taking basically that can be used for purchasing new instrument further. So, that is how you are basically you know given the advantage that whatever depreciation amount is calculated you can further purchase the you know instrument and further do the business.

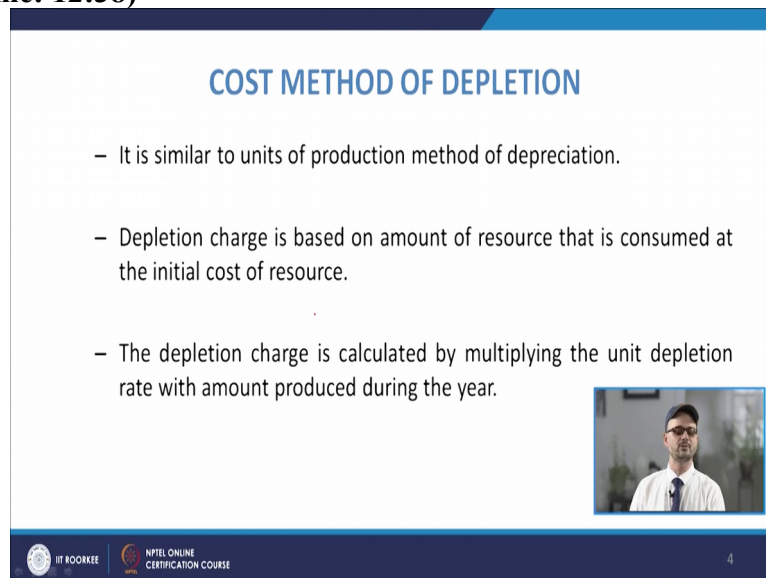
Now in the case of natural resources what happens that you are not going to further get that you know mines because once the mind is over then things are over so you have to just go. So, in those cases you have a process of amortizing this cost of natural resources in

accounting period is known as depletion. So, there we talk because it depleted so we call this term as depletion and there are methods by which these depletion allowances are calculated.

Now there are basically two methods of calculating the depletion charges and the first method is the new cost method and cost method is nothing but it is similar to the units of production method of depletion and here again we are assuming that depending upon the piecewise removal of these you know natural resources whatever amount has been; so suppose some mind is there it has or you have a you know timber you know garden and or you have one area where you have to you are getting the contract for cutting the timber.


And you know that there is fixed amount of timber so depending upon how much you are cutting in a particular period. So, depending upon that the depreciation charges will be you know calculated and that method is known as the cost method of depreciation.

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COST METHOD OF DEPLETION

- It is similar to units of production method of depreciation.
- Depletion charge is based on amount of resource that is consumed at the initial cost of resource.
- The depletion charge is calculated by multiplying the unit depletion rate with amount produced during the year.






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And as you know that it is similar to the units of production method of depreciation and here the depletion charge is based on the amount of resource that is consumed at the initial cost of resource. So, what we do is in these cases the depletion charge is basically calculated by multiplying the unit depletion rate with amount produced during that year. So, if you take the example of such type of the cost method of depletion.

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estimated 1,000,000 barrel of oil
 Initial Investment of \$ 7,000,000
 Unit depletion rate = $\frac{7,000,000}{1,000,000} = \7 per barrel.
 If during any year, 50,000 barrel of oil is produced:
 depletion charge = $7 \times 50,000 = \$350,000$



Suppose you have a producer you know you have a reservoir which produces basically you have estimated about it is there told that it is estimated that there is 10 lakh barrels of oil is the in the reservoir. So, now in that you are putting the initial investment of you know 70 lakh dollars 7 billion dollars. Now what we do is that as you try to take these barrels of oil from the; you know reservoir that there will be shortage of the oil you know continuously.

So, based on that there will be depletion charges you know attributed to that so now if you take that a place of rate so, you will take a unit depletion rate. So, unit depletion rate will be basically you will have to divide this so 7 billion divided by 1 billion so you have; so you get to the dollar 7 per barrel. So, there are 7 upper you know a barrel of oil is the depletion charge. So, now if in any year if the 50,000 is; if during any year 50,000 per barrel and you know if 50,000 barrels of oil are produced. So, in that case you know so is produced so the depletion charge if you have will take you can simply multiply this depletion charge.

So, this part barrel it is dollar 7 so it will be $7 * 50,000$ so it will be dollar 350,000 so this way you will have the depletion charges that will be you know calculated and it will be used and these are used for the tax benefits as you know so this is the method of the cost method.

So, that is known as cost method of depletion.

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PERCENTAGE METHOD OF DEPLETION

- It allows a fixed percentage of gross income produced by the sale of the resources to be the depletion charge.
- It is required that depletion charge for any period should not exceed 50% of all the taxable income for that period (figured without the reduction of depletion).



Then another method of depletion is the percentage method of depletion. Now what happens in the case of percentage method of depletion what we do is we allow a fixed percentage of the gross income produced by the sale of the resources to be the depletion charge. So, in such cases whatever you know income you have by taking. So, these are the one where you do not have the; you do not know how much is the natural resource.

The cost method is more applicable for those cases where you know approximately that that much is the you know number of that much is the volume of the or amount of the natural resources but here you do not have much idea. Now in those cases what we do is normally a fixed percentage of gross income is normally taken as the depletion charge. Now what happens that it may be very high so in those cases there is also a cap that depletion charge for any period it has not to exceed you know 50% of the all the taxable income of that period.

So, you have a total income and then also you have you calculate the taxable income taxable income is calculated by; so if you are investing in terms of you know instruments or equipments then those charges are being debited. So, you get the taxable income and basically the depletion charge for that for any period for each we are going to calculate that should not be more than 50% of the taxable income for that period.

So, this way we are calculating, so if that becomes more than 50% of the taxable income in those cases what we do is we take that 50% as the maximum value we cannot take more than that this is how this is known as the percentage method of; now this percentage fixed percentage is also there it has a particular value for the particular type of minerals like if you go for oil and gas wells so this is referred from the standard book.

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– Allowed percentages for selected mining properties are

Deposits	Percentage
Oil and gas wells	15
Sulfur, uranium, asbestos, lead, zinc, etc.	22
Gold, silver, copper, iron ore, and oil shale	15
Coal, lignite, and sodium chloride	10
Clay and shale to be used in making sewer pipe or bricks	7.5
Clay (used for roofing tile), gravel, sand and stone	5
Most other minerals; includes carbon dioxide produced from well and metallic ores	14



That from oil for oil and gas wells it is 15% sulphur uranium asbestos lead and zinc 22%, the gold silver copper it is 15%, coal lignite sodium chlorite 10%, clay and shale that is 7.5% clays and minerals washes are minerals like carbon dioxide produced from well and metallic ores. All these are 14% so these are the so you know percentages which are to be used. Now how to use that so suppose you are given that this much you are taking out that it has you are selling it at certain rate and then that percentage you know;

So you are that percentage will be basically calculated so and then it will be seen that what is the gross income and then what is the half of that taxable total taxable income so, based on that only we are calculating so we can have an example.

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If price of oil is \$20 per barrel and other expenses are estimated to be \$200,000.

Gross depletion income = $50000 \times 20 = 1000000$

Depletion rate -- 15%

Percentage depletion change: $\boxed{\$150000}$

Total taxable income: Gross income - Other expenses

$= 1000000 - 200000$

$= 800000$

Sup. = $\boxed{400000}$ (MAY VALUE)



Suppose if you look at so what happens that suppose the example of oil which we have seen now in that if the price of oil if price of oil is you know dollar 20, so, per barrel and also you know expenses and other expenses are estimated as 2 lakh dollars other expenses are

estimated to be dollar 2 lakh. Now what happens that you have to calculate the depletion charges. Now the depletion charges what you do is that you get the income.

So, you are getting 50000 barrels so your gross you know depletion income so you know gross depletion income will be 50000 barrels and that was multiplied by 20 so it will be 1 2 3 4 5 so 1 million you know and then now on this you have the percentage so we know that for oil if you look at the oil, for oil it is 15%, so this is 15%. Now so the depletion rate is basically 15% so the depletion charge; so the percentage depletion charges so it is will be 1 lakh 50000 rupees dollars.

Now we have to see that what is the; you know petrol taxable income and it should not exceed the 50% of the total taxable income. So, your gross income is 10 lakh and the expenses which I which are the 2 lakh so your total taxable income if you look at total taxable come it will be gross income minus the other expenses, so it will be gross income minus other expenses okay.

So, that way what we will do is you have you know 10 lakh - 2 lakh so it will be 8 lakh and then 50% of this taxable income will be 50% will be 4 lakh. Now this is quite high as compared to this so basically you can go for this depletion charge because this depletion percentage depletion charge should not exceed 4 lakh maximum of depletion charge will be 4 lakh this is maximum value okay.

So, that way you can go for these depletion charge that will be the answer for such cases so that is how we calculate the depletion charge for using the percentage method. Apart from that you know apart from these depreciation methods which we have discussed there are other methods of depreciation also and one of the methods of depreciation is the amortization method of depreciation.

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Amortization method:



$$A = \frac{CV \cdot r}{1 - (1+r)^{-n}}$$

$$D_k = \frac{[C - S(1+r)^{-n}] \cdot r}{1 - (1+r)^{-n}}$$

$$\frac{r}{1 - (1+r)^{-n}} = \frac{r}{1 - \frac{1}{(1+r)^n}}$$

$$= \frac{r \cdot (1+r)^n}{(1+r)^n - 1}$$



Now what we do in this case is that we are considering the time value of money and we are basically getting so if your first cost is the C and salvage value is S so basically we are getting the present value present worth of the S also so you will have $C - S \cdot 1 + R$ raised to the power $-n$ if R is the interest amount and basically we are amortizing these depreciation charges so that each of these the annual charges what we negate they include not only to a certain extent you know the share of asset cost but also the interest on the book value.

So, that is how you know you know for every depreciating year you have to take that also into the account. So, in this method what we do is that your, depreciating life full life is being taken into consideration and then we are getting these you know discounted scrap value. So, discounted scrap value means the S value is there so its present value will be $S \cdot 1 + R$ raised to the power $-n$ so that will be there.

And then the current values; so for that value you know annual annuity series type of value will be calculated a value which will be taking into account the principle amount as well as the you know the discounted you know scrap value. So, all these value is to be taken into account and for that the A the annuity amount is to be calculated so that way it is calculated and that A basically which we get so that will be as we know that if you have the current value of sum you know so it is given as $CV \cdot R / 1 - 1 + R$ raised to the power $-n$.

So, this is you know the value for the annuity when the current value of a fund is given. Now this current value will be nothing but as you know in this case you will have these depreciation charges if you calculate now in that case you will have if the C is the initial cost and S is the; you know and the salvage value. So, in that case these depreciation charge which we calculate will be $C - S \cdot 1 + R$ raised to the power $-n$ so that value is coming.

And then this factor will be coming out and further you will have $1 + R$ raised to the power $-n$ so it is nothing but it is also the P/A factor basically if you like or A/P factor which we have discussed in the interest cases where you are getting this you know annuity value or the equal annual amount you know that will be there so that is you know being expressed in terms of a and a factor. So, that factor is so if you do that if you look at this factor you know so it will be $1 - (1 + R)^{-n}$ it is nothing but $1 - 1 / (1 + R)^n$.

So, basically it will be $R * 1 + R/n$ and divided by $1 + R/n - 1$ so this is something like you know A/P factor or P/A factor that way so you should reverse so that will be either P/A or A/P factor so that way we calculate these D/K and once we calculate this D/K then also you have to have the you know fraction of the interest on depreciation and then interest on the depreciation on the principle amount is calculated.

Based on that you will have the you know depreciation schedule being calculated and you will have a C and S and finally at the end of the schedule you will be getting the value of S . So, this is the you know method this is the method where these amortization is being carried out and you know so suppose you have been given certain you know first cost and then depreciation you know salvage value at the end of its life and the n is given suppose its life is given and in rate of interest is given.

Then this D/K basically is calculated using this formula and that will be basically the annual depreciation and then the interest on depreciation so that will be calculated basically as a percentage of so whatever the interest rate so whatever is the book value that percentage of that book value will be used as the you know interest on depreciation. So, the D/K will be consisting of two things interest on depreciation interest on principle so depreciation on principle so that way you will have you know two values.

And based on that you have cumulative depreciation being calculated so the precision schedule is being you know found out using the in this method. So, we can solve a problem and get to know about such methods which are used for the depreciation calculations, thank you very much.